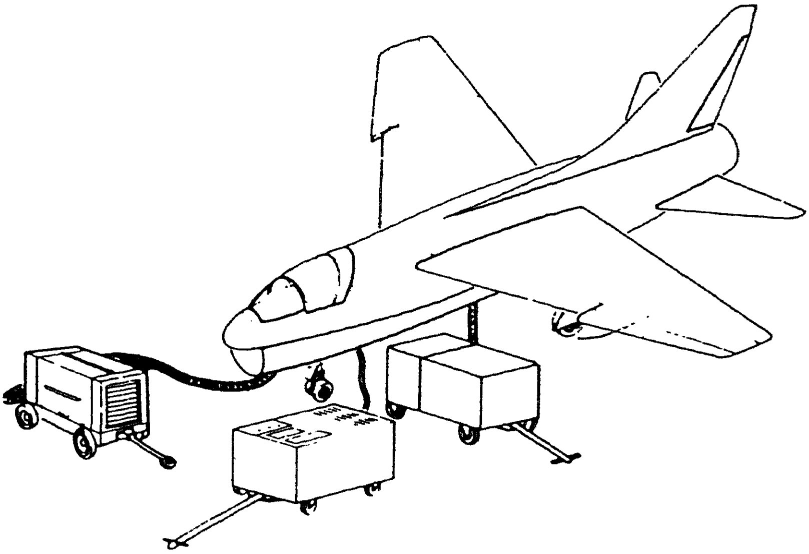
PERSONNEL QUALIFICATION STANDARD

HANDBOOK ON MANAGEMENT AND IMPLEMENTATION PROCEDURES FOR AVIATION PERSONNEL



CHIEF OF NAVAL EDUCATION AND TRAINING

APRIL 1980

0501—LP—221—0020

INTRODUCTION

The purpose of this Handbook is to describe the concept of the Personnel Qualification Standards Program and to provide guidance for its implementation and management as an element of the overall training program for operational units of the Navy.

The information in this handbook is for guidance only; it should not be construed as policy.

Definition. A Personnel Qualification Standard (PQS) is a written compilation of knowledge factors, skills and performance criteria necessary to operate and maintain specific equipment/systems or to perform designated tasks. Each PQS package is an evaluation tool designed to ascertain the depth of the trainee's knowledge/experience and job proficiency.

Background

a. The concept of standards for personnel qualifications is not new to the Navy. Various forms of qualification standards have been in use for years. However, there are many functional areas in which standards have not been developed or are neither consistent in content nor implemented efficiently. The ever-increasing demand for technical knowledge and the rate of personnel turnover mandates a high priority for PQS development.

Development of a PQS is inititated by a request from any command via its Type Commander to CNO in accordance with OPNAVINST 3500.34. Upon approval, priorities are assigned by CNO, and direct authorization for development is granted by CNET.

b. With PQS, a highly structured and formalized program is available for establishing personnel performance standards. These standards are based on job task analyses encompassing virtually all aspects of today's Navy. This program is readily adapted to traditional Navy jobs as well as to tasks previously obscure or unidentifiable. Standards may be developed for many levels of both technical and nontechnical tasks.

c. Current listing of PQS standards and qualification cards can be found in CNETNOTE 3500 and are available through the Naval Supply System.

d. OPNAVINST 1500.11 and OPNAVINST 3500.34 establish policies, responsibilities and procedures for PQS.

Discussion. As previously stated, PQS is a compilation of performance factors necessary to accomplish a designated task. It is not, in itself, a training program, but represents the terminal objectives of a training program. The prime responsibility for training rests with established training/support activities (i.e., formal schools) and traditional senior-to-junior instruction and supervision. Not only does PQS reemphasize the need for comprehensive training, it also provides development guidelines.

As part of a complete training program, PQS is designed for self-study promote initiative and responsibility by providing the trainee with spec fic objectives, job-relevant requirements and readily available resource and assistance.

OVERVIEW

A Personnel Qualification Standard...

...represents, by virtue of the manner in which it is prepared a composite of the collective experience of a broad cross-section of subject matter experts and reference documentation;

...displays in a uniform format the detailed performance standards required for each specified task;

...provides the questions (but none of the answers);

...organizes the task analysis into a repetitive format. By using the same learning terminology throughout, communication between supervisor and trainee is simplified. In addition, the trainee is motivated by the awareness that the studies include far more similarities than differences;

...requires the trainee to master only a small number of questions at a time. Once the format and scope of the question have been absorbed, the trainee can concentrate on acquiring the knowledge and skills necessary to perform assigned duties;

...lists all necessary references and publications the trainee needs to obtain the required information;

...speeds up the training process since each person knows exact what information to obtain from specified reference materials;

...provides a convenient record of accomplishments; and

...provides a means for supervisors to check on the trainee's progress toward qualification.

Page

Introduction i

Chapter I Description of the Personnel Qualification Standard (PQS) 1

Chapter II Department Qualification Organization 7

Chapter III PQS Implementation 9

Chapter IV Maintaining Qualification. 13

Chapter V Administration and Management of PQS 15

Chapter VI Recordkeeping. 23

Appendix A Provisional/Temporary Qualification Form A-1

Appendix B Qualification Petty Officer Notice Form. B-1

Appendix C PQS Evaluation Guideline C-1

Appendix D Work Center PQS Evaluation Guideline D-1

Appendix E VP PQS Model Manager Program E-1

Appendix F Proposed PQS Model Managers. F-1

CHAPTER I

DESCRIPTION OF THE PERSONNEL QUALIFICATION STANDARD (PQS)

Every PQS contains the following sections:

INTRODUCTION
GLOSSARY OF TERMS USED IN PQS
CONTENTS
100 SERIES (FUNDAMENTALS)
200 SERIES (SYSTEMS)
300 SERIES (MAINTENANCE ACTIONS/SUPPORT ACTIONS)
400 SERIES (QUALIFICATION CARDS) (packaged separately)
BIBLIOGRAPHY
PQS INFORMATION REPORT AND SUGGESTION SHEET

INTRODUCTION. The purpose of the Introduction is to explain the format and organization of a PQS in terms of its application and impact.

GLOSSARY OF TERMS USED IN PQS. The PQS glossary defines key terms and phrases as they are specifically intended to be applied to achieve program objectives. The writers of all PQS use these precise definitions to ensure uniform interpretation.

CONTENTS. The Contents page lists, by order of entry, the Fundamentals, Systems, and Maintenance Actions/Support Actions included in the PQS.

100 SERIES (FUNDAMENTALS). Each PQS begins with Fundamental sections covering the basic knowledge and principles needed to understand the equipment or duties to be studied. All references cited for study are selected according to their credibility and availability.

Because safety is a paramount consideration, the final Fundamental section within each booklet describes the precautions which apply throughout the Standard. This permits a subsequent listing in the System sections of specific safety precautions for a given system. Similarly, a portion of any given Fundamental section may be devoted to expanding the learner's vocabulary by calling for definitions of technical terms used throughout the Standard.

Fundamental sections have direct application to the formal school situation or to self-study for the officer or enlisted person who has not attended the specialized school. Common Fundamentals developed from "A" school courses supplement or replace previously identified sections.

200 SERIES (SYSTEMS). In this section, each PQS breaks down the subject equipment or duties into smaller, more comprehensible functional "Systems" as the basic building blocks in the learning process. Each System is written to reflect specific Maintenance Action/Support Action requirements by delineating the equipment/operations most relevant to one or more designated maintenance personnel. The less complex Systems may be identified and covered quickly or relegated to a lower priority to permit greater emphasis on more significant or complex Systems.

Any given System is assembled for learning into two levels. Systems have components and components have parts. A PQS System will list those items which must be understood for operation and/or maintenance. It will not list every item appearing in the technical manual parts list. At times, a number of very broad (overview) Systems are disassembled into their components or parts with the "big picture" as the learning goal. Items listed as components or parts in such a System may then be analyzed as separate Systems with more complete breakdowns of components and parts For example, the TF-30 engine may be listed as a component of the F-14 Aircraft Power Plant System, then broken into smaller Systems on later pa for closer study. As a result, broad principles of propulsion may be vie within the scope of an operator and the internal workings of the TF-30 en presented in depth for the maintenance technician.

The Systems section of a PQS is the most comprehensive and has been signed to follow the law of primacy, which requires the acquisition of kn ledge in correct sequence. If the trainee focuses on learning how the eq ment functions and interrelates during normal operation, the trainee will then be better able to identify potential problems and to take the proper corrective action.

Each PQS System section contains the following:

a. Title of System under discussion

b. References to be used for study

c. Directions for the trainee to:

1. Draw a diagram of the System using all components and component parts, or

2. Refer to a standard print, or

3. Refer to the actual equipment.

d. Questions about specific components and component parts to give the trainee knowledge required before completing the maintenance tas

e. The dynamics of how the components and component parts operate to make the System function.

f. The operating limits that must be monitored during System operation.

g. A study of the relationships between the subject System and other Systems, equipment or external influences.

h. A discussion of safety devices that protect the System, as well as general and unique safety precautions that apply to personnel.

(The System section of each PQS is written following a prescribed format; however, a certain amount of flexibility is required for PQS to fulfill fleet demands.)

300 SERIES (MAINTENANCE ACTIONS). The Maintenance Action section of each PQS tests the technician's readiness to perform a designated task. The goal of the Maintenance Action section is to guide the trainee in categorizing, analyzing and performing the step-by-step procedures required to obtain qualification.

There are three types of Maintenance Actions -- Operator, Maintenance Technician, and Support.

I. Each Operator Maintenance Action contains the following:

a. Routine operations that are performed frequently

b. Indications that the equipment is operating normally

c. Infrequent operations which may be performed as conditions permit or which may be simulated due to safety hazards, lack of funds, etc.

d. Abnormal conditions

e. Emergency conditions which may occur and the action the operator must be prepared to carry out

II. The Maintenance Technician Maintenance Action contains the following:

a. Scheduled maintenance which is performed frequently

b. Unscheduled maintenance which is performed frequently

c. Infrequent scheduled maintenance which may be performed as conditions permit or may be simulated due to their infrequent nature

d. Infrequent unscheduled maintenance which may be performed as conditions permit or may be simulated due to lack of funds, safety hazards or their infrequent nature

e. Emergencies that might occur during maintenance actions (fuel leak, fire, etc.)

III. The Maintenance Support Action contains the following:

a. Support action operations that are performed frequently

b. Infrequent or abnormal support action operations which might occur or be required

c. Emergencies which might occur during support actions

Operating Procedures

The PQS Standard deliberately makes no attempt to specify the cedures to be used to complete a task. The only proper sources of this formation are the technical manuals, MIMS, MRC's, Naval Air Training an Operating Procedures Standardization (NATOPS) or other policymaking doc ments prepared for the specific installation or equipment. Additionall the level of accuracy required of a trainee may vary from Command to Command based upon such factors as mission requirements, equipment configuration, etc. Thus, proficiency may be confirmed only through demonstrated performance at a level of competency that satisfies the Command Officer.

400 SERIES (QUALIFICATION CARD). The Qualification (Qual) Card is an accounting document centrally stored at the work center. As a current record of qualification, it facilitates the assignment of each newly reported person to the most needed area.

Each Qual Card contains the following:

a. Final Qualification Page

b. Qualification Summary Page

c. Fundamentals and Systems Summary Page

d. Performance Sign-off Pages

1. Final Qualification Page

The Final Qualification Page, which identifies the goal, is t form upon which the Commanding Officer certifies competency.

2. Qualification Summary Page

The Qualification Summary Page summarize the Maintenance Acti necessary to achieve Final Qualification. For example, there may be fo Maintenance Actions which require completion before the Commanding Offi certifies Final Qualification. The Summary Page then becomes the recor when and by whom the trainee was recommended to assume full responsibil for any one Maintenance Action. Upon completion of each Maintenance Ac the trainee is examined by the Supervisor. The certifying officer will verify satisfactory accomplishment by signing the Qualification Summary Maintenance Action qualification will result in a Service Record Entry 4) and verification of completion of the applicable practical factors f advancement in rate.

3. Fundamentals and Systems Summary

The third section serves as a record of completion of the var Fundamental and System requirements for overall qualification.

4. Performance Sign-Off Pages

In this final section, all of the requirements leading to qualifi-ation are specified, including prerequisite Maintenance Actions as well as equired Fundamental and System sections.

Instructions for performance and discussion in the Qual Card 400 Series) duplicates those in the Qualification Standard (300 Series). or example:

umber in Qualification Standard	Number in Qualification Card
301	401
302	402
303	403

Separate signature blocks are provided for every discussion and per-ormance item, each of which is assigned a point value of one (1). The oint value tally provides both the trainee and the supervisor with an indication of the trainee's progress. The total points necessary for inal qualification is obtained by summing the individual pages plus the oints for required Fundamental and System sections and is indicated at he end of each Maintenance Action.

Discussion Items

Though actual execution of the task is always preferable to obser-ation or discussion, some items listed in each Maintenance Action may be too hazardous or occur too infrequently to perform in a reasonable time eriod. For the sake of obtaining timely qualification, the PQS trainee ay be permitted to discuss such designated items with a qualified supervisor.

IBLIOGRAPHY

Following the final Maintenance Action is the Bibliography, which lists y title and number all reference materials relevant to the Qualification tandard. The Bibliography not only serves as a supplementary source of information to the trainee, but also as an aid to the command, which will ompile a reference library from the listed entries.

QS INFORMATION REPORT AND SUGGESTION SHEET

The PQS Information Report and Suggestion Sheet (Feedback Form), the last page in the manual, is preaddressed for mailing to the PQS Develop-ent Group. This direct-line response from each Standard user assures that the manual will continue to meet the Navy's current needs. All feedback should be routed via the PQS Model Manager for your specific aircraft type.

CHAPTER II

DEPARTMENT QUALIFICATION ORGANIZATION

Thus far in this book, Personnel Qualification Standards have been reated essentially as single equipment or skill area documents; i.e., PN-153 Technician or Plane Captain, respectively. In effect, the scope f each Standard is developed within specific departmental guidelines, hereby integrating the systems/equipment operated or maintained by the esignated Maintenance Technician/Aircrewman.

ommand-Wide Coding

The preceding chapter described each Qualification Standard as having he following subdivisions:

100 SERIES -- FUNDAMENTALS
200 SERIES -- SYSTEMS
300 SERIES -- MAINTENANCE ACTIONS
400 SERIES -- QUALIFICATION CARDS

To identify the department and the coordinated skill specialty, the ualification Standard for all equipment or skill areas within a Command s organized and indexed within PQS by adding another digit to the "Series" esignators (above). The four-character code is tabulated/interpreted as ollows:

FIRST CHARACTER (Qualification Section Number)	SECOND THRU FOURTH CHARACTER (Qualification Series)
2 = Plane Captain	100 Series = Fundamentals
3 = Power Plants	200 Series = Systems
4 = Hydraulic/Structural	300 Series = Maintenance Actions
5 = Electrical Instrumentation	400 Series = Qualification Cards
6 = Utility/Environmental/Safety	
7 = COMM/NAV/IFF	
8 = Special Equipment	
9 = Weapons/Weapon Release	
0 = Pilot	

Each Fundamental, System, and Maintenance Action is assigned a sequential umber. For example, 7105 of an Aviation package is identified as follows:

7	1	05
Denotes Qualification Section 7 (COMM/NAV/IFF)	Denotes 100 Series - Fundamental Section	Denotes the fifth Fundamental Section of the 100 Series

The Qualification Cards are designated as the 400 Series.

The Qualification Card will be identified by a letter Q and a n
which will denote the Qualification Card number. For example:

NAVEDTRA 43XXX-2Q1 denotes the first Qualification Card in
Qualification Section Two (2).

NAVEDTRA 43XXX-3Q2 denotes the second Qualification Card i
Qualification Section Three (3).

In this manner, the Aviation Personnel Qualification Standard c organized into separate sections for Plane Captain, AD, AMH/AMS, AE, AT, AX/AC/PR, and AO. Each of these sections (1000, 2000, 3000, etc are bound into separate booklets referred to as "Qualification Secti Thus, each command's Qualification Standard is physically divided in integrated booklets, and each trainee receives only the assigned por of the program.

This coded system has been devised as an index to the entire pr and is fully compatible with many of the Navy's computer systems. W each Personnel Qualification Standard, within each division, within department, within each squadron it is possible under this simplifie system to identify any qualification with a minimum number of charac i.e., 3301 Power Plants Maintenance Technician, 3302 Engine Turnup O 6301 Utility/Environmental Technician (AME), 6302 Survival/Safety Te (PR).

CHAPTER III

PQS IMPLEMENTATION

The objective of PQS is to provide a well-defined individual qualification process to indicate when a trainee is ready to assume responsibility for the evolutions of a job. The Qualification Standard program in no way intends to imply that the trainee who has completed a given set of Qualification Cards is as competent as the experienced officer or petty officer. Completion of a given set of Qualification Cards, however, will do much to ensure that the newly qualified person is not a personal or equipment hazard. Having gone through the process of qualification, each trainee is confident as to acquired knowledge and ability. Supervisors will have greater assurance that the trainee will respond effectively. Thus, after an operating unit has implemented the PQS program, no one should be assigned a task alone until qualified on that task.

PREPARING FOR IMPLEMENTATION

In preparation for implementing the PQS Program, Department Heads, Division Officers and the leading enlisted supervisors should meet to study their various qualifications and to determine the acceptable answers to all questions and performance items. Before presenting the program to the crew, the unit's operations and current personnel assignments to duties should be analyzed and the first maintenance qualification assignment for each member should be resolved. Since every member will be assuming a role in the training of future crewmembers, no individual should receive a Final Qualification without actual completion of his own Qualification Cards. It is imperative that specific time periods or time limits for Qualification be established by Department Heads and Division Officers for each Maintenance Action. The time period takes into account the ship's or squadron's deployment schedule and will tell each individual the time they have to become qualified. The time limit also provides a means whereby the Department Head, Division Officer and supervisory personnel can check individual progress. This is a goal to instill motovation in each individual and the spirit of competition with the work center. (See "Tailoring" in Chapter V)

Because each ship and station is unique, the Division Officer and Supervisor must determine the answers required for each qualification signature. The Qualification Standard cannot specify the answers for any ship or command. The proper source for the answers include rate training manuals, manufacturer's instruction and/or technical manuals, MIMS, MRC's and NATOPS.

THE GRACE PERIOD

The grace period is that period of time allowed for experienced personnel to become PQS qualified in a specific task during the unit's initial implementation. The length of this grace period, during which the department operates without any PQS qualified personnel, should be decided upon during the supervisory meeting held prior to introduction of the program to the crew. Experienced personnel onboard should possess most of the knowledge and skills required by the Personnel Qualification Standard; therefore, the transition to PQS training should not take an excessive period of time.

INTERIM QUALIFICATION

In addition to the "grace period," procedures should be establis cover the necessity for interim qualification of personnel under oth ditions. For example, the technician may be assigned an interim qua tion when he must be used to fill a more demanding billet but does n possess a final qualification, or in the case of a newly reported pe officer who must perform maintenance without prior qualification. I an instance, the steps shown below should be followed:

a. Administer oral/written examinations to determine lev qualification.

b. Department Head should recommend to the Commanding Of that an individual be assigned an interim qualification for a specif maintenance task. An accurate appraisal of those qualification requ which have been deferred should accompany such a recommendation.

c. Determine a deadline date by which an individual with qualification must complete previously deferred items.

QUALIFICATION OFFICER/PETTY OFFICER

The role of the officer/senior petty officer in the PQS approac training is extremely important. They must exercise quality control what is learned while also serving as a point of reference to assist trainee in acquiring the knowledge and skills that could not be achi alone. To ensure uniformity and program quality, only a limited num personnel should be selected and authorized to verify completion of 100 and 200 portions of the Qualification Standard. Those officers/ officers authorized to sign the 100 and 200 items should be so desig in writing and be made known to all members of the department. Pers that have completed specific 300/400 areas are technically qualified sign off those areas for nonqualified personnel. However, it is sug that the number of designated qualifiers be limited to ensure positi control within each Work Center. A solution would be designate CDI as PQS qualifiers for 300/400 areas; however this is not mandatory. command has the flexibility of handling the selection of PQS qualifi considered appropriate; qualifiers for the 300/400 areas need not be ignated in writing.

PRESENTING THE QUALIFICATION STANDARD PROGRAM TO THE COMMAND

Following designation of Qualification Officers/Petty Officers assignment of initial Maintenance Actions, Qualification Standards s be distributed and explained, emphasizing the concept, introduction, sary, numbering system and, particularly, the grid network. At this distribution of Qual Cards should be made and the procedures explain assignment to Maintenance Action 100, 200 and 300 Series prerequisit signature blocks, the point system and its cross-referencing to the Qualification Standard (300 and 400 series) should also be explained

HOURS FOR CHECKOUT

It is preferable and more adaptable to the daily routine of an operational unit to have the qualifier who is assigned 100 and 200 Series checkout authority announce to the division that a checkout session will be held at a given time and place. If possible, the qualified petty officer should schedule this session to coincide with a working shift for there is no better training aid than the equipment itself.

Oral examinations should be limited to no more than two trainees on a given qualification element at any one time. In this way, each member of the crew has the opportunity to display individual knowledge and to receive the necessary reinforcement from the supervisor.

If the examinations are challenging and fair, the learning process will be stimulating, the pace of qualification will accelerate, and the trainee will be motivated to perform well. During the first few weeks after implementation, supervisors normally discover a personal reward in that they are automatically renewing and reinforcing their own previous knowledge.

If supervisors follow recommended procedure, they will find that within a short time their roles in the department will change with the increased efficiency of trainees. More time will be spent discussing equipment maintenance with the trainees and less time in actual repair.

CHAPTER I.

MAINTAINING QUALIFICATION

All officers and enlisted personnel should go through some form of qualification or requalification process as they transfer to new commands. As they move from unit to unit, they retain their rate, rank and NEC, but they should not assume automatic qualification on any given task their first day aboard. At the option of the Commanding Officer, however, certain Final Qualifications acquired previously may be validated immediately.

Formal completion of a new set of Qualification Cards at each new duty station is, under normal circumstances, imperative. Requalification progresses very quickly since only those differences between past and present duties must be learned. Usually this involves determining the physical differences between installations. An APS "XYZ" radar at a new station has the same functions and interrelations, but the location of equipment, sources of power, and distribution of output information may vary. Until these dissimilarities are identified and mastered, the newly arrived officer or petty officer will not be a qualified operator or technician capable of assuming responsibility for training personnel reporting from other commands or be able to perform maintenance tasks alone.

Through normal personnel rotation procedures, all officer and petty officer qualifications are updated every few years. Additionally, during any given tour, senior personnel maintain high levels of expertise through the training and counseling of their juniors.

OPERATIONAL MALFUNCTIONS

The cause of some equipment malfunctions and personnel injuries is due to human error, resulting from lack of knowledge, inattention to duty, or willful negligence.

There is no foolproof method of identifying the malcontent who would deliberately impair an operation, nor can potential inattention to duty be easily detected. However, an error due to lack of knowledge is largely preventable since knowledge is measureable. The Qualification Standard program can provide a basis for this measurement and for taking appropriate action.

REQUALIFICATION FOR CAUSE

When an error has occurred, personnel involved should be interviewed at the highest reasonable level in the command. If it appears they did not at that time possess the required knowledge to perform the task, appropriate signatures on their Qualification Cards should be lined out and personnel involved should be required to requalify.

Petty officers who actually signed the Qual Cards should be interviewed to ensure that: (1) They possess the required knowledge; and (2) They took time to verify the trainee actually accomplished the signed item. The command should hold a similar interview with the officers who recommended or qualified the personnel to perform the task.

The Commanding Officer who swiftly and judiciously enacts the requalification requirement for all or a selected portion of a Qual Card will improve the quality of training.

CHAPTER V

ADMINISTRATION AND MANAGEMENT OF PQS

PQS developed to date have proven beneficial to those commands that have fully supported PQS as an element of a well-managed unit training program. In those commands where the PQS program has not been successful, one or more of the following conditions have existed:

1. Ineffective management of the overall unit training program

2. Failure to use sufficient numbers of supervisory personnel to execute the PQS program or the unit's overall training program

3. Failure to establish a program to prepare supervisory personnel as PQS qualifiers

4. Failure to establish and maintain an adequate PQS reference library of technical, procedural, and rate training manuals

5. Failure to establish individual qualification goals and time limits

6. Ineffective or nonexistent monitoring of individual qualification progress

PQS is an element of an overall unit training program for both afloat and aviation communities. As an element of training, PQS is to be incorporated as the keystone program for unit personnel qualification. One of the essential elements in the effective management of the PQS program is the assignment of a specific qualification goal to each individual and the time frame in which the qualification is expected to be completed. In this regard, it is recommended that the Division Officer and Leading Petty Officer conduct individual interviews with newly reported personnel to evaluate their past experience, qualifications, and general background. After the PQS program has been explained, a PQS package is provided and specific completion dates are assigned as to which Fundamentals, Systems, and Maintenance Action Qualifications are to be accomplished. Each trainee must also be given information as to how much time they are expected to spend each week on this training in order to accomplish the established goals. This procedure will permit the individuals to know what they are qualifying for and when they are required to have their qualification completed. The procedures required to conduct a successful PQS program should be identified in a command PQS Instruction and, as with any successful training program, active command attention and an effective organization are required to achieve meaningful results; PQS as an administrative action will not succeed.

Development of a PQS is initiated by a request from any command via its Type Commander to CNO in accordance with OPNAVINST 3500.34. Upon approval, priorities are assigned by CNO, and direct authorization for development is granted by CNET.

To facilitate administration of the PQS program, the following
guidelines are recommended:

1. Outline the specific responsibilities of personnel involved (See COMMAND PQS DUTIES DEFINED which follows).

2. Initiate grace period for each PQS qualification (as required).

3. Establish qualification time frames.

4. Schedule training periods for PQS.

5. Ensure applicability of PQS qualifications, eliminating those portions which do not apply. Tailor the Standard for your type aircraft and mission requirements.

6. Establish guidelines for requalification.

RECOMMENDED COMMAND PQS DUTIES

COMMANDING OFFICER
EXECUTIVE OFFICER
TRAINING OFFICER
PLANNING BOARD FOR TRAINING
PQS COORDINATOR
DEPARTMENT HEAD
DIVISION OFFICER
LEADING PETTY OFFICER
QUALIFICATION PETTY OFFICER
WORK CENTER SUPERVISOR
TRAINEE

Recommended functions and responsibilities follow for the primary individuals involved with the management of a unit's effective PQS program

1. Commanding Officer

   a. Serve as the final qualification authority for the command.

   b. Establish and promulgate an appropriate means of recognizing the achievement of major PQS qualification. These means might include (but are not limited to) the following:

      (1) Meritorious Mast
      (2) Letter of Achievement

   c. Ensure that fitness reports and enlisted evaluation commentaries are consistent with PQS performance of members.

   d. Designate in writing those personnel qualified to certify major qualifications.

e. Determine and promulgate the method of final qualification. This method will vary with the "style" and requirements of the command, but might include any combination of the following:

  (1) Acceptance of recommendations
  (2) Oral/written examination
  (3) Examination board
  (4) Personal observation of performance

f. Set up a regular and personalized means of testing the viability of the unit's program. Establish a personal method of determining if the observed performance of each trainee is correctly documented in the PQS program.

2. Executive Officer

a. Monitor the PQS program through the Training Officer and/or the PQS Coordinator when assigned.

b. Maintain and update the unit's PQS Instruction and the unit's notice designating the "qualifiers."

c. Monitor officer PQS progress at least monthly through reports of progress by Department Heads at the Planning Board for Training Meeting.

d. Reinforce command emphasis on PQS by linking it to routine administrative practices, granting of special requests, recommendations for advancement, etc.

3. Training Officer (PQS Coordinator, when assigned)

a. Maintain the basic system references (OPNAV Instructions, CNET 3500 and other command directives).

b. Maintain system software. Ensure that a "tailored" copy of each PQS Standard reflecting the type aircraft and squadron mission requirements is on file. This can be coordinated through each individual work center as required. Master copies should be held by each work center and Qual cards will be maintained in each individual's training record.

c. Order and store "all-hands" PQS such as NAMP, GSE, and Corrosion Control.

d. Function as the focal point for ordering Department PQS approved for implementation by the Planning Board for Training.

e. Advise the Executive Officer and Planning Board for Traini
on all PQS matters.

f. Prepare all external PQS reports for the command.

4. Planning Board for Training

a. Coordinate and schedule PQS training that impacts on general unit's training and operations.

b. Function as the senior examining board if required.

c. Approve the implementation of newly developed PQS.

d. Assist the Commanding Officer in the formulation and adjustment of PQS goals for the command, which should be reviewed at least monthly.

5. "X" or "I" Division Officer

a. Review with the trainee the command's policies and procedures for PQS, using the unit's instruction and notice as a basis

6. Senior Watch Officer

a. Perform those duties specified for the "X" Division Officer in the case of newly reported officer personnel.

7. Department Head

a. Monitor, supervise, and set goals for the officer trainees assigned, in accordance with command policy.

b. Maintain Progress Chart for officer trainees assigned.

c. Brief the Executive Officer at least monthly on the PQS progress of assigned officer trainees.

d. Set and adjust PQS goals for the Department.

e. Approve Departmental PQS qualifications.

f. Serve as Chairman of the Departmental Examining Board (if board is required).

g. Recommend final qualification of a trainee to the Commandin
Officer.

8. Division Officer/LCPO/LPO/Work Center Supervisor

a. Supervise the Division PQS program.

Determine the "entry level" of newly reported personnel, based upon past duty station, enroute schooling, etc. This determination should be made based upon a review of past Qual Cards and must be made jointly with the trainee's ultimate Division Officer. (See "Tailoring" on following page)

c. Issue "all hands" PQS standards and qualification cards to newly reported personnel.

d. Conduct the PQS indoctrination for newly reported enlisted personnel, as specified in the unit's instruction

e. Assign maintenance tasks and PQS goals to individual trainees IAW departmental guidance.

f. Monitor weekly progress of division personnel towards PQS goals.

g. Brief the Department Head monthly on the status of division personnel and adjust goals accordingly.

h. Integrate PQS status with routine administration of special requests, early liberty approvals, etc.

i. Recommend Maintenance Action qualification to the Department Head.

j. Ensure Page 4 service record documentation of PQS.

9. Qualifiers

NOTE: Qualifiers will normally include the Leading CPO/Petty Officer of the Division and key qualified personnel.

a. Thoroughly research and reference those Fundamental and System items for which they are responsible. Upon receipt of new PQS standards, ensure that the Standard is "tailored" to reflect the unit's specific type aircraft and squadron requirements, eliminating those items that that do not apply.

b. Research those maintenance/aircrew items for which they are responsible.

c. Require all candidates to respond/perform in the prescribed standard manner.

d. Document Qual Cards.

e. Motivate trainees to ensure their Progress Charts are properly documented.

10. Trainees

a. Complete training as required.

b. Ensure Qual Card is properly documented.

c. Ensure Progress Chart is up-to-date.

RECORDS.

In the administration of the PQS program, there is flexibility in the format for required records. The following minimum items of information will be maintained regardless of what records are adopted within a command:

1. Progress Monitoring.

a. The organization by name, number and division/work center
b. Trainees by name and rank/rate
c. Date trainee completed command PQS indoctrination
d. PQS Standard by name or number
e. Date trainee commenced training on a particular qualification
f. Trainee's progress

2. Service Record Entries.

As an individual completes a PQS Maintenance Action and is subsequently qualified by appropriate authority (i.e. Commanding Officer, Department Head, Division Officer) the fact should be recorded on Page Four of the trainee's Service Record.

3. Required Periodic Reports.

Fleet and most Type Commanders do not presently require periodi reports of their individual units. Consult Fleet and Type Commanders' directives for specific guidance.

TAILORING.

Basically there are two types of tailoring: (1) Tailoring the qualification program to meet an individual's particular training needs and goals, and (2) Tailoring a PQS Standard to meet the command's particular system and equipment configurations.

1. Tailoring PQS for Needs and Goals.

a. Newly Reporting Inexperienced Personnel.

When initially introducing new personnel to the division or work center, research must be conducted to determine what training the individual has received. To validate this requirement, formal school have been directed to identify PQS items covered in their curricula. The schools are then asked to document those items to which the student has been exposed and to ensure that this documentation is forwarded to the trainee's duty station. With this information, tailoring of basic Fundamentals and Systems requirements for each individual can be accomplished. (It is recommended that a question and answer period be conducted to establish the trainee's strong and weak areas.) It is suggested that a trainee attending schools or other temporary duty assignments away from the parent command bring the Qual Card along for possible sign-off in the areas covered by the school.

The number of evolutions required to perform a task can be adjusted to fit an individual's training needs. Some trainees may need to perform more than the recommended number of times indicated in the Standard. On the other hand some advanced trainees may not have to repeat the performance items as frequently.

b. Requalification Tailoring.

Each officer and enlisted person should go through some form of qualification or requalification process when they transfer to new commands. As they move from unit to unit, they retain rate, rank and NEC. They are not necessarily qualified to perform specific tasks at the new command, since each command is unique in organization, equipment and mission within the framework of a standard type equipment code. There are some general qualifications which can be transferred from command to command at the discretion of each Commanding Officer. However, under normal circumstances, a previously qualified individual should requalify in installed systems and items unique to the unit. Personnel that fall into this category should have a tailored qualification program laid out for them shortly after arrival at their new command.

2. Tailoring PQS for Systems and Equipment.

It must be kept in mind that changes to a PQS Standard must be reflected in the squadron's standard answer bank. Tailoring a PQS Standard is required by two conditions:

a. When initially implementating a PQS program or receiving a new PQS Standard at the command, the package must be validated to ensure that it actually reflects the unit's equipment, system configurations and task (Maintenance Action) requirements. This may require the addition or deletion of portions of the Standard.

b. If the unit undergoes an overhaul, transition, or extensive changes, the PQS affected must be updated to reflect those changes.

When tailoring a package (adding one or more new areas), there is a possibility that the additions have been previously developed for another Standard. If changes are complex and extensive, it might be necessary to request that a new PQS be developed or existing PQS be updated or revised by the PQS Development Group. If this is the case, a request should be submitted via the chain of command so the Type Commander can decide what should be done and how it will impact on other units.

ORDERING PQS MATERIALS

Distribution of a Standard and its Qualification Cards will be determinec
by and limited to those units configured with the subject equipment/syste
Quantity of Standard and Qualification Cards will, in general, be determ
by the number of personnel aboard affected by the subject matter of the F
The Chief of Naval Education and Training (CNET) publishes CNET NOTICE 35
quarterly listing all PQS material in the Supply System. Individual stoc
numbers are included to assist each unit ordering from the Naval Publicat
and Forms Center, 5801 Tabor Avenue, Philadelphia, Pennsylvania 19120. T
complete PQS catalog is included in the Navy Standard Stock List of Publi
tions and Forms (NAVSUP 2002). Microfiche editions are issued quarterly
(February, May, August, and November). Each edition reflects all curren
COG 01 and 11 publications and forms. Additionally, Section I and Secti
III reflect such information as cancellations and superseded or replaced
items formerly contained in change bulletins and supplements.

Requisitions for COG 01 publications will be submitted to Naval Publicat
and Forms Center, Philadelphia on DD Form 1348. Demand Code "R" (Block
Col. 44 of DD 1348) must be reflected on all requisitions with the basic
publication.

The Progress Chart can be obtained at SERVMART (Rev 11/74-1500/1)
(0115-LP-015-0005) or ordered from Philadelphia.

RECORDKEEPING

Aside from the trainee's Qual Card, the PQS program requires only one training record, the PQS Progress Chart, for each division or work center. (Examples of Progress Charts can be found in the PQS Manager's Guide, NAVEDTRA 43100-1)

PROGRESS CHART

Listed across the top of the chart are all of the maintenance/aircrew qualifications to be completed by each individual in the division or work center. The remaining columns may be used to record and coordinate additional training data to meet the spcific needs of the individual units; i.e., competitive exercises, divisional GMT lectures, required shcools, etc.

Vertical columns at the left side of the chart list the names of each person in the division and/or those who will be included in the training program. The next column is designated to record the last time the chart was updated for each individual.

The Indoctrination column indicates the date on which the trainee received PQS pre-instruction. This record is included to ensure that each trainee is aware of how to use PQS.

Point totals for a qualification are recorded under each qualification titile. Applicable only as management tool to monitor individual progress, the point tallies do not measure the complexity of maintenance/aircrew duties. Totals may vary according to the requirements of each command.

The maintenance action and progress columns are designed for multiple use. The progress column contains the points earned each week according to the golas assigned by the Division Officer. The maintenance action columns are halved diagonally to record both the dates on which the subject qualification was assigned and the projected completion date. The right-hand section is used to indicate the total points accumulated. Using this method, the indivudal will know which Maintenance Action must be worked on and when it is to be completed. By comparing points earned with total point value, and elapsed work time with the planned completion date, the Division Officer and Leading Petty Officer can monitor each individual's progress at a glance. Completed qualification is made visibly notable by blacking out the Progress block.

POWER PLANT SHOP

	LAST UPDATE JULIAN DATE	INDOCTRINA	PLANE CAPTA 2402	PROGRES	ENGINE OPE 3401	PROGRESS	POWER PLANT M TECH 3402	PROGRESS	JET & CALIBR ANALYST 34	PROGRESS	TEST CELL OPERATOR 34	PROGRESS		PROGRES		PROGRESS		PROGRESS		PROGRE
POINT TOTALS			75		190		200		120		200									
ADJC HOPKINS	6345	6009	6058 6151		6120 6210						6341									
ADJ1 BURLISON	6340	6035	6058 6151		6048 6150				6064											
ADJ2 CROOKSHANK	6343	6036	6041 6159		6301 6347		6061 6230													
ADJ2 SYKES	6340	6091	6278 6330		6031 6213															
ADJ3 BELL	6341	6094	6129 6219		6001 6156		6061 6230													

PROVISIONAL/TEMPORARY QUALIFICATION

Date ______________

From: ____________________ Department Head
To: Commanding Officer
Via: PQS Coordinator

Subj: Provisional/Temporary Qualification for Watchstation

Ref: (a) Squadron's PQS Instruction
(b) Wing's PQS Instruction

1. In accordance with reference (a), it is recommended that
_________________________ be awarded a Provisional/Temporary PQS
(NAME/RATE)
Qualification as ____________________________.
(MAINTENANCE ACTION)

2. For the purpose of Provisional Qualification, the following practical factors/qualification requirements will be deferred in accordance with reference (b):

3. For the purpose of Temporary Qualification, the above items will be postponed in accordance with reference (b) (these items will be completed in no more than ________ months) or (not later than ____________ 19_____).

____
DEPARTMENT HEAD

From: PQS Coordinator
To: Commanding Officer

1. Forwarded, having reviewed subject person's Qualification Card for discrepancies.

____
PQS COORDINATOR

From: Commanding Officer
To: ____________________ Department Head

1. The Provisional/Temporary Qualification of _____________________ as
(NAME/RATE)
____________________________ is approved.
(MAINTENANCE ACTION)

____
COMMANDING OFFICER

Copy to:
PQS Training and Evaluation Group

DEPARTMENTAL PQS QUALIFICATION PETTY OFFICER NOTICE

From: ______________________ Department Head
To: ______________________ Department Personnel

Subj: Departmental PQS Qualification Petty Officers; designation of

1. This notice promulgates PQS Qualification Petty Officer designations for the ______________________ Branch.

2. The following ___________ Branch personnel are designated to sign Qualification Cards in Fundamental/System (100/200) sections for the ______________________________ PQS Standard:

____
(Department Head)

Copy to:
PQS Coordinator
Individual's Training Jacket (Optional)

PQS EVALUATION GUIDELINE

1. Administrative Requirements

a. Does the squadron organization manual define or is there a squadron instruction which defines the PQS program?

b. Is the squadron Training/Evaluation Board firmly established and actively involved in assisting and evaluating the squadron training program?

c. What report does the Executive Officer require of the PQS Coordinator and how is it disseminated to the Commanding Officer?

d. Does the PQS Coordinator maintain a current library of the following?

(1) NAVEDTRA 43100-2 (Aviation Handbook on Management and Implementation procedures for Personnel Qualification Standards)

(2) Current listing of PQS Standards (CNETNOTE 3500)

(3) OPNAVINST 3500.34

(4) OPNAVINST 1500.11

(5) CINCLANTFLTINST 3500.19

(6) CINCPACFLTINST 3500.16

(7) COMNAVAIRLANTINST C3500.52

(8) COMNAVAIRPACINST 3500.1

e. Is an explanation of PQS included in the squadron indoctrintion program?

f. Are service record entries being made?

g. Have individualized short-term goals/deadlines been established for personnel in each work center?

h. Have books and qualification cards been issued to each person in the required areas?

i. Are questions and answers for oral/written exams recorded to provide uniformity and continuity within the qualification program?

j. How many personnel onboard have final qualification certification by the Command?

WORK CENTER PQS EVALUATION GUIDELINE

From: PQS Training and Evaluation Group
To: Commanding Officer
ia: Executive Officer

Subj: Personnel Qualification Standards; inspection report of
_______________ Work Center

ART I - General PQS Maintenance

ES NO

___ ___ 1. Are all pertinent Progress Charts posted?

___ ___ 2. Do Progress Charts include all assigned PQS Standards?

___ ___ 3. Are proper entries made on Progress Charts?

___ ___ 4. Are personnel aware of their own record of progress on the chart?

___ ___ 5. Are Qualification Petty Officers assigned by name?

___ ___ 6. Are Service Record entries being made?

___ ___ 7. Are only PQS qualified personnel signing off qualification items?

___ ___ 8. Are required reference books and technical manuals onboard and available?

ART II - Personnel Awareness

andomly select a minimum of two individuals listed on the Work Center rogress Chart.

Names: 1. _______________
2. _______________

___ ___ 1. Can they locate the Progress Chart?

___ ___ 2. Do they know who their Qualification Petty Officers are?

___ ___ 3. Can they locate their Qualification Cards and state the procedure for getting items signed off?

___ ___ 4. Can they locate references and other materials needed to accomplish items on the Qualification Card?

___ ___ 5. Can they demonstrate a knowledge of items included o
Progress Chart?

REMARKS: (Indicate repeated discrepancies. Three or more "no" ans
indicates unsatisfactory PQS performance).

____

____

____

____

____
PQS Training and Evaluatio

Copy to:
________________ Department Head

THE VP PQS MODEL MANAGER PROGRAM

The VP PQS Model Manager Program team was officially established on 3 March 1978. Patrol Squadron THIRTY-ONE was designated as the VP PQS Model Manager and Patrol Squadron THIRTY was designated as the CPWL PQS Coordinator. (A listing of proposed PQS Model Managers is included as Appendix J.) Major objectives and responsibilities of the Model Manager Program follows.

OBJECTIVES:

Develop and maintain a viable personnel qualification system which:

a. Yields the best possible trained individuals in VP Maintenance and aircrew positions by developing/revising PQS materials that reflect changes in aircraft configuration and training concepts.

b. Ensures type, model and series PQS acceptance and standardization within the VP community.

c. Provides timely development and revision of PQS materials by establishing close liaison with COMPATWINGSPAC/COMPATWINGSLANT/VP Fleet Squadrons/Readiness Squadrons/VP Reserve Squadrons and the PQS Development Group, San Diego.

d. Maximizes the efficient use of available training resources (manpower, aircraft, weapon system trainers, facilities) by developing an integrated PQS Program interfaced with CPWP/CPWL Squadron Training Programs.

RESPONSIBILITIES:

a. Administer the VP PQS Model Manager Program.

b. Revise/draft PQS materials where necessary and maintain liaison with the PQS Development Group, San Diego, for the timely incorporation of changes.

c. Disseminate interim PQS changes to applicable users in a timely manner.

d. Maintain PQS master copies incorporating all pertinent changes.

e. Convene an Annual Joint VP PQS Review Conference.

f. Convene, as required, Fleet Squadron VP PQS Review Workshops for establishing inputs to the PQS Program.

g. Solicit and review PQS changes from all VP Units on a continuing basis.

Systems Development Team (ISDT), the P-3 NATOPS Model Manager, FAS
Tactical Training Teams and FRS Training/Maintenance/FRAMP departm
to align and incorporate requirements/revisions to the PQS Program

i. VP-30 acts as the CPWL PQS Coordinator and is designated
assist Atlantic Fleet VP squadrons in the implementation and condu
their PQS Program.

Meeting these objectives requires the full cooperation of all
squadrons. Timely submission of recommended changes and/or new id
innovations is the responsibility of each PQS holder and will ensu
program which is responsive to the needs of the patrol community.

A key to any training program is built on flexibility. The M
Manager Program provides long-term flexibility through Model Manag
and squadron-generated changes.

Short-term flexibility rests with squadron Commanding Officer
PQS program was written for optimum training conditions. Each Com
Officer must exercise the short-term flexibility of the PQS progra
trying to manage PQS in other than optimum conditions.

The overall intent of the PQS program is to provide each squa
with a viable training tool which is standardized throughout the p
community. Success in the use of this tool is directly related to
effort expended by each squadron in the administration of the prog

PROPOSED PQS MODEL MANAGERS

/C	NATOPS MODEL	A/C	NATOPS MODEL
-7	VA 174	T-34B	NAS PNCLA
-6	VA 42	T-34C	VT 3
A-6	VAQ 129	T-28	VT 27
-2B	RVAW 110	C-131 F/G	TACSUPWING DC
-2C	RVAW 120	UH-1N	
-4	VF 121/171	TH-57	HT 8
F-8G	VFP 63	KC-130	USMC
-3	VS 41	EC-130	VA 3
-2	HSL 30	LC-130	VXE 6
-3	HS 1	C-130A	VC 3
-46	USMC (HC-3)*	F-14	VF 124
-53D	HM 12	C-12	COMRESTACSUPWING
-1	VRC 30	T-2	VT-9
-39E/G	VRC 30	T-44	VT 31
-39D	VT 86	E-P3	VQ 1
-2		P-3A/B/C	VP 31
-9	COMRESTACSUPWING	A-3	VAQ 33
-118	COMRESTACSUPWING		

PQS ONLY

☆ U.S. GOVERNMENT PRINTING OFFICE 1985--544--442/10541 Region 4

Information Report and Suggestion Sheet
PQS DEVGRU AUTOVON 957-5367

From ____________________ DATE __________

Activity ____________________

Mailing Address ____________________

____________________ AUTOVON # __________

Qual Standard Affected __________ NAVEDTRA # __________

Section Affected ____________________

Page # __________

Date Commenced Qual __________ Date Completed Qual __________

Remarks/Recommendations (Use additional sheets if necessary)

Suggestions for improving this Qual Standard

DEPARTMENT OF THE NAVY

POSTAGE AND FEES PAID
DEPARTMENT OF THE NAVY
DOD 316

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE, $300

Commanding Officer
Naval Education and Training
Support Center, Pacific
San Diego, CA 92132
Attn: Code N-7

in the direction of the track.

8. Empirical Rules for Thickness of Arch Ring.—With t trial curvature of the intrados decided upon, the next step in t design of a concrete arch is to choose a trial thickness of t ring at the crown and at the springing. Since the crown thic ness depends not only on the thrust, but also upon the locatio of the line of pressure, it is obvious that it is impossible to devis a formula for its determination. Various empirical formula however, have been developed for this trial thickness at the crow and are an aid to the judgment.

Mr. F. F. Weld[1] gives the following formula:

$$h = \sqrt{l} + \frac{l}{10} + \frac{w}{200} + \frac{w'}{400}$$

where

h = crown thickness in inches.
l = clear span in feet.
w = live load in pounds per square foot, uniformly distributed.
w' = weight of dead load above the crown of the arch in pounds per square foot.

Mr. W. J. Douglas gives the following tabulated formulas[2] for different highway spans, the values of h being given in feet:

Under 20 ft. $h = 0.03\ (6 + l)^*$
20 to 50 ft. $h = 0.015\ (30 + l)^*$
50 to 150 ft. $h = 0.00010\ (11{,}000 + l^2)\dagger$
Over 150 ft. $h = 0.016\ (75 + l)\ddagger$

* For railroad arches, add 25 per cent.
† For railroad arches, add 20 per cent.
‡ For railroad arches, add 15 per cent.

Mr. D. B. Luten gives the following formula for solid-spandrel arches:

$$h = 4 + \frac{3l^2\,(r + 3F)}{4000r - l^2} + \left[\frac{(w)l^2}{30{,}000r} \text{ or } \frac{w_c\,(l + 5r)}{150r}\right]$$

where h = crown thickness in inches.
l = clear span in feet.
r = the rise from springing line to intrados in feet.

---

[1] Engineering Record, Nov. 4, 1905, page 529.
[2] Taken by permission from the American Civil Engineers' Pocket Book.

F = the fill over the crown of the extrados in feet.
w = uniform live loads in pounds per square foot.
w_c = concentrated loading consisting of maximum live loading on single track over half-span in tons.

The thickness of an arch should increase from the crown to the springing (except for a hinged arch which is not considered here). The radial thickness of the ring at any section is frequently made equal to the thickness at the crown multiplied by the secant of the angle which the radial section makes with the vertical. For segmental and three-centered curves the radial thickness at the springing (*a–a*, Fig. 11) should be taken from two to three times the crown thickness.

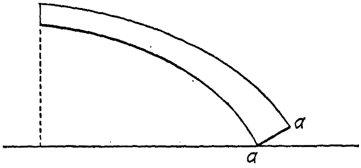


FIG. 11.

The empirical rules given above, it must be remembered, should be used only for trial. The exact shape of the arch ring and the thickness at different sections must be determined by analysis as subsequently explained.

9. Classification of Arch Rings.—Arches may be classified as hinged or hingeless. A hingeless arch is one having fixed ends, while a hinged arch may have a hinge at the crown, a hinge at each end, or a hinge at each end and one at the crown. Arches of one and two hinges are not used to any extent in masonry construction since the three-hinged arch offers the advantage of more definitely fixing the line of pressure throughout the ring and thus makes possible a saving of material. Hinges are, however, often an expensive detail and the three-hinged arch is by no means so common as the concrete arch having fixed ends. Friction on hinges is also an important consideration.

Concrete arches without hinges will be treated in the chapters immediately following and later on will be considered the modifications necessary for three-hinged arches.

CHAPTER II

DEFLECTION OF CURVED BEAMS

Deflection formulas for curved beams (in which the radius of curvature is large as compared with the depth) are employed in the development of arch theory. Hence, it is important for the student to appreciate the meaning of these deflection formulas before taking up the theory of arches, and this appreciation of the formulas can come only by following through their derivation. A proof for these formulas will now be attempted.

Let AB, Fig. 12, be any portion of a curved beam in its *unstrained* form and $A'B$ the same portion in its *strained* form, assuming the beam rigidly fixed at B. Let $X - X$ and $Y - Y$ be rectangular axes with origin at A, and denote the components of A' as Δx and Δy. AO, tangent to the arch axis at A, moves through the angle k. It is desired to derive formulas for the following values: (1) angular change of AO, (2) component Δx of A', (3) component Δy of A'. Only the effects of dead and live loading will be treated in this chapter leaving the effects of temperature change and direct stress for consideration in the chapter on Arch Analysis.

10. Angular Change.—Consider *abcd* as any small portion of the beam included between two consecutive cross-sections at right angles to the axis. Assume the radius of curvature of the beam as large in proportion to the depth so that the length of all fibers may be assumed equal. Now assume the end *ad* as fixed, and let the change in angle between end faces or end tangents of this element (of length s) due to bending be denoted by k'. (See Figs. 12 and 13.) The change in length of a fiber at a distance r from the neutral axis will be equal to rk' (the angle k' being expressed in circular measure) and the deformation per unit length of beam will be equal to $\frac{rk'}{s}$. Now, $E = \frac{\text{unit stress}}{\text{unit deformation}}$ (by definition of modulus of elasticity), or, stress per unit area at distance r from the neutral axis $= (E) \times (\text{deformation}) = E \cdot \frac{rk'}{s}$.

Let a (Fig. 14) be the area of a small element of the cross-section at a distance r from the neutral axis. Then the stress on this area equals $E \cdot \frac{rk'}{s} \cdot a$, and the moment of this stress about the neutral axis equals $E \cdot ar^2 \cdot \frac{k'}{s} \cdot$ The total moment of resistance of the cross-section is the sum of all the moments of resistance for the separate elements, which resolves itself into finding the sum of the expressions (ar^2).

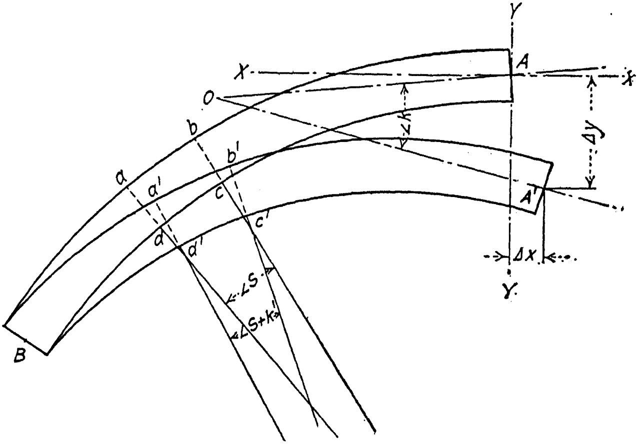


FIG. 12.

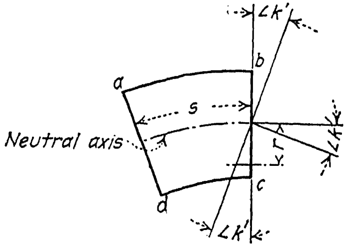


FIG. 13.

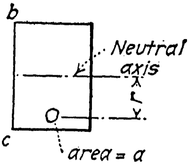


FIG. 14.

The term *moment of inertia, I*, may be defined as follows: The moment of inertia of a plane surface with respect to an axis is the sum of the products obtained by multiplying each elementary area by the square of its distance from that axis. If Σ is taken to mean *the sum of*, then it is evident that Σar^2 between the limits of the section represents the moment of inertia of the section.

Hence,

$$M = EI\frac{k'}{s}, \text{ or } k' = \frac{Ms}{EI}$$

(This expression might have been obtained more directly by employing the formula derived from the common theory of flexure.) But an expression for the angle k is desired, and this must be obtained by a summation of all the angles k' for the elements of the curved beam from A to B. Denote this summation by $\sum_A^B k'$, then

$$k = \sum_A^B \frac{Ms}{EI}$$

In deriving the foregoing equation, a material with constant E has been assumed. For beam of reinforced concrete

$$\begin{aligned} M &= E_cI_c\frac{k'}{s} + E_sI_s\frac{k'}{s} \\ &= (E_cI_c + nE_cI_s)\frac{k'}{s} \\ &= E_c(I_c + nI_s)\frac{k'}{s} \\ &= E_cI\frac{k'}{s} \end{aligned}$$

or

$$k' = \frac{Ms}{E_cI} \qquad (a)$$

and

$$k = \sum_A^B \frac{Ms}{E_cI} \qquad (1)$$

The smaller the elementary lengths of beam considered, the more accurately will Formula (1) apply. The values of M and I have been regarded as constant quantities for each particular elementary length considered. Since this is not true in practice on account of each element having appreciable length, a close approximation to the actual M and I for a given element may be obtained by taking the values of the bending moment and moment of inertia at the mid-point of s.

11. Components of Deflection.—Consider $BCDEFA$, Fig. 15A, to represent the axis of the unstrained form of the beam of Fig. 12 and let the beam pass into its strained form by the bending of each elementary length s in consecutive order. Bending the element BC through the angle k' causes the portion AC to turn through this same angle about C as a center, the point A moving to A_c (with radius u) through a small distance that we will call dv, having the components dx and dy. Fig. 15B shows clearly the method of bending. Bending the element BC through the angle k', if AC is kept stationary, would cause B to move to the point B'. Now revolving the bent beam about C as a pivot until B' is brought back to B, it should be noticed that A moves to A_c. Fig. 15C shows the bending of the element CD. Thus from the bending of CD' the point A_C moves to A_D, etc.

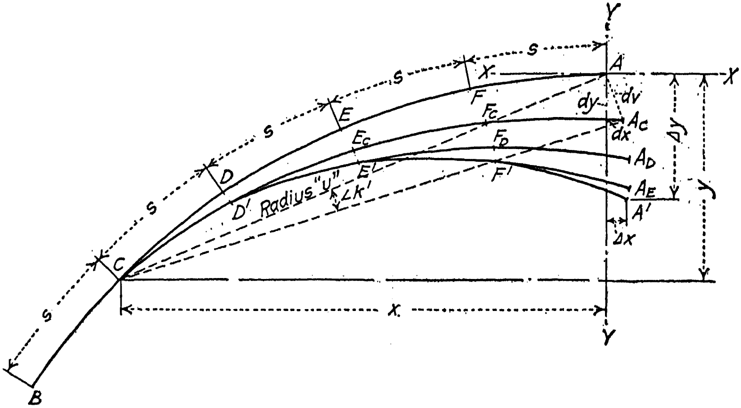


Fig. 15A.

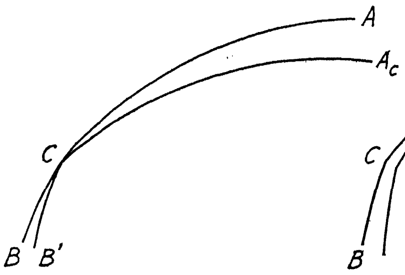


Fig. 15B.

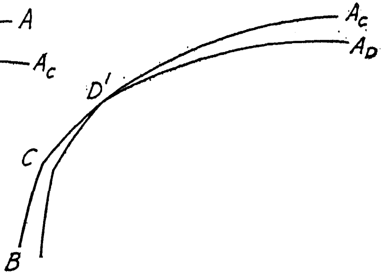


Fig. 15C.

If x and y are the coördinates of any point C, origin at A, we have, by similar triangles (triangles are similar since the side dv is practically at right angles to the radius u, the movement being very small)

$$\frac{dy}{dv} = \frac{x}{u}, \text{ and } \frac{dx}{dv} = \frac{y}{u}$$

Solving for dy and dx and substituting for dv the value uk', we have

$$dy = xk' \quad dx = yk'$$

Substituting the value of k' from Equation (a), we have

$$dy = x\frac{Ms}{E_cI}, \text{ and } dx = y\frac{Ms}{E_cI}$$

Summing up the value of the components for each change of k' in angle, the total change of angle made is k, and dy and dx become Δy and Δx, respectively. Then

$$\Delta y = -\sum_A^B \frac{Mxs}{E_cI} \tag{2}$$

and

$$\Delta x = \sum_A^B \frac{Mys}{E_cI} \tag{3}$$

(As in simple beams, M is considered positive when it tends to increase the compression on the back of the arch. The minus sign is used in Formula (2) because the effect of a positive value of M in any element causes an upward deflection—that is, a minus value of Δy, considering only the effect of bending in the element in question.)

Formulas (1), (2), and (3) are the fundamental formulas employed in the analysis of masonry arch rings by the elastic theory.

CHAPTER III

ANALYSIS OF THE SYMMETRICAL ARCH BY THE ELASTIC THEORY[1]

12. General Discussion.—A concrete arch with fixed ends is statically indeterminate. There are, in all, six unknown quantities—three at each support (the vertical and horizontal components of the reaction, and the bending moment; or, what is the same thing, the magnitude, direction, and point of application of the reaction)—and it is possible to determine only three unknowns by the principles of statics. The three additional equations may be found from the following conditions:

The change in span of the arch $= \Delta x = 0$

The vertical displacement at one end relative to the other end $= \Delta y = 0$

The angle between the tangents to the arch axis at the two ends of the arch remain unchanged, or $\angle k = 0$

These three conditions must be true since the arch is fixed at the abutments.

Instead of actually finding the components of the reactions and the moments at the supports as outlined above, it is simpler for symmetrical arches to take the origin of coördinates at the crown and find the thrust, shear, and moment at that point. The method of doing this will be explained later, but it should be clear that, with these three unknowns determined, each half of arch may then be treated as statically determinate.

The analysis of an arch consists in finding the thrust, shear, and bending moment at the crown and at intermediate sections in the arch ring or arch rib, and then finding the stresses resulting therefrom. A longitudinal slice of the arch is considered, having a thickness of 1 ft. The thrust is here taken to be the normal component of the resultant force on the section, and the shear is the component at right angles to the normal. The bending moment will be considered positive when it tends to increase the compression on the back of the arch, this being the same convention as for beams.

---

[1] Method of analysis is taken by permission from Turneaure and Maurer's "Principles of Reinforced-concrete Construction," 2d edition, pages 335 to 344. Copyright 1907, 1909 by F. E. Turneaure and E. R. Maurer.

A horizontal thrust is produced at the crown when the arch is loaded symmetrically. For non-symmetrical loading, an inclined pressure acts at the crown, but its horizontal component is called the horizontal thrust for that loading. Its vertical component is the shear at the crown. Let us assume the arch as cut at the crown and consider each half to act as a cantilever sustaining exactly the same forces as exist in the arch itself. The external forces holding a semi-arch in equilibrium (Fig. 16) are the loads P_1, P_2, etc.; the horizontal thrust H_c, the vertical shear V_c, and the reaction at the skewback R_a.

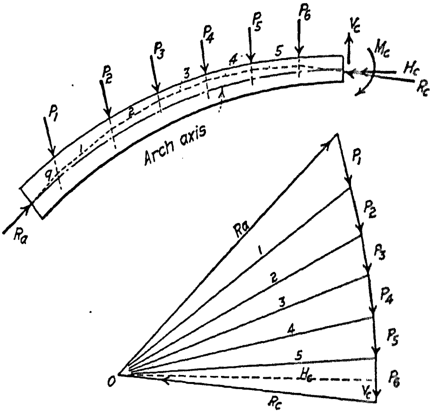


FIG. 16.

After H_c, V_c, and M_c have been computed, the line of pressure (accurately enough represented by the equilibrium polygon) can be constructed by help of the force polygon, Fig. 16. The value of M_c definitely determines the point of application of H_c and makes the construction of the exact line of pressure possible. (For a positive value of M_c, the thrust H_c acts above the arch axis.) From this line of pressure and the accompanying force polygon may be obtained the thrusts, shears, and bending moments at intermediate points of the arch. The force polygon gives directly the thrusts and shears, while the line of pressure makes possible the determination of the bending moment at any section, the bending moment being equal to the resultant pressure at the given point multiplied by the perpendicular distance from the arch axis to the line of pressure. Usually the line of pressure is drawn to serve only as a check on the computations, and the bending moments at the various points are determined algebraically. This algebraic method for finding moment will be employed in the designing work to follow.

The line of pressure of an arch is a continuous curve, but differs very little from an equilibrium polygon for the given loads (Fig. 16). In fact this curve becomes tangent to the equilibrium polygon between the angle points. The greater the number of loads, the nearer the polygon approaches the line of pressure.

With H_c, V_c, and M_c determined, all external forces are known except the reaction at the skewback, and this is determined by the closing line of the force polygon. An equilibrium polygon may then be constructed as already mentioned, the first side being in the line of R_c produced, the second parallel to the ray 5, and so on until the last side through q gives the position of R_a.

13. Notation.—The following notation will be employed in arch analysis:

Let

s = length of a division of the arch ring measured along the arch axis.
n_h = number of divisions in one-half the arch.
l = span of arch axis.
c_a = average unit compression in concrete of arch ring due to thrust.
t_c = coefficient of linear temperature expansion.
t_D = number of degrees rise or fall in temperature.
E_c = modulus of elasticity of concrete.

At the crown, let

H_c = horizontal thrust.
V_c = vertical shear.
R_c = resultant of H_c and V_c.
M_c = bending moment.

At any point on the arch axis, with coördinates x and y referred to the crown as origin, let

N = thrust (normal) on radial section.
S = shear on radial section.
R = resultant force on radial section, resultant of N and S.

x_o = eccentricity of thrust on section, or distance of N from the arch axis.

t = depth of section.

I = moment of inertia of section including steel = $I_c + nI_s$.

A = area of section including steel = $a_c + na_s$.

p_o = steel ratio for total steel at section.

d' = embedment of steel from either upper or lower surface.

M = moment = Nx_o.

m_L = moment at any point on left half of arch axis of all external loads (P_1, P_2, etc.) between the point and the crown.

m_R = moment at any point on right half of arch axis of all external loads between the point and the crown.

m = moment at any point on either half of arch axis of all external loads (P_1, P_2, etc.) between the point and the crown.

14. Formulas for Thrust, Shear, and Moment.[1]—Let Fig. 17 represent a symmetrical arch loaded in any manner and cut at the crown, the halves being separated in order to show the forces acting at the section cut.

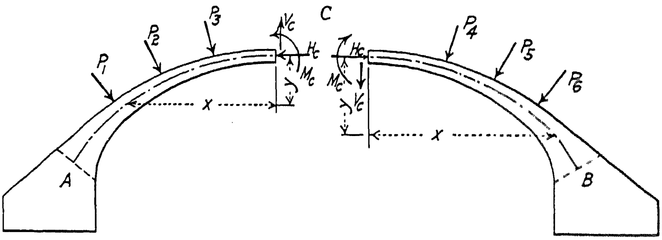


FIG. 17.

Formulas for thrust, shear, and moment at the crown will first be developed and then will follow a formula for moment at any section. Thrust and shear at any section may easily be determined graphically.

The horizontal motion of C as regards the left cantilever, due to bending of the elements between A and C, is the same in amount as the horizontal motion of C as regards the right cantilever, due to bending of the elements between B and C. It should be noticed that a positive bending moment for an element on the left will tend to move C in the opposite direction from a positive bending moment for an element on the right. Then, from Chapter II on "Deflection of Curved Beams," we have

---

[1] Taken by permission from Turneaure and Maurer's "Principles of Reinforced-concrete Construction," 2d edition, pages 338, 339, 343, and 344. Copyright 1907, 1909 by F. E. Turneaure and E. R. Maurer.

$$\sum_C^A My \frac{s}{E_cI} = -\sum_C^B My \frac{s}{E_cI}$$

(The coördinates x and y refer to the center of each short length of arch ring of length s, when M is also taken at the center.) The vertical motions are equal but in the same dirction, hence

$$\sum_C^A Mx \frac{s}{E_cI} = \sum_C^B Mx \frac{s}{E_cI}$$

Also the changes in direction of the tangent to the axis at C are equal but opposite in direction, hence

$$\sum_C^A M \frac{s}{E_cI} = -\sum_C^B M \frac{s}{E_cI}$$

The arch ring will be so divided into sections that $\frac{s}{I}$ will be made a constant. Then E_c also being a constant, the quantity $\frac{s}{E_cI}$ may be placed outside the summation sign. Thus, denoting $\sum_C^A M$ by ΣM_L and $\sum_C^B M$ by ΣM_R, we have

$$\Sigma M_L y = -\Sigma M_R y \qquad (b)$$

$$\Sigma M_L x = \Sigma M_R x \qquad (c)$$

$$\Sigma M_L = -\Sigma M_R \qquad (d)$$

The bending moment at any point may be expressed as follows:

$$M_L = M_c + H_c y + V_c x - m_L$$

$$M_R = M_c + H_c y - V_c x - m_R$$

where M_L and M_R are the moments at the center of any division to the left and right of the crown respectively. Substituting these equations in (*b*), (*c*), and (*d*), combining terms, and noting that ΣM_c for one-half the arch is equal to $n_h M_c$, we have

$$2M_c\Sigma y + 2H_c\Sigma y^2 - \Sigma m_L y - \Sigma m_R y = 0 \qquad (e)$$

$$2V_c\Sigma x^2 - \Sigma m_L x + \Sigma m_R x = 0 \qquad (f)$$

$$2n_h M_c + 2H_c\Sigma y - \Sigma m_L - \Sigma m_R = 0 \qquad (g)$$

Combining (e) and (g)

$$H_c = \frac{n_h \Sigma(m_L + m_R)y - \Sigma(m_L + m_R)\Sigma y}{2[n_h \Sigma y^2 - (\Sigma y)^2]} \tag{4}$$

From (f)

$$V_c = \frac{\Sigma(m_L - m_R)x}{2\Sigma x^2} \tag{5}$$

From (g)

$$M_c = \frac{\Sigma(m_L + m_R) - 2H_c\Sigma y}{2n_h} \tag{6}$$

These are the fundamental equations in the analysis of arch rings. All summations refer to one-half the arch axis, and the signs of m_L, m_R, x, and y being taken into account in the derivation of the formulas, their numerical values should be taken as positive when substituting to obtain the corresponding values of H_c, V_c, and M_c. A positive value of V_c indicates that the line of pressure at the crown slopes upward toward the left; a negative value, downward toward the left.

The values of the thrust, shear, and moment being found from the above equations, the moment of any section of the left cantilever is

$$M = M_c + H_c y + V_c x - m_L \tag{7}$$

and, at any section of the right cantilever,

$$M = M_c + H_c y - V_c x - m_R \tag{8}$$

To the thrusts, shears, and moments in an arch due to loads must be algebraically added the thrusts, shears, and moments due to a change of temperature. For a rise (or fall) of temperature of the arch ring of t_D degrees, the span of arch axis l would *tend* to increase (or decrease) in length an amount expressed by $t_c t_D l$, t_c being the coefficient of linear expansion. The arch being restrained at the abutments, thrust, shear, and moment would be caused at the different sections of the arch ring.

The method of procedure for determining formulas for H_c and M_c due to temperature is similar to that employed in determining the corresponding formulas due to loads. The value Δx of Formula (3) is equal to the amount of the tendency of the half-span to change in length, or $\frac{t_c t_D l}{2}$; and k of Formula (1) is zero since the crown section rises and falls in a vertical line as the material of the arch expands and contracts. Thus, we have, considering either half of the arch,

$$\Sigma My \frac{s}{E_cI} - \frac{t_c t_D l}{2} = 0$$

$$\Sigma M \frac{s}{E_cI} = 0$$

Since there are no external loads to be considered in the matter of temperature and since the arch is symmetrical, $V_c = 0$, and

$$M = M_c + H_c y$$

Substituting this value of M in the above equations and remembering that $\frac{s}{E_cI}$ is to be made a constant, we have

$$M_c \Sigma y + H_c \Sigma y^2 = \frac{t_c t_D l}{2} \cdot \frac{E_cI}{s}$$

$$n_h M_c + H_c \Sigma y = 0$$

From which

$$H_c = \frac{I}{s} \cdot \frac{t_c t_D l n_h E_c}{2[n_h \Sigma y^2 - (\Sigma y)^2]} \tag{9}$$

and

$$M_c = -\frac{H_c \Sigma y}{n_h} \tag{10}$$

The bending moment at any point is

$$M = M_c + H_c y \tag{11}$$

In Formula (9) the value t_D should be inserted as positive for a rise in temperature and negative for a drop in temperature. The value of t_c may be taken at 0.000006 per degree Fahrenheit (see Art. 13, Volume I) and the value of E_c at 2,000,000 lb. per square inch. Moments are usually expressed in foot-pounds and the distance y in feet, in which case the value of E_c should be substituted in Formula (9) in pounds per square foot and the value of l in feet. Since there are no outer loads to consider, the thrust and shear at any point in the arch may be found by resolving H_c normal and parallel to the arch section at that point.

It should be noted in Formula (9) that, for a given span, the horizontal thrust due to temperature varies inversely as $[n_h \Sigma y^2 - (\Sigma y)^2]$, which means that a slight decrease in rise of the arch will produce considerable increase in ring stress.

If the span and rise are kept constant, H_c and likewise M_c vary directly with I, or, in other words, with the cube of the depth of section. On the other hand, fiber stresses for a given moment are known to vary inversely only as the square of the depth of section. Thus in light highway bridges, where temperature moment constitutes a large proportion of the total moment, the resulting fiber stresses may actually be increased by an increase of section.

The thrust acting throughout the arch tends to cause a shortening of the span. Denote c_a as the average unit compression in concrete of arch ring due to thrust. Then the arch span will tend to shorten the amount $\frac{c_a l}{E_c}$. This action tends to produce the same result as a lowering of temperature and the value of the resulting crown thrust may be found by substituting $\frac{c_a l}{E_c}$ for $t_c t_D l$ of Formula (9). Hence,

$$H_c = -\frac{I}{s} \cdot \frac{c_a l n_h}{2\left[n_h \Sigma y^2 - (\Sigma y)^2\right]} \tag{12}$$

and, as for temperature stresses,

$$M_c = -\frac{H_c \Sigma y}{n_h} \tag{13}$$

$$M = M_c + H_c y \tag{14}$$

Thrusts and moments due to rib shortening are usually small except in flat arches, where they may become fairly large.

15. Division of Arch Ring for Constant $\frac{s}{I}$.—Since the depth of the arch ring generally increases from the crown to the springing, the moment of inertia I likewise increases, but much more rapidly; in fact, it increases approximately as the cube of the depth. To maintain a constant $\frac{s}{I}$, the divisions should be made to increase in length as the arch ring deepens, and some preliminary computations are necessary to do this.

The greater the number of divisions selected, the more accurate the results; but for an arch of ordinary span, the number need not be greater than twenty—sixteen or twenty being the common rule. Of course, since each half of arch is considered separately, the number of divisions should be even.

To obtain the value of $\frac{s}{I}$ (the constant), determine first the length of the arch axis. Next, calculate several values of $\frac{1}{I}$ at equal intervals along the arch and determine its mean value. Then

$$\frac{s}{I} = \frac{l'}{n_h} \cdot \frac{1}{I}$$

where l' is one-half the length of arch axis, n_h the desired number of divisions in one-half the arch, and $\frac{1}{I}$ the mean value found as previously explained. In determining this value the steel should be considered.

After once the value of $\frac{s}{I}$ is known, the lengths of the divisions can be readily determined. Assume a length of the first division to one side of the abutment. Determine the value of I at the center of the assumed division. Then divide the length assumed by the value of I determined for the center. If the length of the division was assumed correctly, the result should equal the value of $\frac{s}{I}$ computed above. If not, the necessary adjustment should be made, and so on for each division until the crown is reached. The lengths of the assumed number of divisions should agree with the length of one-half of the arch axis. If an error exists which is not large, it should be divided proportionately among the divisions.

If desired, a graphical means may be employed by which to divide the arch axis into divisions having a constant $\frac{s}{I}$. The method to be given will be used in the arch design which follows.

Fig. 18 illustrates the method in detail. AB is drawn to any convenient scale equal in length to one-half the arch axis. The curve EF is then drawn through points whose ordinates are the values I and whose abscissas are the corresponding distances along the arch axis from the skewback. (In order to make the determine the curve EF are not shown.) A length AH is then assumed, a perpendicular LC erected at its center, and the lines AC and CH determined. Starting from point H, lines are drawn parallel alternately to AC and CH, as shown in Fig. 18. Only three or four trials will usually be required to divide the line AB into the desired number of divisions. The base of each triangle thus formed corresponds to s and its altitude to I. Since all the triangles are similar by construction, the term $\frac{s}{I}$ is constant throughout.

A convenient modification of the above method is to draw a second curve $E'F'$ below AB, using the same ordinates as for EF. AH is then assumed as before and the perpendicular CC' erected at its center. Starting with C', diagonals and verticals are drawn alternately making the diagonals parallel to AC. This method offers the advantage of drawing all the diagonals parallel to the same line.

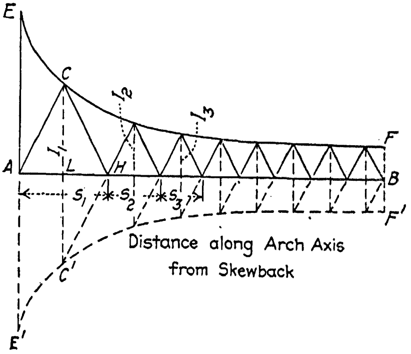


FIG. 18.

16. Loadings to Use in Computations.—Small earth-filled arches should be designed at least for uniform live load over the whole span and the half-span. Large earth-filled arches and arches having the roadway supported on spandrel arches or spandrel columns should be designed for live load over one-third of the span, one-half of the span, two-thirds of the span, the whole span, the middle third of the span, and the end thirds of the span. These uniform loadings are only approximations to the true loadings which produce the maximum stresses. The exact loadings to cause maximum conditions may be found by the use of influence lines, as will be explained in temperature range in concrete structures. Undoubtedly the most important are those which have recently been completed under the direction of the Engineering Experiment Station at Ames, Iowa, on two highway arch bridges of the earth-filled type. These experiments[1] are described in detail in Bulletin No. 30 of the Iowa State College of Agriculture and Mechanic Arts where a summary is also given of the other tests that have been made on internal temperature variation.

In the Iowa experiments, thermometers were embedded in different parts of the arch ring in such a manner that the thermometers were in direct contact with the concrete at the depth of penetration. Both mercurial soil and electrical resistance thermometers were employed.

The tests of the Squaw Creek arch at Ames extended over the greater part of the school years 1909 to 1912. The west arch of this bridge was the one tested, it being easy of access. The arch is unsymmetrical with a span of 45 ft. and a total rise of 11 ft. 3 in. at the center of span.

The tests on the Walnut Street bridge at Des Moines were made in 1911 and 1912—daily readings being taken covering a period of 1 year. This bridge is composed of six 68-ft. spans with a rise of 11 ft. 6 in. The east arch was selected upon which to make the experiments.

Internal Range in Temperature.—The results on the two bridges mentioned were obtained during years in which the extreme atmospheric temperatures, both high and low, were the greatest recorded for many years. With this in mind, the results show that about 75 per cent of the mean atmospheric variation would be effective temperature range in a mass of concrete of less than 5 or 6 ft. in thickness. The atmospheric mean variation at Des Moines was 102°F., 75 per cent of which is 76.5°F. This is only 2.7° less than the average obtained on the Walnut Street bridge during the period January to August, 1912, and but 2.6° less than the average obtained on the Squaw Creek bridge for the same year.

Heat Generated in Setting.—The results obtained in the Walnut Street arch tests show a rise to an average maximum temperature of 94.5°F. in an average time of 33 hours after pouring, while other results quoted in the bulletin show maximum high temperatures in setting varying from 100° to 130°F. attained in time intervals of from 18 hours to 8 days after placing. All considered, the value of 100° may be taken as a safe average of the final temperature attained by concrete in setting in structures typical of the Walnut Street arch.

---

[1] By Messrs. C. S. Nichols and C. B. McCullough.

The following is quoted from the bulletin above mentioned:

Temperature Variation.—"The main point of interest in connection with the temperature rise, due to the setting action, is not so much in the amount of this maximum itself, as in the fact that apparently all the subsequent temperature change is a drop, the stress-producing variation thus becoming more nearly equal to the entire temperature range than to one-half of it. This seems to be the conclusion drawn by Mr. Merriman (from his experiments on the Boonton dam), who says: 'If the maximum temperature attained is 100°F., then the stress-producing range is the difference between 100°F. and the lowest temperature subsequently attained.'

"While at first thought the above conclusion would seem to apply also to concrete arch structures typical of the ones herein described, yet the following facts demand our consideration. The sheer drop in temperature during the first season might induce stresses in the steel of the arch ring exceeding the elastic limit, thus producing a permanent set in the steel. This would operate to raise the elastic limit of the material in the direction of the stress (tension) to about the point of greatest stress, and produce in the arch ring a permanent deformation. The temperature of zero stress would then be, not the initial temperature, but the temperature which originally produced an elongation or distortion in the steel equal to the permanent set. During the subsequent seasonal variations, the variation of temperature would be each way from this new mean. Thus the temperature variation would be reduced to a value more nearly equal to one-half of the temperature range than to the whole range. On the other hand, the elongation of the steel might be so localized at the point of greatest bending as to cause an unsightly crack in the concrete.

"In the Walnut Street arch a temperature drop of the entire range, 87.6°F. plus the entire dead load and rib shortening cause a stress in the steel less than the elastic limit of the material. As a result, there are as yet no cracks in the structure, although it has passed through a season of maximum external temperature variation.

"In the Squaw Creek bridge at Ames, a temperature drop of 80° or even 60° will stress the steel at the spring-line above the elastic limit, with the result that at three of the four corners of the structure there are unsightly cracks at the exact point of computed maximum stress. If this cracking is due to a permanent set in the steel, the effective stress-producing variation is now probably each way from a mean corresponding to the amount of the set produced, and the stresses consequently much lower than during the first year.

"In the Locust Street bridge at Des Moines, the steel is also stressed above the elastic limit with a sheer temperature drop equal to the range, and in proof of the theory above advanced, there are cracks at the crown of both the shore spans at the points where a theoretical elongation in the steel corresponds to a temperature drop.

"In spandrel-filled arches it is doubtful if at the coldest period much of the dead load, aside from that of the concrete itself, comes upon the structure by reason of the arching action of the partially frozen fill. However the amount of the arching action is problematical. This might tend to reduce the mathematical stresses produced by the application of all forces.

Effect of Atmospheric Temperature.—"It will be noted that the range during the first seasonal variations, which include the high temperatures of setting, exceeded the subsequent seasonal range by about 10 per cent of the former. It is obvious that had the concrete been placed at any other than the extreme high temperature of summer, the heat of setting would have been dissipated to a greater extent, and the range would have been less. This is illustrated by a comparison of the setting temperatures attained in different latitudes. The experiments in the tropical Panama climate showed a rise exceeding those of this country by from 10° to 30°. Other conditions being equal, a temperature more nearly normal would seem to be better for the concreting of arch structures typical of the ones herein described.

Lag.—"The data on the *lag* seem to show that in structures of this type the minimum temperatures are attained in time intervals anywhere from less than 1 day to 4 days after the atmospheric minimum. This interval depends upon the position of the portion of the structure considered, and is roughly proportional to the distance from the nearest exposed face."

Agreement with Theory.—Levels were taken over the arch rings of both bridges from time to time to determine the rise and fall of the arch ring with variations in temperature. It was found that when uninfluenced by other factors than atmospheric variation, the rise and fall agreed quite closely with theory.

Conclusions as to Necessary Provision for Temperature Changes.—The writers of the bulletin concluded from the foregoing results and considerations that "to render an arch structurally safe, provision should be made (in the latitude where the bridge tests were conducted) for stresses induced by a temperature variation of *at least* 40°F. each way from an assumed temperature of no stress. Particular circumstances may demand that a greater variation be used for drop in temperature to prevent the appearance of cracks. This will always remain largely a matter of judgment with the designing engineer."

18. Shrinkage Stresses Due to Setting.[1]—In arches constructed in longitudinal ribs, shrinkage in the concrete due to setting tends to deform the arch ring in a manner similar to that resulting from a temperature drop. A lowering of the temperature, however, causes contraction of both steel and concrete, while with shrinkage the concrete alone is contracted. As the concrete contracts, the steel is compressed until an equilibrium is established between the compression in the steel and the tension in the concrete which entirely restrains further shrinkage of the mass.

The contraction which results from shrinkage causes a downward deflection of the arch ring, which induces bending stresses. One side of the arch ring is thrown into tension and the other side into compression. On the one side the tension in the steel is offset by the compression due directly to shrinkage. On the other side the compressive effects in the steel are additive, tending to throw quite heavy compressive stresses in the reinforcement.

The primary effect of shrinkage and the bending effect due to the same cause must be considered in the concrete as well. The primary effect is tension, but the bending stresses which result must be taken into account and added algebraically to the direct shrinkage stresses.

---

[1] The greater portion of this article is taken almost verbatim from Bulletin No. 30 of the Iowa Engineering Experiment Station, by Messrs. C. S.

In connection with the temperature range experiments on the Walnut Street bridge described in the preceding article, observations were made on a block of concrete 36 in. long, 10 in. wide, and 6 in. deep, cast at the same time as the arch ring on the east arch. The object of these measurements was to determine the change in dimensions of the concrete during the period of setting. The block was of a 1:2:4 mixture, cast wet, and unreinforced. The measurements, made at a constantly-maintained room temperature of 72° for 100 days, showed a shrinkage of 0.04 per cent. This is somewhat less than the results of Mr. A. T. Goldbeck (see Concrete Age, August, 1911), who arrived at a coefficient of shrinkage of about 0.0005.

It is possible theoretically to determine the stresses in steel and concrete due directly to shrinkage. If we let c denote the coefficient of contraction of the concrete, the contraction per unit length of the reinforced section may be expressed as follows:

$$c_r = c - \frac{f_c}{E_c} = \frac{f'_s}{E_s}$$

Now, for equilibrium,

$$f_c = pf'_s$$

From these equations we get

$$f_c = cE_c \frac{np}{1 + np}$$

$$f'_s = \frac{f_c}{p}$$

The contraction of the reinforced section may be found by the formula

$$c_r = \frac{f'_s}{E_s}, \text{ or } c_r = \frac{c}{1 + np}$$

which is, of course, somewhat less than that of an unreinforced section. It is this contraction which causes the downward deflection of the arch ring and causes bending stresses throughout. Formulas may be determined for thrust, shear, and moment due to this bending, in the same manner as described for temperature in Art. 13.

The following tables have been taken from Bulletin No. 30 of the Iowa Engineering Experiment Station. They give the shrinkage stresses in the Walnut Street arch based upon the above theory. A value of 0.0004 was taken for c which gave an average value of $c_r = 0.0003\frac{1}{3}$.

STRESSES AT CROWN

Condition of loading	Stress in steel		Stress in concrete	
	Maximum tension	Maximum compression	Maximum tension	Maximum compression
Dead load and shrinkage.......	7,400	6,400	None	513
Dead load only...............	−2,000	3,130	−136	213
Effect of shrinkage bending....	9,400	3,270	136	318
Primary effect of shrinkage.....	−10,000	10,000	130	−130
Net maximum stress induced by shrinkage..................	None	13,270	130	188

− Indicates stress of opposite kind.

STRESSES AT SPRINGING LINE

Condition of loading	Stress in steel		Stress in concrete	
	Maximum tension	Maximum compression	Maximum tension	Maximum compression
Dead load and shrinkage.......	4,200	2,950	None	218
Dead load only...............	None	1,500	None	110
Effect of shrinkage bending.....	4,200	1,450	None	108
Primary effect of shrinkage.....	−10,250	10,250	121	−121
Net maximum stresses induced by shrinkage................	None	11,700	121	None

−Indicates stress of opposite kind.

From the tables the most serious results seem to be the high compressive stresses induced in the steel reinforcement. The ordinary compressive stresses in the steel of arch rings, however, due to dead and live load, temperature, and rib shortening are usually low and, unless these stresses approach the allowable value, the effect of shrinkage need not be considered. The Walnut Street arch has a high percentage of steel and the tables seem to show that in arches lightly reinforced the compressive stresses in the concrete may need to be investigated.

The above discussion is instructive but should not be taken too seriously. For example, it is doubtful if such large initial stresses occur in concrete (restrained by reinforcing material only) as is represented by a value of $c_r = 0.0003\frac{1}{3}$. Considère in his experiments observed a contraction in 1:3 mortar reinforced with $5\frac{1}{2}$ per cent of steel to be only 0.01 per cent, or one-fifth the amount his tests showed on plain mortar. By the preceding formulas a contraction of about 0.03 per cent could be expected. It is reasonable, then, to conclude that a gradual adjustment takes place in the concrete during the process of hardening which results in less internal stress than theory would indicate. It is also unlikely that the shrinkage in any actual structure would be as great as that found on the laboratory specimen. Investigations of the actual shrinkage in concrete arch bridges will be needed before any very exact computations can be made for shrinkage stresses.

19. Deflection at Any Point.—The deflection at any point in an arch may be found by Formula (2), Art. 11, or

$$\Delta y = -\frac{s}{E_cI}\Sigma Mx$$

The arch should be assumed as cut at the point in question, and either portion of the arch may be considered. The cantilever selected should be subjected to exactly the same forces as exist in the arch itself.

If the deflection of the crown of a symmetrical arch is desired, the value of M due to loading for any section of the left cantilever may be found from Formula (7), Art. 14; or, substituting this value in the above equation, we have

$$\Delta y = -\frac{s}{E_cI}(M_c\Sigma x + H_c\Sigma xy + V_c\Sigma x^2 - \Sigma m_Lx)$$

For temperature changes, Formula (11) of Art. 14 may be substituted in place of Formula (7), or

$$\Delta y = -\frac{s}{E_cI}(M_c\Sigma x + H_c\Sigma xy)$$

$$= -\frac{t_ct_Dl(n_h\Sigma xy - \Sigma x\Sigma y)}{}$$

20. Reliability of the Elastic Theory.—Formulas based on the elastic theory and actual experiments on arches have been found to give results which agree within practical limits. Whatever error occurs in applying the formulas is due chiefly to the inaccurate assumption that the entire cross-section of the concrete of each section of the arch ring is effective and that the location of the neutral axis, due to bending only, does not change throughout the arch. The neutral axis without any doubt shifts about under the action of different loads (see Art. 39, Volume I) and, as the elastic theory assumes the bending to take place about the neutral axis of each element (see Fig. 13), the values of x and y should properly be measured to some kind of an undulating curve. This curve, however, cannot be determined conveniently, so that the assumption is made in arch analysis that the arch axis passes through the neutral axis of each section. This is undoubtedly far from the truth where tension occurs and the steel takes all the tensile stress. Such a condition of stress is limited, however, to small isolated portions of the span and the error in the total result is undoubtedly small.

A comparison of some theoretical and experimental results on arches is given in the Railroad Age Gazette, March 26, 1909, in an article by Malverd A. Howe entitled "Experimental Verification of Arch Formulas."

Attention should be called to the uncertainty as to the fixedness of the ends of the arch. This uncertainty can be reduced or entirely eliminated by taking the skewback for purposes of analysis at a plane where the ends of the arch are virtually fixed. Whenever the abutments are of such a form that there is no pronounced change of section at the springing lines, then the analysis should include the whole structure down to the point where the distortion due to the live load on the arch will be inappreciable. In some cases this may be the very bottom of the abutment.

21. Method of Procedure in Arch-ring Design.—The main steps that need to be taken in the design of an arch ring may be enumerated as follows:

1. Assume a thickness for the arch ring at the crown and at the springing, using empirical formulas, if desired, as an aid to the judgment.

2. Lay out the curve assumed for the intrados.

3. Lay out a curve for the extrados to give as nearly as possible the assumed ring thickness at the springing.

4. Draw the arch axis between the extrados and intrados.

5. Divide the arch axis into an even number of divisions such that the ratio $\frac{s}{I}$ is constant for all.

6. Compute the dead and live loads, and indicate these loads properly on the drawing.

7. Compute H_c, V_c, and M_c at the crown for the different conditions of loading.

8. Draw the force polygons for the different conditions of loading and the corresponding equilibrium polygons, or lines of pressure.

9. Determine the thrusts, shears, bending moments, and eccentric distances at the centers of the $\frac{s}{I}$ divisions of the arch ring for the different conditions of loading.

10. Compute the thrust and moment at the crown due to variation in temperature; also the moments on the various sections, and the corresponding thrusts and shears by resolving the crown thrust into tangential and radial components.

11. Where necessary, compute the thrust and moment at the crown, and the thrust, shear, and moment at various sections due to rib shortening.

12. Combine the thrusts, shears, and moments due to the different conditions of loading with the thrusts, shears, and moments due to temperature and rib shortening. (The results usually show that the shearing unit stresses are very small and need not be considered.)

13. Compute the maximum stresses—compression in the concrete and tension in the steel—due to the thrusts and moments. If the stresses are either too small or too large, the dimensions or even the shape of the arch ring must be changed and the computations repeated.

Note.—A book of multiplication tables such as Dr. A. L. Crelle's "Rechentafeln" is convenient when making computations in arch analysis. The student is advised to procure a book of this nature.

22. Skew Arches.—Skew arches may be treated exactly as right arches, the span being taken parallel to the center line of roadway and not at right angles to the springing lines of the arch.

CHAPTER IV

DESIGN OF AN EARTH-FILLED ARCH BRIDGE

23. Data.—It is proposed to design an electric-railway arch bridge in accordance with the following data:

Rise, 9 ft. 0 in.
Span, 48 ft. 0 in.
Depth of earth filling over crown, 12 in.
Depth of ballast above earth filling (at crown) to base of rail, 16 in.

Dead load:
Earth filling, 120 lb. per cubic foot.
Concrete, including reinforcement, 150 lb. per cubic foot.
Ballast, ties, and rails, 150 lb. per square foot of roadway.

Live load:
A uniform load equivalent to 50-ton electric cars in train. In view of the considerations presented in Art. 7, it will be sufficiently accurate and on the safe side to consider a uniform load of 200 lb. per square foot extending over a 12-ft. width of arch at the crown. Live load will be applied over only the whole span and half-span. No allowance will be made for impact.
Latitude of Ames, Iowa.

Conditions of calculations:
f_c = 600 lb. per square inch in arch ring for a 1 :2 :4 concrete—temperature variation of 40° F. each way from an assumed temperature of no stress.
f_s = 16,000 lb. per square inch—temperature variation of 40°F. each way from an assumed temperature of no stress.
E_s = 30,000,000 lb. per square inch.
E_c = 2,000,000 lb. per square inch.
n = 15.

Allowable pressure on foundations shall not exceed 5 tons per square foot.

Steel in the arch ring will be placed in two layers—one layer near the intrados and the other a corresponding distance from the extrados. The writer can give no assurance that the steel arrangement to be used in this design does not infringe one or more of the existing patents on concrete bridges. Assurance of this nature would be unwise since engineers well versed in patent law claim that an efficient, safe, and economical concrete arch bridge cannot be designed at the present time without patent infringement. In this connection the reader is urged to study carefully Chapter XII of this volume.

24. Selection of the Trial Arch.—A trial depth of 9 in. will be taken at the crown with a radial depth of 21 in. at the springing. The arch assumed is shown on Designing Sheet No. 1. As explained in Art. 4, the intrados for an earth-filled arch with a ratio of rise to span of less than one-fourth will lie between an ellipse and a segment of a circle, and the trial shape of arch ring must be chosen according to judgment.

25. Dead Loads and Their Action Lines.—Now that the trial arch ring has been assumed, the dead loads may be determined.

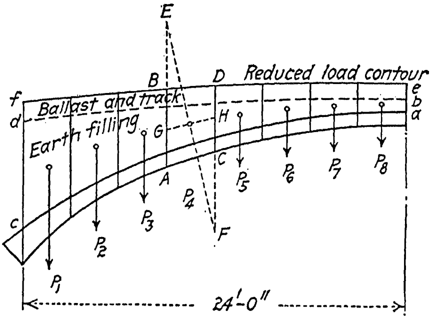


FIG. 19.

The earth filling and ballast, ties, and rails should first be reduced to an equivalent height of masonry, as shown in Fig. 19. For example, since the earth filling reaches 1 ft. above the extrados at the crown, the vertical distance *ab* should be laid off equal to $\frac{120}{150} = 0.8$ ft. In a similar manner the distance *be* should be laid off equal to $\frac{150}{150} = 1$ ft. Points *d* and *f* should be determined for the loading at *c*, and similar points should also be found for other places along the arch ring, a sufficient number being taken to fully determine the curved line *fe*. This line is called the *reduced-load contour*.

The arch with its load should now be divided by vertical lines into trapezoids, or what are nearly so. For testing the trial arch by the approximate method (to be presented in the following article), the horizontal distance between springing lines may be conveniently divided into divisions of equal horizontal length and, for the trial arch at hand, the number will be made sixteen, that is, eight divisions on each side of the crown.

The next step is to determine the area and center of gravity of each trapezoid. With the area known, the load corresponding to each trapezoid is found by multiplying by the weight of a cubic foot of concrete. (The student must carefully bear in mind that the arch considered is included between two longitudinal vertical planes 1 ft. apart.) The center of gravity for each of the trapezoids may be found as follows: Extend *AB* (Fig. 19) so that *BE* = *CD*, and in the opposite direction extend *CD* so that *CF* = *AB*. The intersection of *EF* and the median *GH* is the center of gravity sought.

26. Approximate Method of Testing Trial Arch.—Since the dead load of earth-filled arches usually controls the shape of the arch ring, the next step will be the testing of the trial arch for this loading, employing an approximate graphical method. In the method referred to, that form of arch in which the line of pressure and the arch axis most closely approach each other is considered to be the best that can be designed. A few words of explanation are needed for the student to understand the limits of this method.

There are two classes of theories of the stability of the masonry arch—the line-of-thrust theories and the elastic theories. The line-of-thrust theories do not consider the elastic properties of the material and are usually employed for arches made up of wedge-shaped stones, called voussoirs. For instance, stone arches are generally calculated by these theories and the stability of the arch ring is considered as depending upon the friction and the reaction between the several arch stones. Various assumptions are made in arriving at these different line-of-thrust theories, and the results derived from them are not so accurate as where the elastic properties of the material are employed. They may be made use of, however, in determining the proper shape of the arch ring before applying the elastic theory to monolithic arches.

or even before applying the elastic theory to voussoir arches, since any voussoir arch, whether of stone or brick, will act as an elastic arch as long as the line of resistance remains within the middle third.

A line-of-thrust theory by Winkler may be stated in a general way as follows: "The true line of pressure is the one lying nearest to the center line of the arch ring." The proof of this theorem depends on the assumption that the load is vertical, that the equilibrium polygon coincides with the line of pressure, and that the arch ring has a constant cross-section. This theorem for an earth-filled arch will evidently be more reliable the flatter the arch, since for such a type of arch the less are the conjugate horizontal forces produced on the arch ring by the earth filling. The matter of regarding the equilibrium polygon as coinciding with the line of pressure is assumed in all theories, but a constant cross-section of the arch ring is not usually the case in practice and generally increases, as it should, toward the springing as the thrust increases.

Now if an equilibrium polygon be passed through the centers of the arch ring at the crown and springing, this line will be very near the true line of pressure. In the trial arch mentioned above, if the equilibrium polygon and the arch axis do not closely approach each other there will surely be another shape of arch ring which will fulfill this condition, and hence have probably lower maximum stresses resulting from the live and dead loads. Thus, the method of obtaining the best form of arch is to find for what form the line of pressure for the dead load and the arch axis most nearly coincide. It should be clear that this method will aid us in getting pretty near, at any rate, to the proper form for the arch before applying the elastic theory.

In order to pass an equilibrium polygon through the centers of the arch ring at the crown and springing, the load line should first be laid off as shown on Designing Sheet No. 1 and then any convenient pole O' selected in the horizontal through the point k in the load line, this point representing the end of the load nearest the crown. The next step is to draw the rays to the force polygon and then construct the corresponding equilibrium polygon beginning at the center of the arch ring at the skewback (point A). If now the first and last strings are prolonged to an intersection at B, a vertical line is determined which contains the

The equilibrium polygon which we have constructed does not pass through the center of the arch ring at the crown and is not the one required. We do know, however, that the first string which passes through A must intersect the last string in the vertical through B and that the last string must be horizontal due to symmetrical loading. Drawing a horizontal line through the center of the arch ring at the crown determines the point C which is the intersection of the first and last strings of the equilibrium polygon required. A line drawn through m of the force diagram parallel to CA determines the true pole O on the horizontal through k. If the force diagram is now completed for this pole, the required equilibrium polygon may be easily drawn. The line of pressure is seen to follow the arch axis very closely and the trial arch will be accepted for analysis.

In both preliminary and final analysis the arch ring should be laid out to a scale of 1 in. = 3 ft., and care should be used in the drafting work so as to have the dimensions exact and the lines sharp.

27. Division of Arch Ring for Constant $\frac{s}{I}$. The method of dividing the arch ring into divisions having constant $\frac{s}{I}$ is given on Designing Sheet No. 2, and the divisions themselves are shown on Designing Sheet No. 4. The procedure should be clear from a study of Art. 15.

28. Moments and Thrusts.—A complete set of formulas, as shown on Designing Sheet No. 3, should be at hand before starting the arch analysis. The notes on this sheet are given for convenience and may aid in preventing errors due to the use of wrong algebraic signs. The formulas show, first of all, that the cantilever bending moments are required at the centers of the $\frac{s}{I}$ divisions caused by the given external loads (considering the arch as two cantilevers). These moments are found on Designing Sheets Nos. 5 and 5A by both algebraic and graphical methods. The algebraic method should be clear, but the graphical or equilibrium polygon method of finding these moments may need explanation.

ACB represents the arch axis and the load line is acb. O is any convenient pole on a horizontal line through the point c — a point on the load line at the junction of the crown loads P_8.

The equilibrium polygon corresponding is *mnop* and, since $Oc = H$ is horizontal, the segment *no* is horizontal, and the desired bending moment at any point such as 5 is equal to the intercept on the vertical through this point (included between the equilibrium polygon and the horizontal line *no* produced) multiplied by the pole distance H, or $m = Hk_5$.

If desired, the cantilever moments due to live loading may be found somewhat more accurately than indicated in Designing Sheet No. 5. This may be done either by drawing the parabolic curve for moment due to uniform loading or by computing the value of the live moment at the center of each $\frac{s}{I}$ division in the same manner as for any point in a simple cantilever beam.

Designing Sheet No. 4 shows that the dead load is divided into separate forces in a different manner than in the preliminary analysis of the arch. This second division of the dead-load forces is not common in practice since, theoretically, there is no relation between the divisions of the arch ring and the points of application of the loads, the only requirement being to have such a small subdivision of the arch ring and of the load that the errors of approximation may be neglected. It seems to the writer, however, that taking the separating load-planes through the centers of the $\frac{s}{I}$ divisions results in more accurate values of the bending moments, which actually bend the separate elements of the arch ring, than are obtained by the common method of load subdivision. Besides, the usual division of the arch requires the perpendicular distance from the arch axis to the line of pressure at a division point to be measured *at* or *near* the load, and this is a most unfortunate selection if the eccentric distances at the sections are to be checked graphically. The load arrangement here proposed eliminates any error in scaling these distances, since between load points the equilibrium polygon and line of pressure coincide.

The live and dead loads near the crown act practically in the same vertical line and hence no sensible error results from assuming them to do so. Near the springing, however, where the horizontal distance between division points is considerable, the live and dead loads may need to be considered separately. In the arch in question a separation of dead and live load is made between division points 1 and 2.

Moments and thrusts for the different conditions of loading and for temperature and rib shortening are given on Designing Sheet No. 6. The writer has tried to arrange the results on this sheet in such a form that the operations involved will be clear and will need but little explanation. The values of x and y were scaled, and the values of m for dead load and for live plus dead load were carried forward from Designing Sheet No. 5. The magnitudes of the thrusts, N, were scaled from the force polygons on Designing Sheet No. 4, and the computed eccentricities x_o used simply as a check on the resultant line of pressure determined from the computations. The values of M and N for rib shortening were obtained directly from the moments and thrusts due to a fall of temperature by multiplying by the decimal 0.337.

In finding the average unit compression in the concrete due to thrust, only the total thrusts caused by dead load plus live load on right half of span have been considered, and these only at five sections of the arch ring. For light arches, such as the one we are designing, this method is sufficiently accurate for all practical purposes. For heavy arches, however, the effect of temperature (rise and fall) and rib shortening may need to be taken into account. There are two cases to be considered in such arches: (A) That due to dead load, live load, *fall* of temperature, and rib shortening; and (B) that due to dead load, live load, *rise* of temperature, and rib shortening. The method of finding the average unit compression in this more accurate manner will be shown by making the computations for the arch in question, adopting the same position of the live load as previously mentioned.

Case A. Assume c_a = 155 lb. per square inch.

$$H_c = \frac{-(0.016)(155)(144)(49.2)(8)}{490.8} = -287 \text{ lb.}$$

$$c_a \text{ (at crown)} = \frac{22{,}400 - 890 - 287}{(1)(0.75) + 0.092} = 25{,}250 \text{ lb. per square foot.}$$

$$c_a \text{ (at point 3)} = \frac{\frac{1}{2}(23{,}600 + 23{,}900) - 850 - \frac{850}{890}(287)}{(1)(0.80) + 0.092}$$
$$= 25{,}400 \text{ lb. per square foot.}$$

$$c_a \text{ (at springing)} = \frac{\frac{1}{2}(29{,}200 + 31{,}600) - 620 - \frac{620}{890}(287)}{(1)(1.75) + 0.092}$$
$$= 16{,}050 \text{ lb. per square foot.}$$

$$c_a \text{ (for arch)} = \frac{25{,}250 + 25{,}400 + 16{,}050}{3} = 22{,}200 \text{ lb. per square foot.}$$

$(155)\,(144) = 22{,}300$ lb., assumed value of c_a is satisfactory.

$$M_c = -\frac{(-287)(14.65)}{8} = +526 \text{ ft.-lb.}$$

Case B. Assume $c_a = 160$ lb. per square inch.

$$H_c = (-287)\left(\frac{160}{155}\right) = 296 \text{ lb.}$$

$$c_a \text{ (at crown)} = \frac{22{,}400 + 890 - 296}{0.842} = 27{,}300 \text{ lb. per square foot.}$$

$$c_a \text{ (at point 3)} = \frac{\frac{1}{2}(23{,}600 + 23{,}900) + 850 - \frac{850}{890}(296)}{0.892}$$
$$= 24{,}300 \text{ lb. per square foot.}$$

$$c_a \text{ (at springing)} = \frac{\frac{1}{2}(29{,}200 + 31{,}600) + 620 - \frac{620}{890}(296)}{1.842}$$
$$= 16{,}700 \text{ lb. per square foot.}$$

$$c_a \text{ (for arch)} = \frac{27{,}300 + 24{,}300 + 16{,}700}{3} = 22{,}700 \text{ lb. per square foot.}$$

$(160)\,(144) = 23{,}000$ lb., assumed value of c_a is satisfactory.

$$M_c = -\frac{(-296)(14.65)}{8} = +542 \text{ ft.-lb.}$$

TABLE OF SHEARS

Pt.	Live load on right half of span		Live load on whole span	Temperature (fall of 40°, rise of 40°, shears of opposite sign)	Rib shortening	Max. shear	Cross-section (sq. in.)	Max. shear (lb. per sq. in.)
	Left half	Right half						
Cr.	− 820	− 820	0	0	0	− 820	108	8
Spg.	+3200	+1000	+1300	−640	−220	+3620	252	14
1	+1900	− 200	+1100	−535	−180	+2255	161	14
2	− 200	−1600	− 600	−360	−120	−2080	117	18
3	+ 100	− 900	− 300	−280	− 95	−1275	115	11
4	+ 100	− 300	− 100	−230	− 80	− 610	114	5
5	+ 50	+ 100	......	−175	− 60	+ 215	111	2
6	− 150	+ 400	......	−120	− 40	+ 480	109	4
7	− 400	+ 600	......	− 75	− 25	+ 650	108	6
8	700	+ 800		− 25	− 10	+ 815	108	8

It is not customary to determine the shears at the centers of the $\frac{s}{I}$ divisions. The shears for the arch here analyzed are given above in order to show the student that the values obtained are actually very small.

29. Combining of Moments and Thrusts.—The method used in combining moments and thrusts for maximum stresses will be explained by giving all the computations needed in finding the maximum stresses at point 1. Reference should be made to Designing Sheets Nos. 6 and 7. Moments and thrusts due to a *fall* of temperature will usually combine with the moments and thrusts on right half of span (live load on right half of span) to give one set of maximum conditions. Moments and thrusts due to a *rise* of temperature will usually combine with the moments and thrusts on left half of span (live load on right half of span) to give a second set of maximum conditions. A third set due to live load on the whole span will need consideration at the crown and at sections near the quarter points. It is convenient to combine rib-shortening moments and thrusts with those for temperature as a preliminary operation. If the values of M, N, and x_0 (or $\frac{x_0}{t}$) of any given set are lower than corresponding values in some other set for the same section, then the first-mentioned set of values need receive no further consideration. Thrusts are likely to control at the crown and moments near the springing.

The necessary computations in combining moments and thrusts at section 1 are as follows:

Fall of Temperature:

	M	N
Temp.	−3900	−710
R. short.	−1300	−240
	−5200	−950

M	N
−5200	−950
−5200	28,000
−10,400	27,050

M	N
−5200	−950
+900	31,650
−4300	30,700

Rise of Temperature:

	M	N
Temp.	+3900	+710
R. short.	−1300	−240

M	N
+2600	+ 470
+5500	26,500
+8100	26,970

M	N
+2600	+ 470
+ 900	31,650
+3500	32,120

It is quite evident that the other possible combinations could not give maximum conditions. In fact, an experienced designer of arches would recognize immediately that the first set only needs consideration. The set having a value of $M = +8100$ ft.-lb. should be eliminated at once since the values of M, N, and x_0 of this set are lower than the corresponding values of the first set.

To comply with some types of specifications it is also necessary to find the maximum moments and thrusts with those due to temperature variation excluded, the idea being not to allow the line of pressure to depart from the middle third for such conditions. The method of doing this should be clear from the above example in which the moments and thrusts due to temperature are taken into account.

30. Maximum Stresses.—Maximum stresses at each section should be found for the entire arch, using Diagrams 13, 14, and 15 of Volume I, which are based on an embedment of the rods from each surface equal to one-tenth the depth. (Diagrams for other depths of embedment will be given in Chapter V, but these will not be considered here.) In light arches the embedment near the crown will be proportionately much greater than assumed and the stresses as determined by the diagrams of Volume I will be lower than the actual. Those sections where maximum stress is likely to result should then be again considered and the stresses computed accurately by using the formulas on Designing Sheet No. 7 which are reproduced from Volume I for convenience. The determination of k must be by trial.

The method of finding the *exact* compressive stress at any section when tension exists over part of section (Case II) will be explained by giving the computations for finding the stress at section 1. Since $k = 0.620$ for one-tenth embedment of the rods, a good value to use for a trial would be 0.600. Substituting in the formula for k, this value, however, does not satisfy the equation. A value of $k = 0.596$ is then readily determined. Substituting in the formula for L gives a value for this term of 0.1066. Then

$$f_c = \frac{(10{,}400)(12)}{} = 540 \text{ lb.}$$

The steel stresses are evidently very small and need be computed in but few cases. The concrete stress at sections 3, 7, and 8 will evidently be lower than that in adjoining sections having approximately the same value of t. The arch assumed we will consider as satisfactory.

When the stresses in an arch are found to be altogether too small or too large, the dimensions or even the shape of the arch ring should be changed and the computations repeated. Small changes, however, may be made without refiguring the whole arch. For example, the thickness of the arch ring may be decreased throughout by changing the moment of inertia of all sections in the same ratio—that is, by keeping $\frac{s}{I}$ constant. This may be done since the dead- and live-load stresses will remain sensibly unchanged, and the temperature and rib-shortening stresses will be but slightly modified. The point in the arch where the unit stress is the greatest should be selected, and the depth t determined by trial which will give the required allowable stress. Of course, this depth should be determined by considering M, N, and x_0 the same as for the unchanged section. The depth at every other section, then, must be changed by such an amount that the percentage change in the moment of inertia will be a constant throughout the arch.

If desired, the modified arch ring may be again tested for maximum stresses. This may be accomplished by finding the new moments due to temperature and rib shortening and combining these with the moments due to loading which can be considered to remain unchanged. The values of the moments and thrusts for temperature change will be increased or decreased in the same ratio as the value of I. The values of the moments and thrusts for rib shortening will increase or decrease inversely as I, but obviously not in the same ratio, and will need to be determined in the same manner as in the previous computations.

31. Design of Abutments.—The shape of abutment must be such that the load on the foundation will not exceed the allowable and will be well distributed. The shape that will be assumed is shown on Designing Sheet No. 8, the top of the abutment being drawn somewhat above a tangent to the extrados at the skewback.

In general, the so-called lines of pressure through the abutment should be drawn for th

1. Dead load and live load on the half span opposite to the abutment.
2. Dead load and live load on the half span adjacent to the abutment, with live load *on* the abutment.
3. Dead load and live load over the whole span, with live load *on* the abutment.
4. In some cases, depending upon the method of procedure adopted in arch construction, an analysis should also be made for dead load only of the arch ring, without earth fill either above or back of the abutment.

The lines of pressure for all cases should be as near to the center of the base as possible in order to obtain a nearly uniform distribution of load over the entire foundation. The analysis and design of the abutment is given on Designing Sheet No. 8, with the exception that for clearness the combination of forces for condition (4) is not shown.

The forces that may act on the foundation consist of (1) the reaction of the arch; (2) the weight of the abutment; (3) the weight of the earth, ballast, and live load directly above the abutment; and (4) the lateral earth pressure. To find the maximum lateral earth pressure, the live-load surcharge should be determined, also the surcharge due to ballast, ties, and rails. The height of the live-load surcharge may be found by dividing the live load per square foot by the weight of a cubic foot of earth. The surcharge due to ballast, ties, and rails may be found in a similar manner. The horizontal pressure on AC' when the live load is acting is thus equal to the difference between the pressures on AG and $C'G$, and may be determined by the method explained in Art. 3 of Volume II. If we assume an equivalent fluid weight of 25 lb. per cubic foot, then this horizontal pressure

$$P = \tfrac{1}{2}(25)(\overline{19.23^2} - \overline{12.33^2}) = 2720 \text{ lb.}$$

and acts at a distance of

$$\frac{\overline{6.90^2} + (3)(6.90)(12.33)}{3(6.90 + 2 \times 12.33)} = 3.19 \text{ ft.}$$

above the base. Without the live load, $P = 2470$ lb. and acts at practically the same distance above the base as when the live load is included. In this design a lateral pressure of 2700 lb.

The weight of the prism of earth whose cross-section is $BGHC$ and thickness 1 ft. is

$$\frac{(12.33 + 6.90 - 1.00) + 12.33}{2} \times 9.50 \times 120 = 17{,}500 \text{ lb.}$$

The weight of the prism represented by $BKLC = 17{,}500 - (9.50)(1.67)(120) = 15{,}600$ lb. The center of gravity of these prisms may be found by the method outlined in Art. 25. The forces representing the weights of the prisms act practically in the same vertical line, and for simplicity we shall use a force of 17,500 lb. for both cases acting 4.45 ft. from the inner edge of abutment.

The weight of the masonry is readily found by dividing the outline of the abutment into a number of triangles and placing the weight represented by a given triangle at its center of gravity. (The center of gravity of a triangle is at the intersection of the medians.) The resultant of all the vertical forces acting (including the weight of the prism of earth) may then be determined by the principle of moments which is explained fully in Art. 2 of Volume II. The resultant is found to have a magnitude of 25,000 lb. and acts 4.94 ft. from the back edge of the abutment.

Designing Sheet No. 8 shows the method of combining forces to find the thrusts on the foundation for the three conditions of loading. The maximum and minimum pressures for any loading may be obtained by using the formulas of Art. 9, Volume II. The true maximum equals

$$p_1 = \frac{51{,}200}{7.50}\left(1 + \frac{6 \times 0.65}{7.50}\right) = 10{,}400 \text{ lb. per square foot}$$

The method of design described above is considerably on the safe side. The student should note that in using the reactions from the arch ring the assumption is made that the live load of 200 lb. per square foot extends over the entire 16 ft. width of bridge, when, in fact, a load of this intensity covers approximately only 12 ft. at the crown, spreading out at other points so that the unit live load on the back of the arch ring near the abutment is very much less than the maximum. In other words, a much more accurate set of reactions could be obtained with which to analyze the arch abutment by assuming a live load of 200 lb. per square foot at th abutment is approached. Some allowance for this condition of loading is sometimes made in the abutment design, but since the elastic theory fails if the foundation is not unyielding, the added factor of safety is not usually considered.

Many engineers believe (the writer included) that in many, if not the majority of cases, the dead and live load per linear foot of arch on a strip 1 ft. wide may be determined for purposes of analysis as equal to the total dead load plus total live load divided by the total width of arch. This loading is adopted on the assumption that a 1-ft. strip will not fail separately but will be assisted by the adjoining strips so as to produce average conditions.

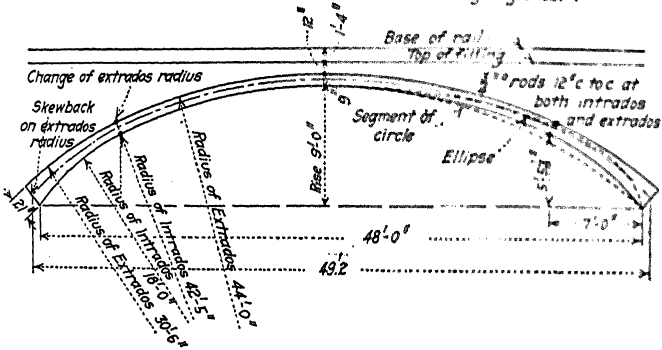
32. Design of Spandrel Walls.—The details of the spandrel walls are shown on Plate I which also gives the complete design of bridge. Since cantilever retaining walls would bring indeterminate stresses in the arch near the edges of the ring, which would not be at all desirable in such a thin arch, the curtain-wall type of construction with cross walls was adopted. The method of design is the same as for counterforted retaining walls which is explained in Art. 12 of Volume II.

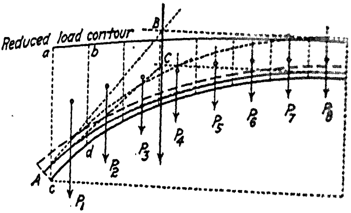
Expansion joints in the spandrel walls are made at the springing lines (in larger arches additional joints should be made throughout the span) and may be made mere planes of weakness or as actual joints filled with one or more layers of felt or some other partially elastic material. Such expansion joints are desirable to prevent unsightly cracks due to rise and fall of the arch caused by temperature changes. If the arch centers are not struck until after the spandrel walls are constructed, then expansion joints are also necessary to prevent cracking due to settlement of the crown of the arch when the centers are lowered.

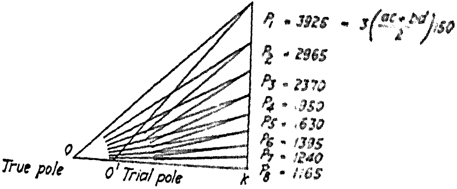
Since an arch bridge with a level coping will always appear to sag in the middle, a camber of 4 in. is provided. This slight camber, of course, was not considered in the previous loading calculations.

A batter is given to the back of the cantilever retaining walls resting on the abutments so as to give a low compressive stress per square inch on the abutment concrete, this concrete being only a 1:3:6 mixture. The back of the arch and the lower parts of the spandrel and cross walls should be waterproofed in order to prevent seepage of water through the arch ring and to facilitate drainage.

Designing Sheet 1.


12"
1'-4"
Base of rail
Top of filling
Change of extrados radius
Skewback on extrados radius
3/4" rods 12" c to c at both intrados and extrados
9
Segment of circle
Ellipse
Rise 9'-0"
Radius of Extrados 44'-0"
Radius of Intrados 42'-5"
Radius of Intrados 18'-0"
Radius of Extrados 30'-6"
5'-5"
7'-0"
21"
48'-0"
49.2'



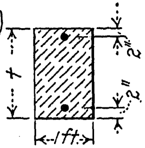


Designing Sheet 2.

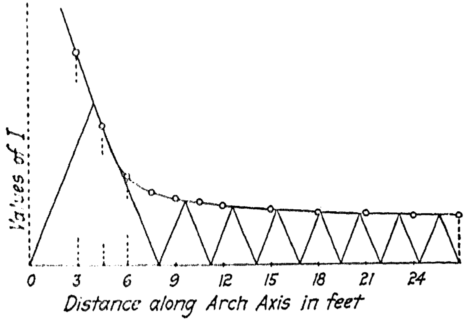
Division of Arch Ring for Constant $\frac{s}{I}$

Measured length of $\frac{1}{2}$ arch axis = 26.82
Rods - $\frac{3}{4}$″°, 12″ c to c.
Steel area = 0.44 × 2 = 0.88 sq. in

$p_o = \frac{0.88}{(12)(9)} = 0.00815$ (at crown)
$a_s = \frac{0.88}{144} = 0.00611$ sq. ft
$15a_s = 0.092$



Distance along the axis from skewbk	t (ft.)	$I_c = \frac{t^3}{12}$	$\frac{t}{2}$	$\frac{t}{2} - d'$	$15a_s(\frac{t}{2}-d')^2 = 15I_s$	$I = I_c + 15I_s$	Division	Lengths of divisions to make $\frac{s}{I}$ constant (by diagram)
3.00	1.23	.155	.615	.448	.019	.174	1	8.00
4.50	1.07	.102	.535	.368	.012	.114	2	3.23
6.00	.92	.065	.460	.293	.008	.073	3	2.91
7.50	.86	.053	.430	.263	.006	.059	4	2.74
9.00	.83	.048	.415	.248	.006	.054	5	2.60
10.50	.82	.046	.410	.243	.005	.051	6	2.52
12.00	.80	.043	.400	.233	.005	.048	7	2.44
15.00	.78	.040	.390	.223	.005	.045	8	2.38
18.00	.77	.038	.385	.218	.004	.042	$\frac{I}{s}$ = 0.016 (from diagram)	
21.00	.76	.037	.380	.213	.004	.041		
24.00	.75	.035	.375	.208	.004	.039		
26.82	.75	.035	.375	.208	.004	.039		



Designing Sheet 3

Formulas Required

Loading:

$$H_c = \frac{n_h \Sigma(m_L+m_R)y - \Sigma(m_L+m_R)\Sigma y}{2[n_h \Sigma y^2 - (\Sigma y)^2]} \qquad (4)$$

$$V_c = \frac{\Sigma(m_L - m_R)x}{2\Sigma x^2} \qquad (5)$$

$$M_c = \frac{\Sigma(m_L+m_R) - 2H_c\Sigma y}{2n_h} \qquad (6)$$

$$M = M_c + H_c y + V_c x - m_L \qquad (7)$$

$$M = M_c + H_c y - V_c x - m_R \qquad (8)$$

All values of m_L, m_R, x, and y should be substituted as positive. All summations refer to one-half of the arch axis. Positive value of V_c indicates that the line of pressure slopes upward towards the left; a negative value, downward towards the left. Positive value of M_c indicates that the thrust H_c acts above the arch axis. Signs preceding terms M_c and $V_c x$ in formulas (7) and (8) depend upon the results of (5) and (6)

Temperature:

$$H_c = \frac{I}{s}\cdot\frac{t_c\, t_D\, \ell\, n_h\, E_c}{2[n_h\Sigma y^2 - (\Sigma y)^2]} \qquad \left\{\begin{matrix}t_D \text{ should be} \\ \text{inserted as} + \\ \text{for a rise;} \\ - \text{for a drop.}\end{matrix}\right\} \qquad (9)$$

$$M_c = -\frac{H_c\Sigma y}{n_h} \qquad (10)$$

$$M = M_c + H_c y \qquad (11)$$

The value of t_D should be inserted as plus for a rise of temperature; minus (-) for a drop. Signs preceding H_c in formulas (10) and (11) depend upon the result of formula (9). Sign preceding M_c in formula (11) depends upon the result of formula (10). Thus for fall of temperature, thrust and moment are of opposite sign from those for a rise. ℓ = span of arch axis.

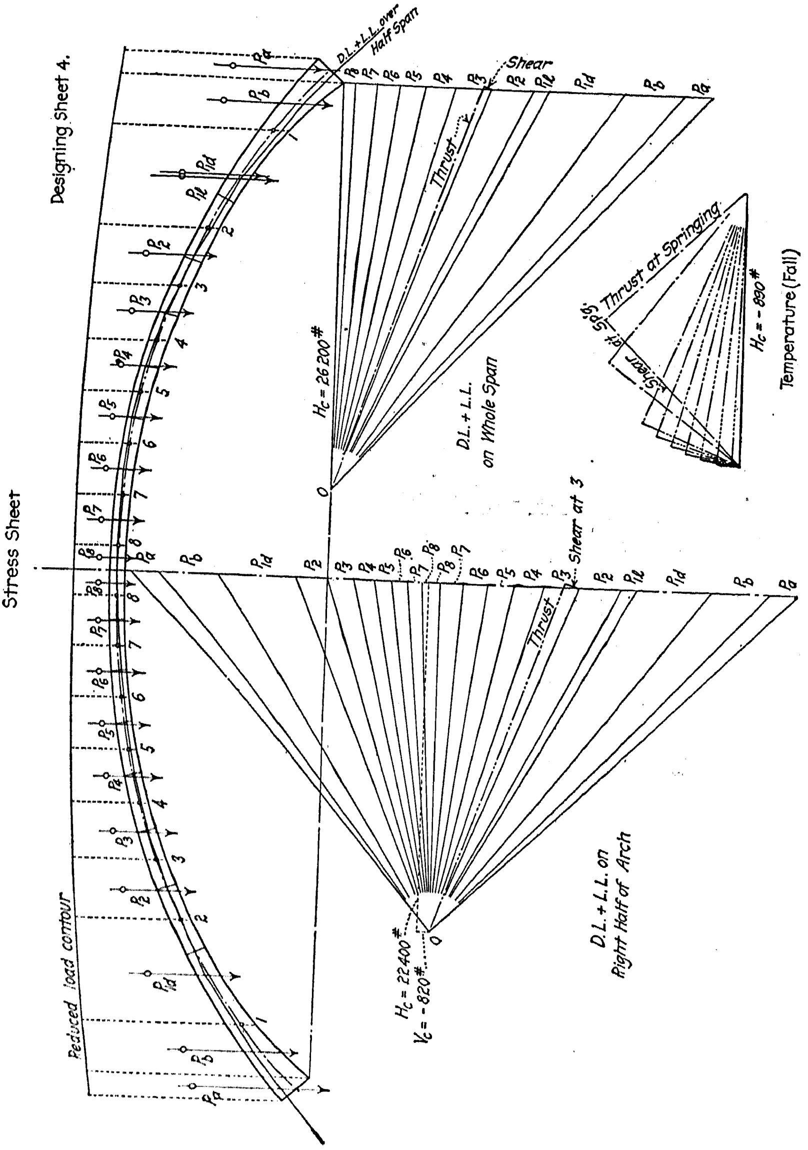
Rib Shortening:

$$H_c = -\frac{I}{s}\cdot\frac{c_a\, \ell\, n_h}{2[n_h\Sigma y^2 - (\Sigma y)^2]} \qquad (12)$$

$$M_c = -\frac{H_c\Sigma y}{n_h} \qquad (13)$$

$$M = M_c + H_c y \qquad (14)$$

Values of moments and thrusts for rib shortening are of same sign as for temperature fall. ℓ = span of arch axis.


Designing Sheet 4.
Stress Sheet
Reduced load contour
D.L. + L.L. over Half Span
Shear
Thrust
Pa
Pb
P1d
P1l
P2
P3
P4
P5
P6
P7
P8
1
2
3
4
5
6
7
8
Hc = 26 200#
0
D.L. + L.L. on Whole Span
Thrust at Springing
Shear at Spg.
Hc = -890#
Temperature (Fall)
Shear at 3
Hc = 22400#
Vc = -820#
D.L. + L.L. on Right Half of Arch

Cantilever Moments
(Algebraic Method)

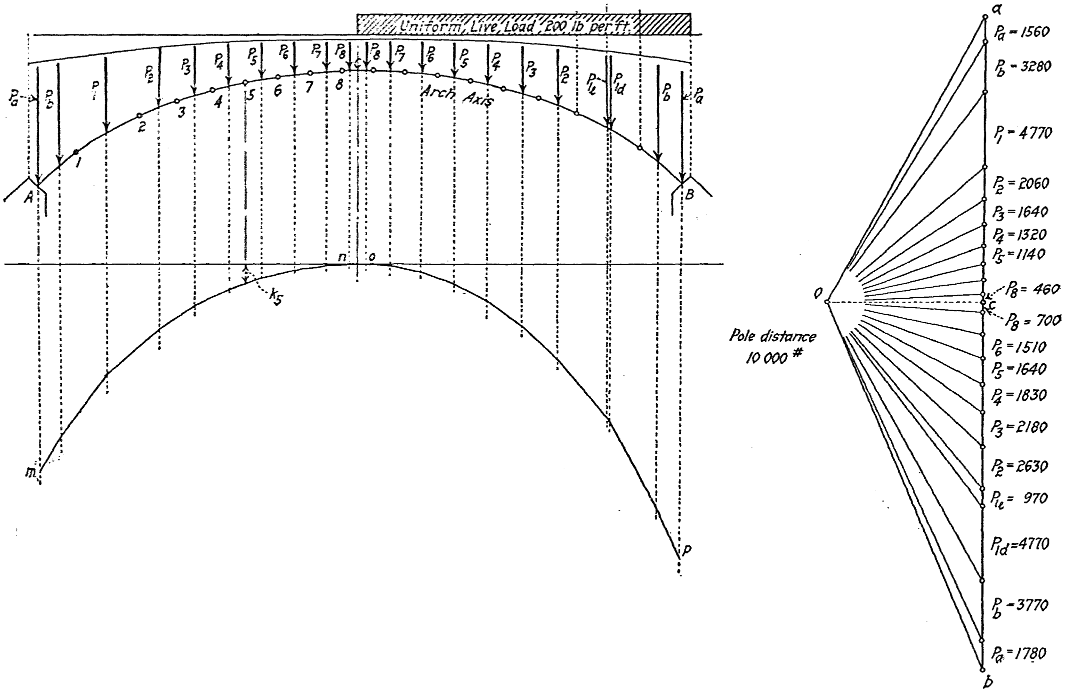
Designing Sheet 5.

			Dead Load				Dead Load plus Live Load				
Point	Hor. dist. from crown	Hor. distance between points	Dead Loads	Summation of Dead Loads	Increment of Moment (Dead)	Moments (Dead)	Live Loads	Sum. of Live Loads	Increment of moment	Moments	
										Live Load	LL+DL
P_8	.61		460	460			240	240			
8	1.19	.58			270	270			140	140	410
P_7	2.41	1.22	940	1400	560	830	480	720	290	430	
7	3.60	1.19			1670	2500			860	1290	3790
P_6	4.87	1.27	1020	2420	1780	4280	490	1210	910	2200	
6	6.07	1.20			2900	7180			1450	3650	10830
P_5	7.37	1.30	1140	3560	3150	10330	500	1710	1570	5220	
5	8.57	1.20			4270	14600			2050	7270	21870
P_4	9.89	1.32	1320	4880	4700	19300	510	2220	2260	9530	
4	11.12	1.23			6000	25300			2730	12260	37560
P_3	12.51	1.39	1640	6520	6780	32080	540	2760	3090	15350	
3	13.82	1.31			8540	40620			3620	18970	59590
P_2	15.28	1.46	2060	8580	9520	50140	570	3330	4030	23000	
2	16.66	1.38			11840	61980			4600	27600	89580
P_{1d}	19.26	2.60	4770	13350	22310	84290					
P_{1l}	19.09	2.43					P_{1l} 970	4300	8090	35690	
1	21.53	P_{1d} 2.27 P_{1l} 2.44			30300	114590			10490	46180	160770
P_b	22.86	1.33	3280	16630	17760	132350	490	4790	5720	51900	
P_a (Spg.)	24.55	1.69	1560	18190	28100	160450	220	5010	8100	60000	220450
		24.55	18190		160450			5010	60000		

(25.05)(200) = 5010

Designing Sheet 5(A)

Cantilever Moments
(Graphical Method)



MOMENTS AND THRUSTS

Designing Sheet 6

Pt.	x	Y	x²	Y²	Dead Load			Live plus Dead Load			Dead Load plus Live Load on Right Half of Span H_c = 22400 V_c = −820 M_c = −370								D.L. plus L.L. on Whole Span H_c=26,200 V_c=0 M_c=−50				Temperature (Fall of 40° F.: rise of 40° F., moments and thrusts of opposite sign) H_c=−890 M_c=+1630			Rib Shortening H_c=−300 M_c=+550	
													Left Half			Right Half											
					m	mx	my	m	mx	my	$H_c y$	$V_c x$	M	N (Scaled)	x_0 (For check)	M	N (Scaled)	x_0 (For check)	$H_c y$	M	N (Scaled)	x_0 (For check)	$H_c y$	M	N (Scaled)	M	N
Cr													−370	22400	−.02	−370	22400	−.02		−50	26200	−.002		+1630	−890	+550	−300
Spg	24.55	8.82	602.7	77.79	160 450	3939 000	1415 200	220450	5412 000	1944 400	197600	−20 100	+16700	29200	+.57	−3100	31600	−.10	231 100	+10 600	34900	+.30	−7800	−6200	−620	−2100	−210
1	21.53	6.17	463.5	38.07	114 590	2467 100	707 000	160 770	3461 400	992 000	138 200	−17700	+5500	26500	+.21	−5200	28000	−.19	161 700	+900	31 650	+.03	−5500	−3900	−710	−1300	−240
2	16.66	3.35	277.6	11.22	61 980	1032 600	207 600	89 580	1492 400	300 100	75000	−13700	−1000	24300	−.04	−1300	24900	−.05	87800	−1800	28800	−.06	−3000	−1400	−820	−500	−280
3	13.82	2.28	191.0	5.20	40 620	561 400	92 600	59 590	823 590	135900	51 100	−11 300	−1 200	23600	−.05	+2400	23900	+.10	59 700	+100	27800	+.004	−2000	−400	−850	−100	−290
4	11.12	1.45	123.7	2.10	25 300	281 300	36 700	37 560	417 700	54 500	32500	−9100	−2300	23100	−.10	+3700	23300	+.16	38 000	+400	27200	+.015	−1300	+300	−860	+100	−290
5	8.57	.86	73.4	.74	14 600	125 100	12 600	21 870	187 400	18 800	19 300	−7000	−2700	22850	−.12	+4100	22800	+.18	22 500	+600	26700	+.02	−800	+800	−870	+300	−290
6	6.07	.40	36.8	.16	7 180	43 600	2 900	10 830	65 800	4 300	9 000	−5000	−3600	22650	−.16	+2800	22600	+.12	10 500	−400	26 500	−.015	−400	+1200	−880	+400	−300
7	3.60	.12	13.0	.01	2 500	9 000	300	3 790	13 600	500	2 700	−3000	−3200	22500	−.14	+1500	22450	+.07	3100	+700	26300	+.03	−100	+1500	−890	+500	−300
8	1.19	.02	1.1	.00	270	300	—	410	500	—	400	−1000	−1200	22400	−.05	+600	22400	+.03	500	0	26200	.0	—	+1600	−890	+500	−300
Σ		14.65	1180.4	57.50	267 040	4520 400	1059 700	383400	6462300	1506100																	

Live Load on Right Half of Span:

$$H_c = \frac{8(1,059,700 + 1,506,100) - (267,040 + 383,400)(14.65)}{2[8(57.50) - (14.65)^2]}$$

$$= \frac{10,997,500}{490.8} = 22,400 \text{ lb.}$$

$$V_c = \frac{4,520,400 - 6,462,300}{2(1180.4)} = -820 \text{ lb.}$$

$$M_c = \frac{(267,040 + 383,400) - 2(22,400)(14.65)}{2(8)} = \frac{-5880}{16} = -370 \text{ ft. lb.}$$

Live Load on Whole Span:

$$H_c = \frac{(8)(2)(1,506,100) - 2(383,400)(14.65)}{490.8}$$

$$= \frac{12,864,000}{490.8} = 26,200 \text{ lb.}$$

$$V_c = 0$$

$$M_c = \frac{2(383,400) - 2(26,200)(14.65)}{16}$$

$$= -\frac{850}{16} = -50 \text{ ft. lb.}$$

Temperature (fall):

$$H_c = \frac{-(.016)(.000006)(40)(49.2)(8)(2,000,000)(144)}{490.8}$$

$$= -890 \text{ lb.}$$

$$V_c = 0$$

$$M_c = +\frac{(890)(14.65)}{8} = +1630 \text{ ft. lb.}$$

Rib Shortening (live load on one-half span):

Assume c_a = 160 lb. per sq. in.

$$H_c = \frac{-(.016)(160)(144)(49.2)(8)}{490.8} = -300$$

$$c_a \text{ (at crown)} = \frac{22400}{(1)(.75) + .092} = 26600$$

$$c_a \text{ (at point 3)} = \frac{1}{2} \cdot \frac{23,600 + 23,900}{(1)(.80) + .092} = 26600$$

$$c_a \text{ (at springing)} = \frac{1}{2} \cdot \frac{29200 + 31600}{(1)(1.75) + .092} = 16500$$

$$3\overline{)69\,700}$$

$$23\,200$$

(160)(144) = 23000 lb./sq ft or.

Rib Shortening (cont)

$$M_c = -\frac{(-300)(14.65)}{8} = +550 \text{ ft. lb.}$$

$$\text{Ratio of } \frac{H_c \text{ for rib shortening}}{H_c \text{ for temperature}} = \frac{300}{890} = .337$$

Designing Sheet 7.

Maximum Stresses

Point	t	p_o	Combination of Moments and Thrusts and Resulting Stresses				
			M	N	$\frac{x_o}{t}$	f_c	f_s
Crown	0.75	.00815	*+2130	25010	.113 (f)	*335	
Springing	1.75	.0035	+20800 *+14700	29610 35310	.401 (r) .238 (r)	472 *324 (Exact 540)	6100
1	1.12	.0054	−10400	27050	.343 (f)	516	2800
2	0.81	.0075	*−3700	27700	.165 (f)	*408	
3	0.80	.0076	+2700	24460	.138 (r)		
4	0.79	.0077	+4100	22150	.235 (f)	406 (Exact 572)	
5	0.77	.0079	+5200 −3200	21640 23430	.312 (f) .177 (r)	508 371	
6	0.76	.0080	−4400	23230	.249 (r)	458	
7	0.75	.0081	−4200	23090	.243 (r)		
8	0.75	.00815	−3300 *+2100	22990 25010	.192 (r) .112 (f)		

*means due to live load on whole span

Formulas for Bending and Direct Stress

Compression over entire section

$$f_c = \frac{N}{bt}\left(\frac{1}{1+np_o} + \frac{6x_o t}{t^2 + 12np_o r^2}\right) = \frac{N}{bt}(K)$$

Tension over part of section

$$k^3 - 3\left(\frac{1}{2} - \frac{x_o}{t}\right)k^2 + 6np_o k\frac{x_o}{t} = 3np_o\left(\frac{x_o}{t} + 2\frac{r^2}{t^2}\right)$$

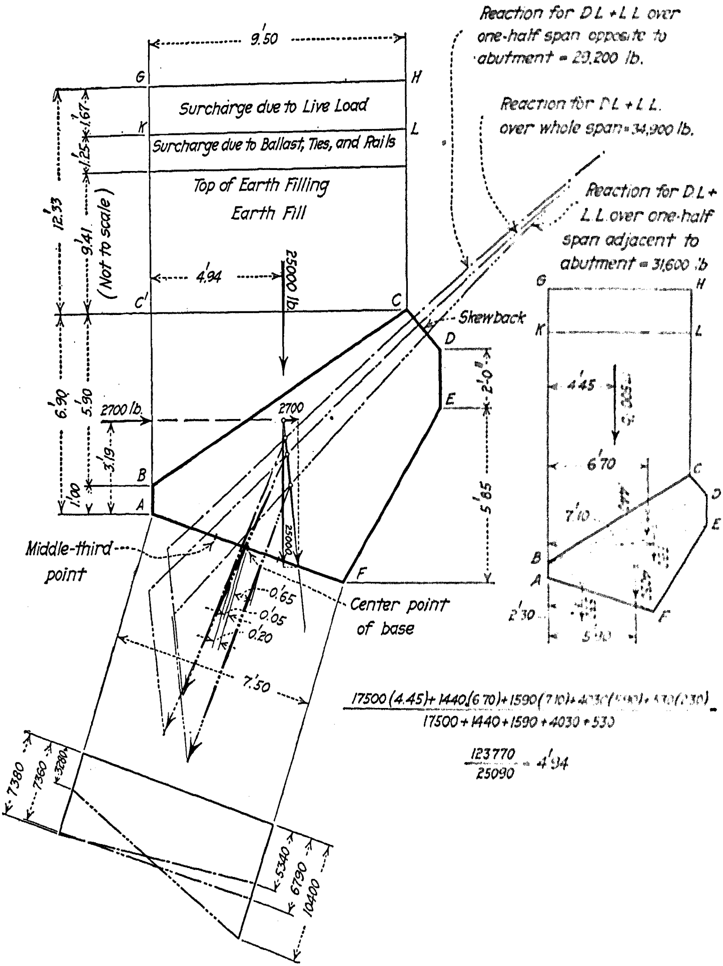
$$L = \left[\frac{np_o r^2}{kt^2} + \frac{k}{12}(3-2k)\right]$$

$$f_c = \frac{M}{Lbt^2}$$

$$f_s = nf_c\left(\frac{d}{kt} - 1\right)$$

Analysis of Abutment

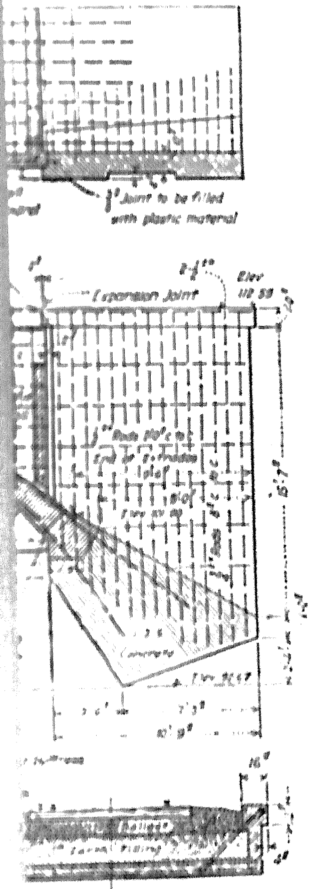
Designing Sheet 8.


Reaction for D.L.+L.L. over one-half span opposite to abutment = 29,200 lb.
Reaction for D.L.+L.L. over whole span = 34,900 lb.
Reaction for D.L.+L.L. over one-half span adjacent to abutment = 31,600 lb.
9'50
G
H
K
L
Surcharge due to Live Load
Surcharge due to Ballast, Ties, and Rails
Top of Earth Filling
Earth Fill
1'67
1'25
12'33
9'41
(Not to scale)
4'94
25000 lb.
C'
C
Skewback
D
E
2'0"
5'85
6'90
5'90
2700 lb.
2700
3'19
1'00
B
A
F
25000
Middle-third point
Center point of base
0'65
0'05
0'20
7'50
7380
7360
3280
5340
6790
10400
G
H
K
L
4'45
6'70
7'10
C
D
E
B
A
F
2'30
5'90

$$\frac{17500(4.45)+1440(6.70)+1590(7.10)+4030(5.90)+530(2.30)}{17500+1440+1590+4030+530}=$$

$$\frac{123770}{25090}=4'94$$

PLATE 1



(Facing Page 66.)

S

ıon prac-
ling over
ıat these
, in the
ned that
greatest
ı general
ınd con-
of pro-
'uctures,
resses is
method.
ınity at
'awn for

ıximum
'ance in
ll show
.milarly
ccurate
lines as
definite
luence-
ısen for
degree

tion.—
; prac-
ring in
ısverse
divid-

The

K·7380

CHAPTER V

USE OF INFLUENCE LINES IN ARCH ANALYSIS

33. Advantage of Using Influence Lines.—It is common practice in arch design to consider the live load as extending over certain definite portions of the span and to assume that these loadings produce the maximum effects. For example, in the design of the arch in the preceding chapter it was assumed that by loading either the whole span or the half span the greatest possible stresses at any given section were obtained. In general this assumption is only a very rough approximation, and considerable inaccuracy may result from such a method of procedure. In fact, in the case of large and important structures, the only satisfactory way to analyze for maximum stresses is by what might be called a *unit-load* or *influence-line* method. By this method the arch is first analyzed for a load of unity at the several load points and then influence lines[1] are drawn for either moment and thrust or for fiber stress.

The position of the live load on an arch to cause maximum stress at any given section cannot be determined in advance in the common method of analysis. An investigation will show that different loadings are required for sections similarly located in arches of different proportions. The only accurate method, then, is to draw a proper number of influence lines as above described. In arches continuously loaded no definite load points exist at which to place the load of unity in influence-line analysis, but in arches of this class points may be chosen for this purpose sufficiently close together to give any desired degree of accuracy.

34. Analysis of an Arch Ring in Open-spandrel Construction.—The use of influence lines will be illustrated by giving practically complete computations for the design of an arch ring in open-spandrel construction supporting a series of transverse walls. The only computations omitted will be those for dividing the half arch into ten divisions having constant $\frac{s}{I}$. The arch to be analyzed is shown in Fig. 20. Loads apply to a longitudinal strip of arch ring 12 in. in width. The notation that will be used is given in Art. 13.

---

[1] Influence lines are treated in "*The Elements of Structures*," by the same author.

Designing Sheet No. 9 gives the computations needed in determining the moments and thrusts at the crown for a load of unity placed successively at the load points L_1, L_2, L_3, and L_4. In a symmetrical arch such as the one under analysis, load points need be taken only to one side of the center of span. The quantity m (cantilever moment) at a given section for a load of unity at a given load point is equal to the value of x for the section in question minus the value of x for the given load point. For example, the value of m for section 1 with a unit load at L_1 equals 31.70 − 30.62 = 1.08 ft.-lb. Also the value of m for section 1 with a unit load at L_4 equals 31.70 − 4.37 = 27.33 ft.-lb. The values of the moments and thrusts at the crown were determined by the formulas given on Designing Sheet No. 3.

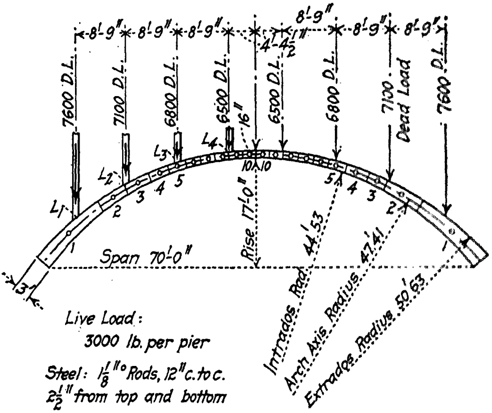


Fig. 20.

Designing Sheets Nos. 10 and 10A give the computations and graphical work required in determining the moments and thrusts at the various sections due to the above loadings. The moments and thrusts could all be determined algebraically, but the graphical method is much the simpler. The method of procedure in constructing Designing Sheet No. 10A will be explained by considering a unit load at L_4.

The values of H_c, V_c, and x_o (at crown) for this loading are known, having been computed on Designing Sheet No. 9. These values determine the magnitude, direction, and point of application of the reaction at the crown. The point of application, designated as *a* on Designing Sheet No. 10*A*, is 1.06 ft. from the arch axis and should be laid off above the same, since the sign is plus. The value of V_c is positive and indicates that the line of pressure slopes upward toward the left. The force polygon for the unit load at L_4 is designated as *AOC*. The side *OA* is determined by laying off *AB* on the vertical side *AC* equal to V_c, and making the horizontal distance *BO* equal to H_c. With *AC* as one side of the force polygon made equal to the load of unity, and with *OA* equal to the reaction on the right of the arch, the reaction to the left of the point L_4 must equal *OC*, or the closing side of the polygon. In order for the forces acting on the arch to be in equilibrium, the forces must act in order around the triangle. This fact shows that *OC* acts upward toward the right, as was to be expected.

The equilibrium polygon may thus be determined by drawing a line through the point *a* parallel to *OA*, and then drawing a line on the left portion of the arch parallel to *OC*, both lines intersecting in the load vertical at *b*. The bending moment at any section, due to the unit load at L_4, is therefore equal to the vertical ordinate measured from the arch axis to the equilibrium polygon, multiplied by the force H_c. If the vertical ordinates be designated in general as v_d, then, at any section, $M = H_c v_d$. The thrusts and shears at any section may be found by resolving the proper reaction into components perpendicular and parallel respectively to the section in question. The values of the moments and thrusts are given in the proper columns on Designing Sheet No. 10 Moments and thrusts for other loadings may be determined in a similar manner.

Influence lines could now be drawn for moment and thrust, but maximum fiber stress does not occur in general for either maximum moment or maximum thrust. Obviously, the true maximum stress occurs for the loading that makes the algebraic *sum* of the stresses due to moment and thrust a maximum. Influence lines, therefore, should often be drawn directly for outer fiber stress, as such procedure eliminates any necessity for trial loadings.

Before influence lines are plotted, there is no way of telling, for a given maximum condition of loading, whether the stress analysis of a given section will fall under Case I or under Case II (see Arts. 75 and 76, Volume I)—that is, whether compression will be produced over the entire section or whether a tension will exist over part of the section greater than the allowable tensile strength of the concrete. If the latter condition (Case II) should result, an influence line for fiber stress cannot be employed to determine the corresponding maximum stress, due to the fact that it is impossible, when computing values of the fiber stress for a load of unity at separate load points, to take properly into consideration the exact amount of tensile stress in the concrete that would be neglected if the analysis should be made for the maximum loading using total moment and total thrust. This is not serious, however, since no matter whether maximum stress is desired at an upper or lower fiber, the loading that will produce a maximum value, assuming a Case I distribution of stress, will always be the same loading that will cause maximum stress for Case II distribution. Consequently, the influence lines should all be drawn assuming Case I distribution, and, if tension is found greater than the allowable tensile strength of the concrete, the Case II conditions should be separately considered by drawing influence lines for moment and thrust and finding the values of the moments and thrusts that correspond to the Case I loadings for maximum stress.

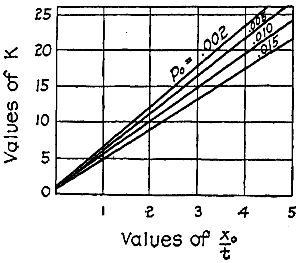


DIAGRAM 1.

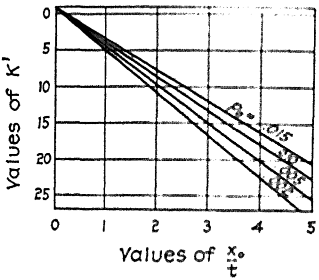


DIAGRAM 2.

(If no tension greater than the tensile strength of the concrete is likely to result at any section of the arch, then Designing Sheet No. 10 may be omitted. The values of x_o and N may be scaled and used directly in Designing Sheets Nos. 11 and 11A. In fact, even with considerable tension at some sections, not all of Designing Sheet No. 10 will be needed and only those values should be determined as are later found necessary. The designing sheet is given in full here, however, for convenience and for instructive purposes.)

The following formulas refer to Case **I**:

$$f_c = \frac{NK}{bt}$$

$$f'_c = \frac{NK'}{bt}$$

in which, for reinforcement symmetrical about the center of section,

$$\begin{matrix}(K)\\(K')\end{matrix} = \left[\frac{1}{1+np_o} \begin{matrix}(+)\\(-)\end{matrix} \frac{6x_ot}{t^2+12np_or^2}\right]$$

For an embedment of rods equal to one-tenth the depth, and $n = 15$,

$$\begin{matrix}(K)\\(K')\end{matrix} = \left[\frac{1}{1+15p_o} \begin{matrix}(+)\\(-)\end{matrix} \frac{x_o}{t} \cdot \frac{6}{1+28.8p_o}\right]$$

Diagrams 1 and 2 give values of K and K' for various values of p_o and $\frac{x_o}{t}$, and for one-tenth embedment. These diagrams as presented here are too small for actual use, but illustrate the construction of diagrams which should be drawn to a large scale in practice for at least three depths of embedment—that is, for d' equal to $0.05t$, $0.10t$, and $0.15t$. The size of the diagrams should be approximately 30 in. square to obtain a reasonable degree of accuracy. The few values above $\frac{x_o}{t} = 5.0$ may easily be computed directly from the formulas.

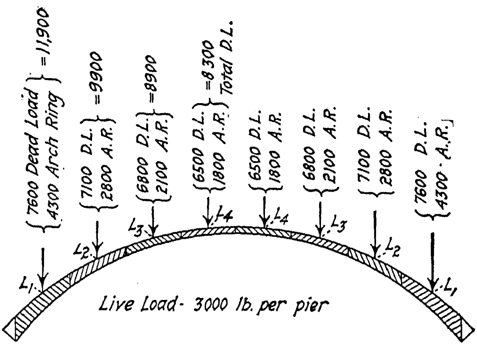


FIG. 21.

minimum fiber stress (Case I) remain constant for any g
section, influence lines drawn for the quantities NK and
will serve as influence lines for fiber stress. (See Art. 44
another method of determining fiber stresses.) The value
K and K' on Designing Sheets Nos. 11 and 11*A* have been
termined directly from the diagrams, making the assumption
simplicity, that $d' = 0.10t$ at all sections. The influence l
resulting from multiplying these values by the thrusts are sh
on Designing Sheet No. 12, making no attempt to obtain sm
curves. The full lines are for upper fiber stress and the dot-a
dash lines for lower fiber stress. Ordinates above the base
indicate compression for both upper and lower fibers.

The maximum fiber stresses (assuming Case I to apply a
sections) are given on Designing Sheet No. 13, considering
weight of the arch ring as applied at the centers of the suppo
piers (Fig. 21). The computations are given on this sheet for
maximum stresses at the crown and springing. Notice t
tension exists at the springing for loading No. 1—that is,
loading for maximum compression in upper fiber—and influe
lines for moment and thrust should be drawn if the stresses
to loading only are to be determined.

The student should note that where all the loads on an arch
at load points, the actual influence lines need not be dra
In such a case a table as on Designing Sheet No. 14 may be c
venient, the values being taken directly from Designing Sh
Nos. 11 and 11*A*.

The stresses given on Designing Sheet No. 13 are for loa
only, assuming Case I. When stresses are required inclu
temperature and rib shortening, these stresses due to loa
should be combined with those for temperature and rib shorten
and the resulting stresses computed. When tension exists at
section, due to Case I loading combined with temperature
rib shortening, then influence lines for moment and thrust sho
be drawn, whether or not such lines are required for loading alo

Designing Sheet No. 15 gives the total maximum stresses
the four conditions of loading shown, assuming Case I to apply
all sections. The last column gives the sections and loadi
that need to be considered for a Case II distribution of str
Although the depth of section gradually decreases toward
crown, it is obvious that, for Case II conditions, the stresses

Designing Sheet 9.

Determination of Moments and Thrusts at Crown
(Unit Load at Definite Points)

Pt.	x	y	x^2	y^2	Unit Load at L_1			Unit Load at L_2			Unit Load at L_3			Unit Load at L_4		
					m	mx	my	m	mx	my	m	mx	my	m	mx	my
L_1	30.62															
L_2	21.87															
L_3	13.12															
L_4	4.37															
1	31.70	12.16	1004.9	147.87	1.08	34.2	13.13	9.83	311.6	119.53	18.58	589.0	225.93	27.33	866.4	332.33
2	24.18	6.63	584.7	43.96				2.31	55.9	15.32	11.06	267.4	73.33	19.81	479.0	131.34
3	19.61	4.25	384.6	18.06							6.49	127.3	27.58	15.24	298.9	64.77
4	16.08	2.80	258.6	7.84							2.96	47.6	8.29	11.71	188.3	32.79
5	13.06	1.85	170.6	3.42										8.69	113.5	16.08
6	10.33	1.13	106.7	1.28										5.96	61.6	6.73
7	7.80	.64	60.8	.41										3.43	26.7	2.20
8	5.43	.30	29.5	.09										1.06	5.8	.32
9	3.18	.12	10.1	.01												
10	1.05	.02	1.1	.00												
Σ		29.90	2611.6	222.94	1.08	34.2	13.13	12.14	367.5	134.85	39.09	1031.3	335.13	93.23	2040.2	586.56

Unit Load at L_1

$$H_c = \frac{10(13.13) - (1.08)(29.90)}{2[10(222.94) - (29.90)^2]} = \frac{99.0}{2670.8} = 0.037$$

$$V_c = \frac{34.2}{2(2611.6)} = \frac{34.2}{5223.2} = 0.007$$

$$M_c = \frac{(1.08) - 2(.037)(29.90)}{(2)(10)} = -0.057$$

$$x_0 \text{ (at crown)} = \frac{-.057}{.037} = -1.54$$

Unit Load at L_2

$$H_c = \frac{10(134.85) - (12.14)(29.90)}{2670.8} = \frac{985.5}{2670.8} = 0.369$$

$$V_c = \frac{367.5}{5223.2} = 0.070$$

$$M_c = \frac{12.14 - (2)(.369)(29.90)}{20} = -0.496$$

$$x_0 \text{ (at crown)} = \frac{-0.496}{0.369} = -1.35$$

Unit Load at L_3

$$H_c = \frac{10(335.13) - (39.09)(29.90)}{2670.8} = \frac{2182.5}{2670.8} = 0.818$$

$$V_c = \frac{1031.3}{5223.2} = 0.197$$

$$M_c = \frac{(39.09) - 2(.818)(29.90)}{20} = -0.491$$

$$x_0 \text{ (at crown)} = \frac{-0.491}{0.818} = -0.60$$

Unit Load at L_4

$$H_c = \frac{10(586.56) - (93.23)(29.90)}{2670.8} = \frac{3078.0}{2670.8} = 1.151$$

$$V_c = \frac{2040.2}{5223.2} = 0.391$$

$$M_c = \frac{93.23 - (2)(1.151)(29.90)}{20} = +1.22$$

$$x_0 \text{ (at crown)} = \frac{+1.22}{1.151} = +1.06$$

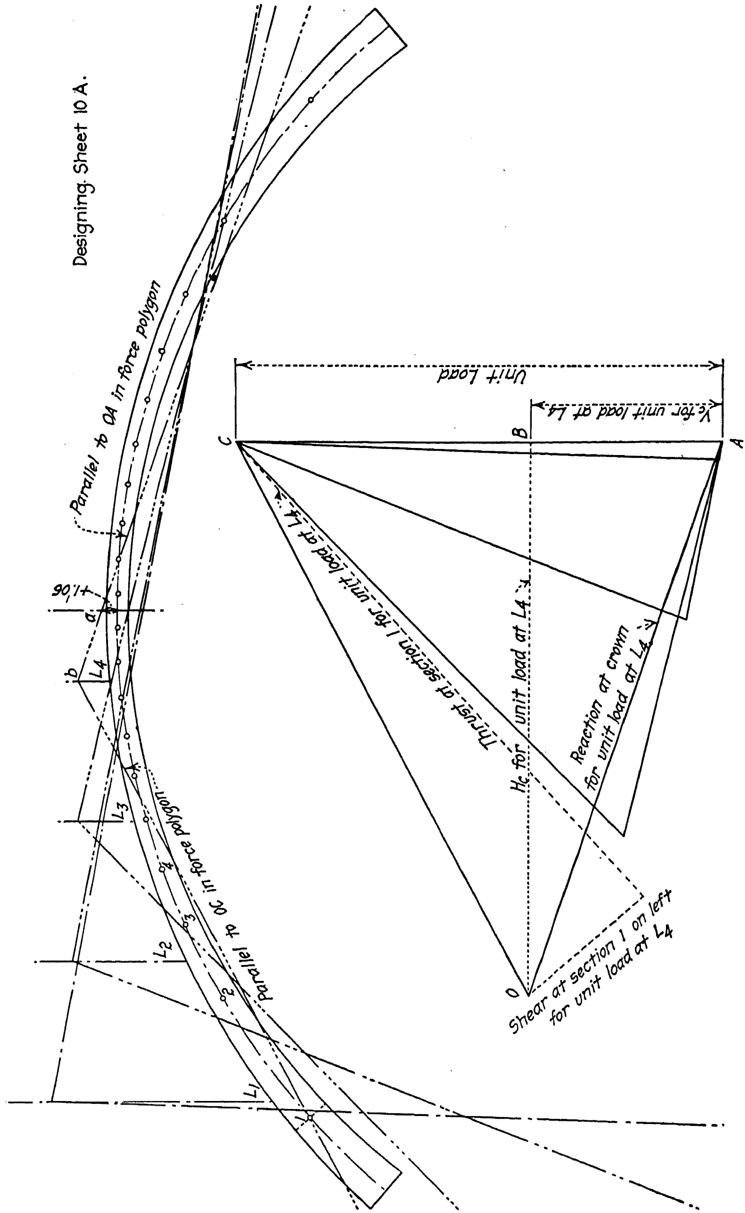
Moments and Thrusts at the Various Sections

(Unit Load at Definite Points)

Designing Sheet 10

Pt.	Unit Load at L_1						Unit Load at L_2						Unit Load at L_3						Unit Load at L_4					
	$H_c = .037$	$V_c = .007$	$M_c = -.057$				$H_c = 0\ 369$	$V_c = 0\ 070$	$M_c = -0496$				$H_c = 0.818$	$V_c = 0.197$	$M_c = -0.491$				$H_c = 1.151$	$V_c = 0.391$	$M_c = +1.22$			
	Left Half			Right Half			Left Half			Right Half			Left Half			Right Half			Left Half			Right Half		
	V_d	$M = H_c V_d$	N	V_d	$H_c V_d$	N	V_d	$M = H_c V_d$	N	V_d	$H_c V_d$	N	V_d	$M = H_c V_d$	N	V_d	$M = H_c V_d$	N	V_d	$M = H_c V_d$	N	V_d	$M = H_c V_d$	N
Cr	- 1 54	- 057	.037				- 1. 35	- .496	.369				- .60	- .491	.818				+ 1.06	+1.220	1.151			
Spg	-125 60	-4 647	.783	+ 8 62	+.319	.030	-16 27	-6 004	.950	+8.46	+3.122	.293	-3.21	-2.626	1.142	+7.32	+5.988	.680	+2.45	+2.820	1.211	+5.50	+6.331	1 044
1	- 11 95	- .442	.692	+ 4.87	+ 180	035	- 9.74	-3.594	.897	+ 4.75	+1.753	.321	- 3.42	-2.798	1.145	+3.85	+3 149	.741	+0 26	+.299	1.263	+ 2.40	+2.762	1 119
2	+ 9.39	+ .347	.028	+ 0.70	+ 026	.036	+ 3 58	+1 321	.795	+ 0 67	+.247	353	- 1.62	-1 325	1.114	+ 0.15	+ .123	.804	-1 30	-1.496	1.301	- 0 55	- .633	1.188
3	+ 6 23	+ 231	030	- 0.84	- 031	.038	+ 6.62	+2.443	.308	- 0.82	-.303	365	+ 0.45	+ .368	1.077	- 1.11	- .908	.826	- 1.23	-1.416	1.298	-1 37	-1 577	1.207
4	+ 4 14	+ .153	032	- 1.63	- 060	038	+ 4 52	+1 668	324	- 1 60	-.590	.371	+ 2.49	+2 037	1 043	- 1 70	-1 391	.836	-0.80	- .921	1.288	-1 60	-1 842	1 215
5	+ 2 63	+ 097	034	- 2 07	- 077	.038	+ 2 95	+1 089	337	- 2 01	-.742	374	+ 4.41	+3.607	.731	- 1.95	-1 595	.841	-0.20	- .230	1.274	- 1 55	-1 784	1 214
6	+ 1.45	+ .054	035	- 2.25	-.083	.038	+ 1.77	+ .653	.345	- 2 18	-.804	.375	+ 3 05	+2 495	.755	- 1 97	-1 611	.840	+0 56	+ 645	1.255	- 1.30	-1 496	1.207
7	+ 0.49	+ 018	036	- 2 30	- 085	038	+ 0 78	+ 288	353	- 2 19	-.808	376	+ 1 83	+1.497	.776	- 1 83	-1.497	.839	+ 1.42	+1.634	1.237	- 0 92	-1 059	1.200
8	- 0 25	- 009	037	- 2 20	- 081	.037	0 00	.000	358	- 2 06	- 760	.375	+ 1 05	+ .859	.791	- 1.60	-1.309	.836	+ 2.35	+2 705	1.213	- 0.45	- .518	1 187
9	- 0.83	- .031	.037	- 2 01	-.074	.037	- 0 60	- .221	.363	- 1.82	-.672	373	+ 0 34	+ .278	804	- 1.24	-1.014	.830	+ 2.30	+2 647	1.122	+ 0.12	+ .138	1 174
10	- 1.37	- 051	.037	- 1 72	-.064	.037	- 1.15	- .424	.368	- 1 54	- .568	.370	- 0.35	- .286	.815	- 0.83	-.679	.824	+ 1.40	+1.611	1.142	+ 0.72	+ .829	1.159

Graphical work on Designing Sheet 10 A.


Designing Sheet 10 A.
Parallel to OA in force polygon
+1.06
a
b
L4
L3
L2
L1
4
3
2
Parallel to OC in force polygon
Unit Load
Y_c for unit load at L4
C
B
A
Thrust at section 1 for unit load at L4
H_c for unit load at L4
Reaction at crown for unit load at L4
O
Shear at section 1 on left for unit load at L_4

Designing Sheet 11·

Data for Influence Lines

Values of NK and NK' in formulas $f_c = \frac{NK}{bt}$ and $f_c' = \frac{NK'}{bt}$ respectively

(Unit Load at Definite Points)

Point	t	P_o	Unit Load at L_1										Unit Load at L_2									
			Left Half					Right Half					Left Half					Right Half				
			$\frac{x_o}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *	$\frac{x_o}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *	$\frac{x_o}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *	$\frac{x_o}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *
Cr.	1.33	.0104	-1.16	6.23	-.231	4.49	+.166						-1.01	5.52	-2.037	3.79	+1.399					
Spg.	3.00	.0046	-1.98	11.42	-8.942	9.54	+7.470	+3.55	19.74	+.592	17.86	-.536	-2.11	12.10	-11.471	10.22	+9.689	+3.56	19.77	+5.793	17.89	-5.242
1	2.50	.0055	-0.26	2.27	-1.571	0.43	+.298	+2.06	11.61	+.406	9.77	-.342	-1.60	9.22	-8.270	7.38	+6.620	+2.18	12.20	+3.916	10.38	-3.332
2	1.97	.0070	+6.29	32.27	+.904	30.45	-.853	+0.37	2.76	+.099	0.94	-.034	+0.84	5.10	+4.055	3.28	-2.608	+0.36	2.70	+.953	0.88	-.311
3	1.74	.0079	+4.42	22.45	+.674	20.77	-.623	-0.47	3.14	-.119	1.46	+.055	+4.56	23.14	+7.127	21.46	-6.610	-0.48	3.19	-1.164	1.51	+.551
4	1.56	.0089	+3.06	15.51	+.496	13.75	-.440	-1.01	5.71	-.217	3.95	+.150	+3.30	16.80	+5.443	15.04	-4.873	-1.02	5.76	-2.137	4.04	+1.499
5	1.47	.0094	+1.94	10.04	+.341	8.28	-.282	-1.38	7.39	-.281	5.63	+.214	+2.20	11.26	+3.795	9.50	-3.202	-1.35	7.25	-2.712	5.49	+2.053
6	1.42	.0097	+1.09	5.98	+.209	4.24	-.148	-1.54	8.09	-.307	6.35	+.241	+1.33	7.11	+2.453	5.37	-1.853	-1.51	7.95	-2.981	6.21	+2.329
7	1.37	.0101	+0.37	2.59	+.093	0.85	-.031	-1.63	8.45	-.321	6.71	+.255	+0.60	3.66	+1.341	1.92	-.730	-1.57	8.17	-3.105	6.43	+2.443
8	1.35	.0102	-0.18	1.70	-.063	0.04	+.001 (comp.)	-1.62	8.37	-.310	6.63	+.245	0.00	0.87	+.311	0.87	-.311 (comp.)	-1.50	7.81	-2.929	6.07	+2.276
9	1.33	.0104	-0.63	3.78	-.140	2.04	+.075	-1.50	7.80	-.289	6.06	+.224	-0.46	3.00	-1.089	1.26	+.457	-1.35	7.11	-2.652	5.37	+2.003
10	1.33	.0104	-1.04	5.66	-.209	3.94	+.146	-1.72	6.57	-.254	5.15	+.191	-0.87	4.88	-1.796	3.16	+1.163	-1.15	6.17	-2.283	4.45	+1.647

+ indicates upper fiber – indicates lower fiber * indicates tension except as noted.

Designing Sheet 11A.

Data for Influence Lines

Values of NK and NK' in formulas $f_c = \frac{NK}{bt}$ and $f_c' = \frac{NK'}{bt}$ respectively

Unit Load at Definite Points

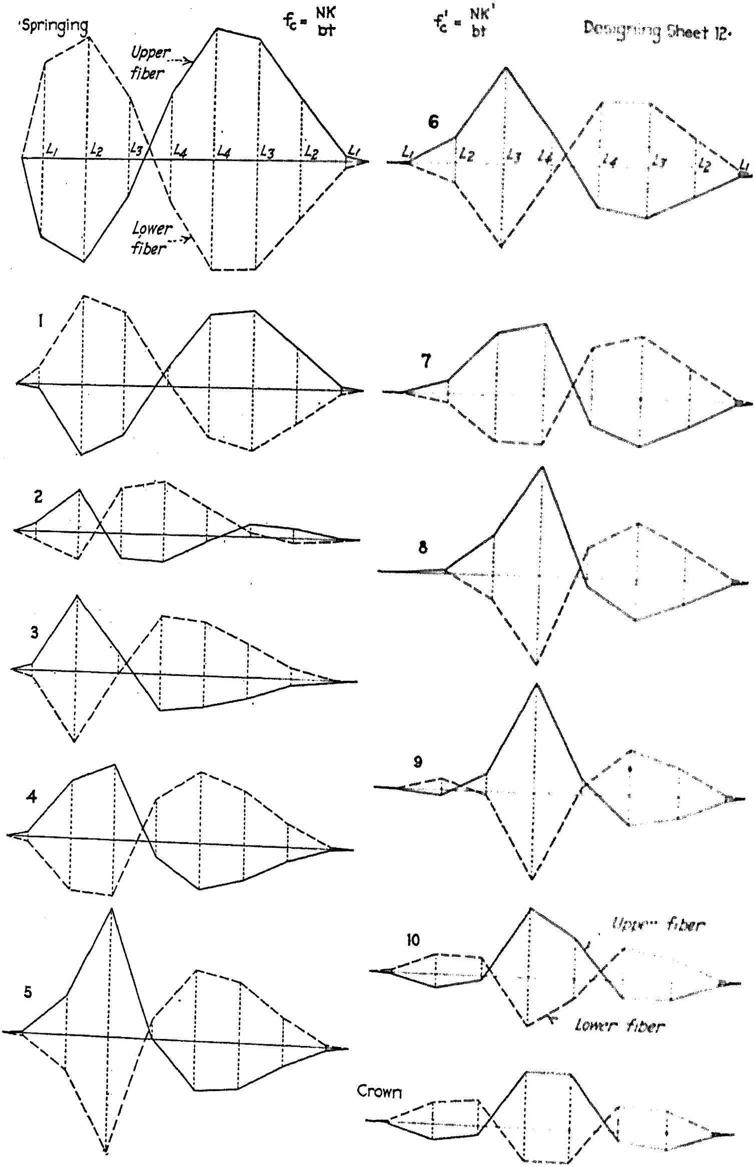
Pt.	t	P_0	Unit Load at L_3										Unit Load at L_4									
			Left Half					Right Half					Left Half					Right Half				
			$\frac{x_0}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *	$\frac{x_0}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *	$\frac{x_0}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *	$\frac{x_0}{t}=\frac{M}{Nt}$	K	NK	K'	NK' *
Cr.	1.33	.0104	-0.45	2.95	-2.413	1.21	+.990						+0.80	4.56	+5.249	2.82	-3.246					
Spg.	3.00	.0046	-0.77	5.01	-5.721	3.14	+3.586	+2.94	16.51	+11.227	14.63	-9.948	+0.78	5.01	+6.067	3.19	-3.863	+2.02	11.62	+12.131	9.74	-10.169
1	2.50	.0055	-0.98	6.00	-6.870	4.16	+4.763	+1.70	9.74	+7.217	7.90	-5.854	+0.09	1.39	+1.756	0.46	-.581 (comp.)	+0.99	6.05	+6.770	4.21	-4.711
2	1.97	.0070	-0.60	3.90	-4.345	2.08	+2.317	+0.08	1.31	+1.053	0.51	-.410 comp.	-0.58	3.80	-4.944	1.98	+2.576	-0.27	2.26	-2.685	0.44	+ .523
3	1.74	.0079	+0.20	1.82	+1.960	0.14	- .151	-0.63	3.92	-3.238	2.24	+1.850	-0.63	3.92	-5.088	2.24	+2.908	-0.75	4.51	-5.444	2.83	+3.416
4	1.56	.0089	+1.25	6.86	+7.155	5.10	-5.319	-1.07	6.00	-5.016	4.24	+3.545	-0.46	3.08	-3.967	1.32	+1.700	-0.97	5.47	-6.646	3.71	+4.508
5	1.47	.0094	+3.36	16.74	+12.237	14.98	-10.950	-1.29	6.97	-5.862	5.21	+4.382	-0.12	1.45	-1.847	0.31	+.395 (comp.)	-1.00	5.60	-6.798	3.84	+4.662
6	1.42	.0097	+2.32	11.75	+8.871	10.01	-7.558	-1.35	7.20	-6.048	5.46	+4.586	+0.36	2.56	+3.213	0.82	-1.029	-0.87	4.95	-5.975	3.21	+3.874
7	1.37	.0101	+1.41	7.43	+5.766	5.69	-4.415	-1.30	6.91	-5.797	5.17	+4.338	+0.97	5.38	+6.655	3.64	-4.503	-0.65	3.89	-4.668	2.15	+2.580
8	1.35	.0102	+0.81	4.62	+3.654	2.88	-2.278	-1.16	6.24	-5.217	4.50	+3.762	+1.65	8.51	+10.323	6.77	-8.212	-0.32	2.35	-2.789	0.61	+ .724
9	1.33	.0104	+0.26	2.07	+1.664	0.33	- .265	-0.92	5.12	-4.250	3.38	+2.805	+1.77	9.05	+10.154	7.31	-8.202	+0.09	1.29	+1.514	0.45	- .528 (comp)
10	1.33	.0104	-0.26	2.06	-1.679	0.34	+ .277	-0.62	3.72	-3 065	2.00	+1.648	+1.06	5.76	+6.578	4.04	-4.614	+0.54	3.35	+3.883	1.63	-1.889

+ Indicates upper fiber — Indicates lower fiber * Indicates tension except as noted

Influence Lines for NK and NK'

$f_c = \frac{NK}{bt}$ $f_c' = \frac{NK'}{bt}$

Designing Sheet 12.



Maximum Stresses Due to Loading Assuming Case I to Apply at All Sections.

ximum Stresses (Loading)

No. 1 Loading Max Comp in Upper Fiber		No. 2 Loading Max Comp in Lower Fiber	
Upper	Lower	Upper	Lower
362	81	118	329
477	- 189	84	199
354	- 5	104	253
161	198	40	364
200	131	- 22	400
281	106	- 47	488
489	-76	97	351
380	44	2	424
376	59	20	417
510	- 65	187	270
521	- 52	248	214
362	77	118	324

Crown:

No. 1 Loading

Upper

$$-2(.166)(11{,}900) - 2(1.399)(9{,}900) - 2(.990)(8{,}900) + 2(5.249)(11{,}300) = 2(34{,}700)$$

$$f_c = \frac{2(34{,}700)}{(1.33)(144)} = 362$$

Lower

$$2(.231)(11{,}900) + 2(2.037)(9900) + 2(2.413)(8900) - 2(3.246)(11{,}300) - 2(7700)$$

$$f_c' = \frac{2(7{,}700)}{(1.33)(144)} = 81$$

No. 2 Loading

Upper

$$-2(.166)(14{,}900) - 2(1.399)(12{,}900) - 2(990)(11{,}900) + 2(5.249)(8{,}300) = 2(11{,}300) = 2(34{,}700) - 2(.166 + 1.399 + .990 + 5.249)(3000)$$

$$f_c' = \frac{2(11{,}300)}{(1.33)(144)} = 118$$

Lower

$$2(.231)(14{,}900) + 2(2.037)(12{,}900) + 2(2.413)(11{,}900) - 2(3.246)(8300) = 2(31{,}500) = 2(7700) + 2(231 + 2.037 + 2.413 + 3.246)(3000)$$

$$f_c = \frac{2(31{,}500)}{(1.33)(144)} = 329$$

Springing:

No. 1 Loading

Upper

$$-(7.470)(11{,}900) + (.592)(14{,}900) - (9.689)(9900) + (5.793)(12{,}900) - (3.586)(8900) \pm (11.227)(11{,}900) + (18.198)(11{,}300) = 206{,}100$$

$$f_c = \frac{206{,}100}{(3.00)(144)} = 477$$

Lower

$$(8.942)(11{,}900) - (.536)(14{,}900) + (11.471)(9900) - (5.242)(12{,}900) + (5.721)(8{,}900) - (9.948)(11{,}900) - (14.032)(11{,}300) = -81{,}700$$

$$f_c' = \frac{-81{,}700}{(3.00)(144)} = 189 \text{ lb. tension}$$

etc.

Designing Sheet 14.

No. 1 Loading

(For method of computing No. 2 Loading see Designing Sheet 13)

Point	Fiber	L₁				L₂				L₃				L₄			
		D.L.=11,900		D.L.+L.L.=14,900		D.L.=9900		D.L.+L.L.=12,900		D.L.=8,900		D.L.+L.L.=11,900		D.L.=8,300		D.L.+L.L.=11,300	
		Comp.	Ten.	Comp.	Ten.	Comp.	Ten.	Comp.	Ten.	Comp.	Ten	Comp	Ten	Comp	Ten	Comp.	Ten
Crown	Upper		2(.166)				2(1.399)				2(.990)					2(5 249)	
	Lower	2(.231)				2(2.037)				2(2.413)							2(3 246)
Spg.	Upper		7.470	.592			9.689	5.793			3.586	11.227				6 067 2 131	
	Lower	8.942			.536	11.471			5.242	5.721			9.948				3.863 10 169
1	Upper		.298	.406			6.620	3.916			4.763	7.217				1 756 6 770	
	Lower	1.571			.342	8.270			3.332	6.870			5.854			.581	4.711
2	Upper			.904 .099				4.055 .953			2.317	1.053			2 576 .523		
	Lower				.853 .034				2.608 .311	4.345		.410		4.944 2 685			
3	Upper		.055	.674			.551	7.127			1.850	1.960			2.908 3 416		
	Lower	.119			.623	1.164			6.610	3.238			.151	5088 5 444			
4	Upper		.150	.496			1.499	5.443			3.545	7.155			1 700 4.508		
	Lower	.217			.440	2.137			4.873	5.016			5.319	3 967 6.646			
5	Upper		.214	.341			2.053	3.795			4.382	12.237			4 662	.395	
	Lower	.281			.282	2.712			3.202	5.862			10 950	6 798		1 847	
6	Upper		.241	.209			2.329	2.453			4.586	8.871			3 874	3 213	
	Lower	.307			.148	2.981			1.853	6.048			7 558	5 975			1 029
7	Upper		.255	.093			2.443	1.341			4.358	5.766			2 580	6 655	
	Lower	.321			.031	3.105			.730	5.797			4.415	4 668			4 503
8	Upper		.245	.001			2.276	.311			3.762	3 654			.724	10 323	
	Lower	.310		.063		2.929		.311		5.217			2 278	2 789			8 212
9	Upper		.075 .224				.457 2.003				2.805	1.664				10 154 1 514	
	Lower	.140 .289				1.089 2.652				4.250			.265			.528	8 202
10	Upper		.146 .191				1.163 1.647				.277 1.648					6 578 3 883	
	Lower	.209 .254				1.796 2.283				1.679 3.065							4 614 1 889

the sections shown on Designing Sheet No. 16 will control. In fact, section 5 might well be omitted, but is given here with the others to show the influence of the depth of section upon the resulting fiber stresses. Notice that although the maximum stress at the springing, given for loading (*b*) on Designing Sheet No. 15, is very much less than at section 9 for loading (*a*), the effect of neglecting the tension is much greater for the deeper section, and the actual springing stress is not far from the maximum for the entire arch.

The formulas for fiber stress for Case II are as follows:

$$f_c = \frac{M}{Lbt^2}$$

$$f_s = nf_c\left(\frac{d}{kt} - 1\right)$$

The value of L may be determined by means of Diagrams 3 to 6 inclusive. These diagrams are similar to Diagrams 14 and 15

Total Maximum Stresses Assuming Case I to Apply at All Sections

Point	y	t	p_0	Temperature Fall of 20°. Moments and thrusts for rise of 20° of opposite sign			Rib Shortening		Fall of Temperature and Rib Shortening $f_c=\frac{NK}{bt}$ $f_c'=\frac{NK'}{bt}$							Rise of Temperature and Rib Shortening $\left(\frac{-1+.795}{1+.795}=-.11\right)$		Total Maximum Stresses (Case I)								Sections and Loadings for which Case I does not apply
				$H_c=-1150$ $M_c=+3440$			$H_c=-910$ $M_c=+2700$							Stress in		Stress in		*(a) Loading		*(b) Loading		*(c) Loading		*(d) Loading		
				$H_c y$	M	N (Scaled)	M	N	M	N	$\frac{x_0}{t}$	K	K'	Upper Fiber	Lower Fiber	Upper Fiber	Lower Fiber	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	
Cr.		1.33	.0104		+3 400	-1150	+2 700	-910	+6 100	-2060	-2.23	11.17	9.43	+113	-120	-12	+13	475	-39	350	94	231	209	106	342	
Spg.	16.75	3.00	.0046	-19,270	-15,800	-750	-12,600	-600	-28,400	-1350	+7.01	38.04	36.16	-119	+113	+13	-12	358	-76	490	-201	-35	312	97	187	(a) (b)
1	12.16	2.50	.0055	-13,980	-10,500	-860	-8 400	-680	-18 900	-1540	+4.91	26.37	24.53	-113	+105	+12	-12	241	100	366	-17	-9	358	116	241	
2	6.63	1.97	.0070	-7,620	-4,200	-990	-3 300	-790	-7 500	-1780	+2.14	11.59	9.77	-73	+61	+8	-7	88	259	169	191	-33	425	48	357	
3	4.25	1.74	.0079	-4,890	-1,400	-1050	-1 100	-840	-2 500	-1890	+0.76	4.56	2.88	-34	+22	+4	-2	166	153	204	129	-56	422	-18	398	(c)
4	2.80	1.56	.0089	-3,220	+200	-1090	+160	-870	+400	-1960	-0.13	1.50	-0.26	-2	-13	0	+1	279	93	281	107	-49	475	-47	488	
5	1.85	1.47	.0094	-2,130	+1 300	-1110	+1 000	-880	+2 300	-1990	-0.79	4.61	2.85	+27	-43	-3	+5	516	-119	486	-71	124	308	94	356	(a) (b)
6	1.13	1.42	.0097	-1,300	+2 100	-1130	+1 700	-900	+3 800	-2030	-1.32	7.06	5.32	+53	-70	-6	+8	433	-26	374	52	55	354	-4	432	
7	.64	1.37	.0101	-740	+2 700	-1140	+2 200	-910	+4 900	-2050	-1.75	9.01	7.27	+76	-94	-8	+10	452	-35	368	69	96	323	12	427	
8	.30	1.35	.0102	-350	+3 100	-1140	+2 500	-910	+5 600	-2050	-2.02	10.22	8.48	+90	-108	-10	+12	600	-173	500	-53	277	162	177	282	(a) (b)
9	.12	1.33	.0104	-140	+3 300	-1140	+2 600	-910	+5 900	-2050	-2.16	10.85	9.11	+98	-116	-11	+13	619	-168	510	-39	346	98	237	227	(a)
10	.02	1.33	.0104	-20	+3 400	-1150	+2 700	-910	+6 100	-2060	-2.23	11.16	9.44	+101	-120	-11	+13	463	-43	351	90	219	204	107	337	

* Loading (a) - No.1 Loading, fall of temperature, and rib shortening
" (b) - " " rise " " " " "
" (c) - No.2 loading, fall of temperature, and rib shortening
" (d) - " " rise " " " " "

Temperature (fall):

$$H_c = \frac{-(.123)(.000006)(20)(TE.2)(10)(2,000,000)(144)}{2\left[10(222.94)-(29.90)^2\right]} = \frac{-3,069,100}{2670.8} = -1150 \text{ lb.}$$

$$M_c = +\frac{(1150)(29.90)}{10} = +3440 \text{ ft. lb.}$$

Rib Shortening (for live load on one-half span, using Designing Sheet 10):

$$c_a(\text{at crown}) = \frac{.037\,(11,900+14,900)+.369\,(9,900+12,900)+.818\,(8,900+11,900)+1.151\,(8,300+11,300)}{1(1.33)+15\left(\frac{199}{144}\right)} = 31,900$$

$$c_a(\text{at point 4}) = \frac{1}{2}\cdot\frac{(.032+.038)(26,800)+(.324+.371)(22,800)+(1.043+.836)(20,800)+(1.288+1.215)(19,600)}{1(1.56)+.207} = 30,000$$

$$c_a(\text{at springing}) = \frac{1}{2}\cdot\frac{(.783+.030)(26,800)+(.948+.293)(22,800)+(1.142+.680)(20,800)+(1.211+1.044)(19,600)}{1(3.00)+.207} = 20,600$$

$$3\,)\overline{82,500}$$
27,500 lb. per sq. ft.

$$H_c = \frac{-(.123)(27,500)(72.2)(10)}{2670.8} = -914$$

$$\text{Ratio of } \frac{H_c \text{ for rib shortening}}{H_c \text{ for temperature}} = \frac{914}{1150} \doteq 0.795$$

Total Maximum Stresses for Case II Conditions Designing Sheet 16.

Point	t	p_0	Type of Loading	Moments and Thrusts										Total Maximum Stresses (Case II)				
				Loading				Fall of Temperature and Rib Shortening		Rise of Temperature and Rib Shortening		Totals						
				No. 1 Loading		No. 2 Loading												
				M	N	M	N	M	N	M	N	M	N	$\frac{k_0}{t}$	k	L	f_c	f_s
Spg.	3.00	.0046	(b)	+81,600	76,670					+3,200	+150	+84,800	76,820	368	.57	.108	606	5,300
5	1.47	.0094	(a).	+18,600	49,700			+2,300	-1,990			+20,900	47,710	298	74	124	540	1,800
8	1.35	.0102	(a).	+16,200	49,700			+5,600	-2,050			+21,800	47,650	.339	.68	129	644	3,100
9	1.33	.0104	(a)	+15,700	51,070			+5,900	-2,050			+21,600	49,020	331	695	129	658	2,900

of Volume I, but are plotted in such a manner as to give greater accuracy.

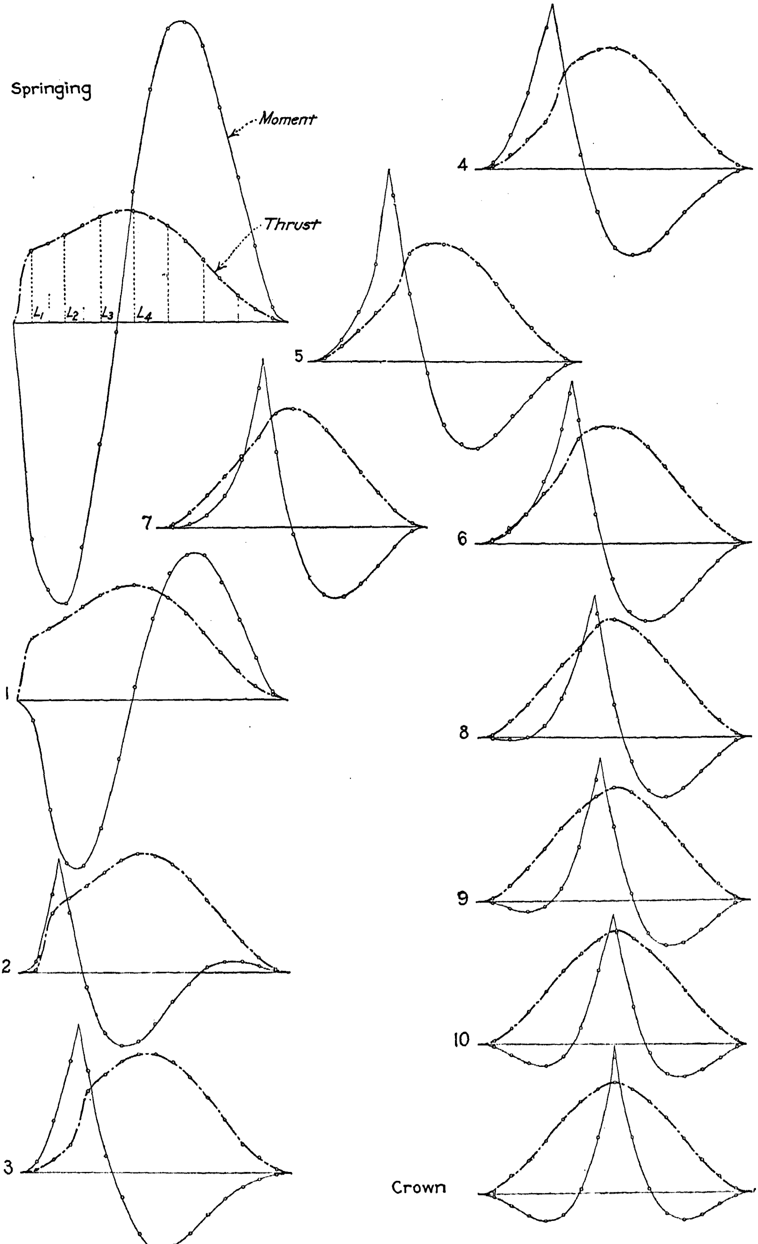
Designing Sheet No. 17 gives the influence lines for moment and thrust for all sections. Although these lines are not actually needed in the problem under consideration, they are presented for instructive purposes. In order to show smooth curves, additional load points have been taken intermediate between the first set, and values of moment and thrust have been computed and plotted at these points.

If the dead weight of the arch ring had been considered as concentrated at seventeen load points (Fig. 21*A*) instead of eight as in the preceding analysis, a maximum fiber stress at section 9 of 640 lb. per square inch would have been the result. This gives some idea of the increased accuracy that may be obtained by doubling the number of load points. A greater number of such points, however, than just sufficient to determine accurately smooth curves for the influence lines are entirely unnecessary. This is due to the fact that, with influence lines properly drawn, loads may be placed at any point and the values scaled with a reasonable degree of accuracy. Thus for continuous loadings, such as occur in earth-filled arches, four to eight load points on each half of the arch may be sufficient to determine properly the curves of the influence lines, but the continuous loading should be broken up into a greater number of loads than represented by the number of load points in order to obtain accurate values of the maximum fiber stresses at the various sections.

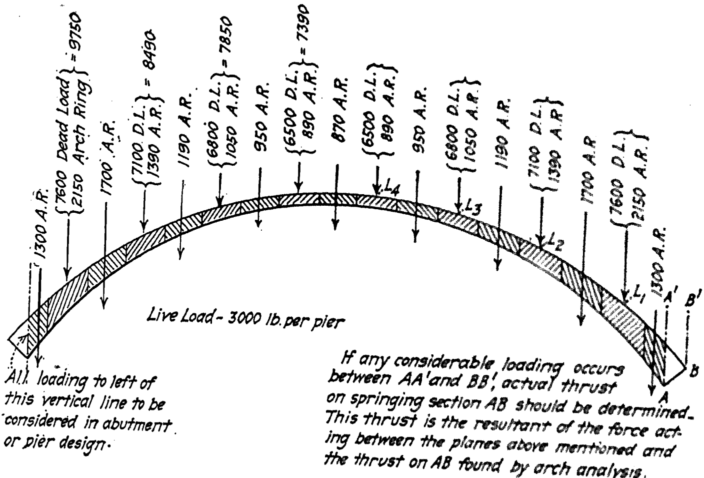
A general discussion in regard to the use of influence lines for moment and thrust may serve to make this part of the subject more clear. Consider influence lines as shown in Fig. 22 and assume uniform live loading. A load placed anywhere on the structure is seen to cause a positive thrust at the given section. For this reason it might seem on first thought that

Influence Lines for Moment and Thrust

Designing Sheet 17



maximum compression on the *upper* fiber will be obtained if the live load extends from C to F. Loads to the left of C and to the right of F, however, will also be found to give compression in the upper fiber, although the resultant moment for such loads is negative. The live load, for example, should extend to points such as B and G where a load placed at such points will be found to give a zero stress at the upper fiber. Likewise, for maximum compression in the *lower* fiber, the live loading should extend from A to D and from E to H. Thus, the termination of the proper steel percentage curve in Diagram 13 of Volume I will give the value of $\frac{M}{Nt} = \frac{x_o}{t}$ for which there is no tension or compression at the outer fiber, and points B, D, E, and G may be determined by trial when influence li



All loading to left of this vertical line to be considered in abutment or pier design.

If any considerable loading occurs between AA' and BB', actual thrust on springing section AB should be determined. This thrust is the resultant of the force acting between the planes above mentioned and the thrust on AB found by arch analysis.

Fig. 21A.

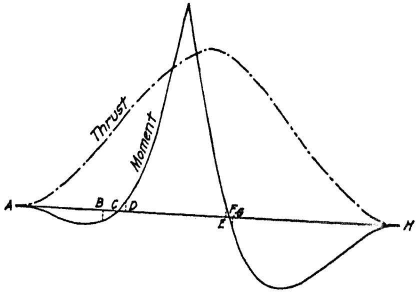
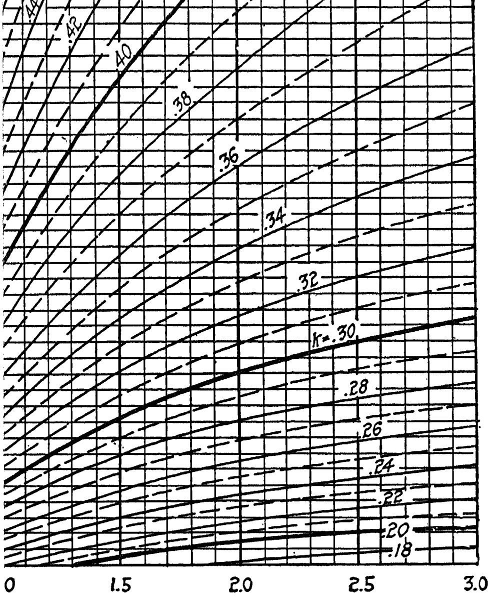
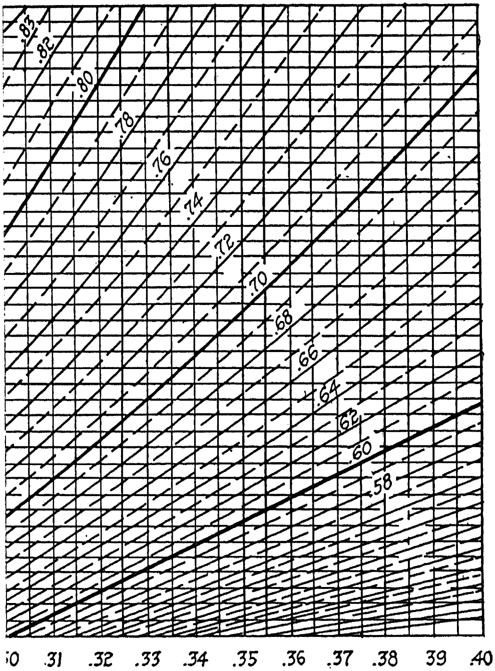


Fig. 22.





(*Facing Page* 84.)

ling
rre-
sec-
1*A*.
s in
ı *D*
is is
ı *A*
wer
om-

lear
axi-
lb.
oad
the
nch
hus
the
s in
has
to
ess.
ions
the
ions
oad

maxi
live l
right
the u

All
this
consi
or p

nega
such
to gi
comp
A to

steel

value

the c

by t

stress. With load points widely separated, as in the preceding analysis, it is not often that load points enter the spaces corresponding to *BD* and *EG*, but this sometimes occurs, as at sections 1, 2, 5, 8, and 9 on Designing Sheets Nos. 11 and 11*A*. For the section represented in Fig. 22, maximum tensile stress in the lower steel would occur for the live load extending from *D* to *E*, and for the upper steel from *A* to *B* and *G* to *H*. This is obvious when it is considered that a load anywhere between *A* and *D* and between *E* and *H* will cause compression in the *lower* steel, while a load anywhere between *D* and *E* will cause compression in the *upper* steel.

Referring to Designing Sheet No. 16, it should now be clear that the steel stresses there given are not necessarily the maximum stresses. For example, a greater steel stress than 2900 lb. per square inch may be obtained at section 9 if only the dead load is placed at point L_4 on the right half of the arch. Removing the live load at this point causes a stress of 640 lb. per square inch in the concrete and a tensile stress of 3300 lb. in the steel. Thus adding the live load is seen to cause a greater stress in the concrete at the upper fiber, but decreases the tensile stress in the lower steel. In some analyses, where the steel stress has its greatest value under such conditions, it may be necessary to consider the exact loading which will give the maximum stress.

Maximum and minimum pressures on abutment foundations are not necessarily obtained for any of the positions of the live load which give maximum stresses at the various sections in the arch. For abutment design, consideration of live load over the half span and the whole span will usually suffice.

CHAPTER VI

UNSYMMETRICAL ARCHES

Unsymmetrical arches are sometimes desirable in the end spans of a series of two or more arches in order to reduce material in abutments and, at the same time, to provide ample waterway area over streams. Also, arches of this type are often necessary under other conditions, as, for example, when a river in a deep ravine is bordered by a railway requiring maximum clearance near the abutments.

35. Method of Analysis.—In the analysis of unsymmetrical arches, the entire arch ring should be divided into a sufficient number of $\frac{s}{I}$ divisions to obtain the desired degree of accuracy. The origin of coördinates may then be taken at the center of any one of the sections occurring between the divisional lengths, but, for convenience in scaling the values of x and y, this origin should be placed at one of the sections near the crown—which we shall call the crown section. The X and Y axes should be drawn perpendicular and parallel respectively to this section so as to permit the crown thrust to be determined directly without composition and resolution of forces. Fig. 23 shows how these axes should be drawn.

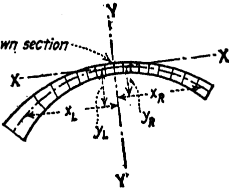


FIG. 23.

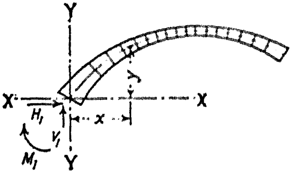


FIG. 24.

The flexure formulas for H_c, V_c, and M_c for unsymmetrical arches, considering the origin of coördinates near the crown, are exceedingly complex and inconvenient for use in practice. The best plan is to use formulas similar to those designated as (*e*), (*f*), and (*g*), Art. 14, and to solve simultaneously for the above values after the numerical values of the coefficients are substituted. Following are the formulas to be solved in this way (see also Designing Sheet No. 18):

$$H_c\Sigma y^2 + V_c(\Sigma x_L y_L - \Sigma x_R y_R) + M_c\Sigma y - \Sigma my = 0$$
$$H_c(\Sigma x_L y_L - \Sigma x_R y_R) + V_c\Sigma x^2 + M_c(\Sigma x_L - \Sigma x_R) - \Sigma m_L x_L + \Sigma m_R x_R = 0$$
$$H_c\Sigma y + V_c(\Sigma x_L - \Sigma x_R) + nM_c - \Sigma m = 0$$

The subscripts L and R in these formulas refer to summations to the left and right of the crown section respectively. No subscript indicates that the summation is to be taken for the entire arch.

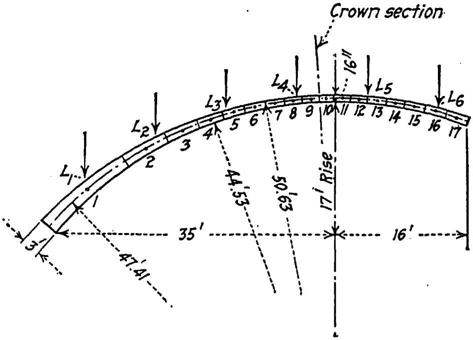


FIG. 25.

The three corresponding equations for temperature are as follows:

$$H_c\Sigma y^2 + V_c(\Sigma x_L y_L - \Sigma x_R y_R) + M_c\Sigma y - \frac{I}{s}\cdot t_c t_D l E_c = 0$$
$$H_c(\Sigma x_L y_L - \Sigma x_R y_R) + V_c\Sigma x^2 + M_c(\Sigma x_L - \Sigma x_R) = 0$$
$$H_c\Sigma y + V_c(\Sigma x_L - \Sigma x_R) + nM_c = 0$$

The value of t_D should be inserted as plus (+) for a rise of temperature; minus (−) for a drop.

Rib shortening causes the same effect as a lowering of the temperature. By referring to Art. 14, it should be clear that the following formula may be employed to solve for the equivalent temperature drop:

$$\frac{c_a l}{E_c} = t_c t_D l, \text{ or } t_D = \frac{c_a}{E_c t_c}$$

The writer believes that for unsymmetrical arches it is convenient to assume the origin of coördinates at the center of the section through the springing. Fig. 24 shows how the coördinates x and y should be measured. The directions of H_1, V_1, and M_1 are shown for values considered as positive in the formulas given on Designing Sheet No. 19. Special note should be made of the fact that values of y measured below the axis $X - X$ should always be considered as negative.

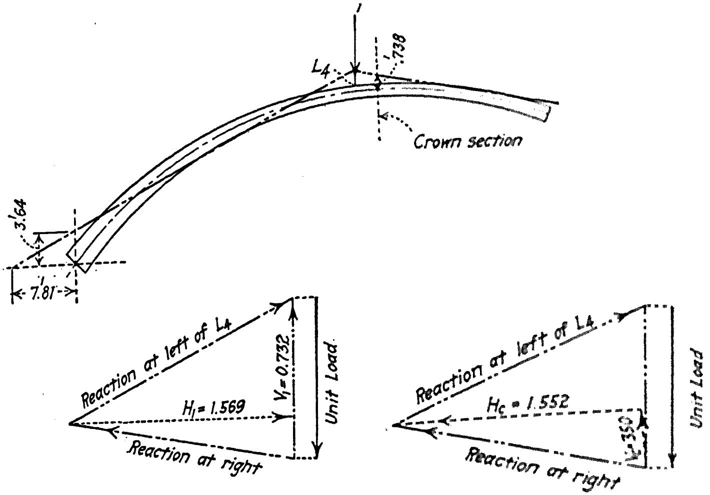


FIG. 26.

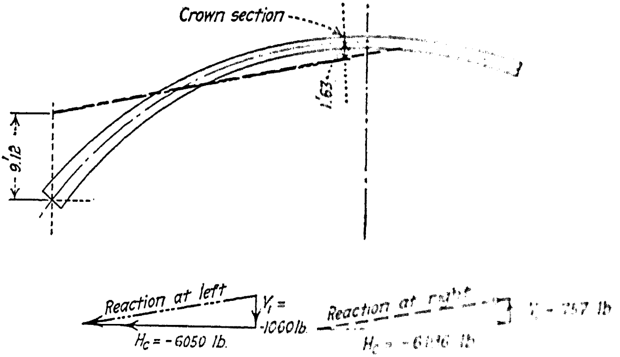


FIG. 27

36. Examples of Computations in Unsymmetrical Arch Analysis.—In order to show the method of analyzing unsymmetrical arches, the computations are given on Designing Sheet No. 20 for finding the thrust, shear, and moment at the crown section of the arch shown in Fig. 25 due to a unit load at L_4. (This unsymmetrical arch was obtained by simply shortening the arch of Fig. 20.) Similar computations are given on Designing Sheet No. 21 for finding H_1, V_1, and M_1 at the springing due to the same loading. Fig. 26 shows the agreement between the two methods, the equilibrium polygon being drawn for the unit load at L_4.

Designing Sheets Nos. 20*A* and 21*A* show the temperature computations by the two methods for the arch in question, assuming a drop of temperature of 40°F. The equilibrium polygon for temperature is a straight line, and Fig. 27 shows the method of determining this line by the two methods.

Designing Sheet 18.

Formulas Required for Unsymmetrical Arches
Origin of Coördinates near Crown

Loading:

$$H_c \Sigma y^2 + V_c(\Sigma x_L y_L - \Sigma x_R y_R) + M_c \Sigma y - \Sigma m y = 0$$

$$H_c(\Sigma x_L y_L - \Sigma x_R y_R) + V_c \Sigma x^2 + M_c(\Sigma x_L - \Sigma x_R) - \Sigma m_L x_L + \Sigma m_R x_R = 0$$

$$H_c \Sigma y + V_c(\Sigma x_L - \Sigma x_R) + n M_c - \Sigma m = 0$$

$$M_L = M_c + H_c y_L + V_c x_L - m_L$$

$$M_R = M_c + H_c y_R - V_c x_R - m_R$$

All values of m_L, m_R, x_L, x_R, y_L, and y_R should be substituted as positive. The subscripts L and R refer to summations to left and right of the crown section respectively. No subscript indicates that summation is for entire arch. Positive value of M_c indicates that the thrust H_c acts above the arch axis. Considering the crown section as vertical, a positive value of V_c indicates that the line of pressure slopes upward towards the left; a negative value, downward towards the left. Signs preceding terms M_c, $V_c x_L$, and $V_c x_R$ in the last two formulas depend upon the signs of M_c and V_c resulting from the three simultaneous equations.

Temperature:

$$H_c \Sigma y^2 + V_c(\Sigma x_L y_L - \Sigma x_R y_R) + M_c \Sigma y - \frac{I}{s} \cdot t_c t_D l E_c = 0$$

$$H_c(\Sigma x_L y_L - \Sigma x_R y_R) + V_c \Sigma x^2 + M_c(\Sigma x_L - \Sigma x_R) = 0$$

$$H_c \Sigma y + V_c(\Sigma x_L - \Sigma x_R) + n M_c = 0$$

$$M_L = M_c + H_c y_L + V_c x_L$$

$$M_R = M_c + H_c y_R - V_c x_R$$

The value of t_D should be inserted as plus (+) for a rise of temperature; minus (−) for a drop. Signs preceding terms M_c, $H_c y_L$, $H_c y_R$, $V_c x_L$, and $V_c x_R$ in the last two formulas depend upon the signs of M_c, H_c, and V_c resulting from the three simultaneous equations. l = span of arch axis measured parallel to X axis.

Rib Shortening:

$$t_D = \frac{c_a}{E_c t_c}$$

Rib shortening causes the same effect as a lowering of the temperature. Solving for t_D gives equivalent temperature drop.

Designing Sheet 19:

Formulas Required for Unsymmetrical Arches

Origin of Coördinates at Left Springing

Loading:

$$H_1\Sigma y^2 - V_1\Sigma xy - M_1\Sigma y + \Sigma my = 0$$

$$H_1\Sigma xy - V_1\Sigma x^2 - M_1\Sigma x + \Sigma mx = 0$$

$$H_1\Sigma y - V_1\Sigma x - nM_1 + \Sigma m = 0$$

or

$$H_1 = \frac{ab-cd}{ae-cf} \qquad V_1 = \frac{d-H_1f}{a} \qquad M_1 = \frac{H_1\Sigma y - V_1\Sigma x + \Sigma m}{n}$$

in which

$$a = \Sigma x\Sigma y - n\Sigma xy \qquad d = \Sigma m\Sigma y - n\Sigma my$$

$$b = \Sigma mx\Sigma y - \Sigma x\Sigma my \qquad e = \Sigma x\Sigma y^2 - \Sigma xy\Sigma y$$

$$c = \Sigma x^2\Sigma y - \Sigma x\Sigma xy \qquad f = n\Sigma y^2 - (\Sigma y)^2$$

Then $M = M_1 + V_1x - H_1y - m$

All values of m and x should be substituted as positive. Values of y below the x – x axis should be taken as negative. The summations refer to entire arch. Positive value of M_1 indicates that the reaction acts to the left of the arch axis at the springing. Positive values of H_1 and V_1 indicate that the reaction acts upward to the right. Signs preceding terms M_1, V_1x, and H_1y in the last formula depend upon the signs of M_1, V_1, and H_1 resulting from the preceding equations.

Temperature:

$$H_1\Sigma y^2 - V_1\Sigma xy - M_1\Sigma y = \frac{I}{s}\cdot t_c t_D l E_c$$

$$H_1\Sigma xy - V_1\Sigma x^2 - M_1\Sigma x = 0$$

$$H_1\Sigma y - V_1\Sigma x - nM_1 = 0$$

or

$$H_1 = \frac{a\Sigma x - nc}{ae-cf}(k) \qquad V_1 = \frac{k\Sigma x - H_1e}{c} \qquad M_1 = \frac{H_1\Sigma y - V_1\Sigma x}{n}$$

in which

$$k = \frac{I}{s} t_c t_D l E_c$$

Then

$$M = M_1 + V_1x - H_1y$$

The value of t_D should be inserted as plus (+) for a rise of temperature; minus (–) for a drop. Signs preceding terms M_1, V_1, and H_1 in the last formula depend upon the signs of M_1, V_1, and H_1 resulting from the preceding equations. l = span of arch axis measured horizontally; that is, parallel to X axis.

Rib Shortening:

$$t_D = \frac{c_a}{E_c t_c}$$

Rib shortening causes the same effect as a lowering of the temperature. Solving for t_D gives equivalent temperature drop.

Determination of Moments and Thrusts at Crown Section - Unsymmetrical Arch
(Unit Load at Definite Points)

Designing Sheet 20

	Point	x	y	xy	x^2	y^2	Unit Load at L_4		
							m	mx	my
Left	1	29.94	10.62	317.96	896.4	112.78	26.30	787.4	279.31
	2	22.26	5.52	122.88	495.5	30.47	18.85	419.6	104.05
	3	17.59	3.37	59.28	309.4	11.36	14.30	251.5	48.19
	4	14.00	2.10	29.40	196.0	4.41	10.79	151.1	22.66
	5	10.95	1.28	14.02	119.9	1.64	7.77	85.1	9.95
	6	8.22	0.72	5.92	67.6	.52	5.09	41.8	3.66
	7	5.74	0.37	2.12	32.9	.14	2.60	14.9	.96
	8	3.36	0.12	.40	11.3	.01	0.26	0.9	.03
	9	[illegible]	[illegible]	.00	[illegible]	.00	0.00	0.0	.00
Σ Left		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	85.96	1752.3	468.81
Right	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]			
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
	5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
	[illegible]	[illegible]	3.22	[illegible]	[illegible]	[illegible]			
Σ Right		[illegible]	8.9[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.0	0.00
Σ Totals			[illegible]		2950.6	180.36	85.96		468.81

$H_c \Sigma y^2 + V_c(\Sigma x_L y_L - \Sigma x_R y_R) + M_c \Sigma y - \Sigma my = 0$

$H_c(\Sigma x_L y_L - \Sigma x_R y_R) + V_c \Sigma x^2 + M_c(\Sigma x_L - \Sigma x_R) - \Sigma m_L x_L + \Sigma m_R x_R = 0$

$H_c \Sigma y + V_c(\Sigma x_L - \Sigma x_R) + n M_c - \Sigma m = 0$

$180.36\, H_c + 431.73\, V_c + 33.01\, M_c - 468.81 = 0$ (a)

$431.73\, H_c + 2950.6\, V_c + 43.60\, M_c - 1752.3 = 0$ (b)

$33.01\, H_c + 43.60\, V_c + 17\, M_c - 85.96 = 0$ (c)

Multiplying (a) by 17 and (c) by 33.01

$3066.12\, H_c + 7339.41\, V_c + (33.01)(17)\, M_c - 7969.77 = 0$

$1089.66\, H_c + 1439.24\, V_c + (33.01)(17)\, M_c - 2837.54 = 0$

$1976.46\, H_c + 5900.17\, V_c - 5132.23 = 0$ (d)

Multiplying (b) by 17 and (c) by 43.60

$7339.41\, H_c + 50{,}160.2\, V_c + (43.60)(17)\, M_c - 29{,}789.1 = 0$

$1439.24\, H_c + 1900.96\, V_c + (43.60)(17)\, M_c - 3{,}747.9 = 0$

$5900.17\, H_c + 48{,}259.24\, V_c - 26{,}041.2 = 0$ (e)

Multiplying (d) by 48,259.24 and (e) by 5900.17

$95{,}382{,}457\, H_c + (48{,}259.24)(5900.17)\, V_c - 247{,}677{,}519 = 0$

$34{,}812{,}006\, H_c + (48{,}259.24)(5900.17)\, V_c - 153{,}647{,}507 = 0$

$60{,}570{,}451\, H_c = 94{,}030{,}012$

$H_c = 1\,552$ lb

Substituting value of H_c in (d)

$3067.47 + 5900.17\, V_c - 5132.23 = 0$

$V_c = \dfrac{2064.76}{5900.17} = 350$ lb

Substituting values of H_c and V_c in (c)

$51.23 + 15.26 + 17\, M_c - 85.96 = 0$

$M_c = \dfrac{19.47}{17} = 1.145$ ft.lb. $\qquad x_o = \dfrac{1.145}{1.552} = .738$ ft.

Determination of Moment and Thrust at Crown Section- Unsymmetrical Arch
(Temperature Fall of 40° F.)

Designing Sheet 20 A·

$$H_C \Sigma y^2 + V_C(\Sigma x_L y_L - \Sigma x_R y_R) + M_C \Sigma y - \frac{I}{S} \cdot t_C t_D \ell E_C = 0$$
$$H_C(\Sigma x_L y_L - \Sigma x_R y_R) + V_C \Sigma x^2 + M_C(\Sigma x_L - \Sigma x_R) = 0$$
$$H_C \Sigma y + V_C(\Sigma x_L - \Sigma x_R) + n M_C = 0$$

$$180.36\ H_C + 431.73\ V_C + 33.01\ M_C + (.125)(.000006)(40)(52.95)(2{,}000{,}000)(144) = 0 \quad ($$
$$431.73\ H_C + 2950.6\ V_C + 43.60\ M_C = 0 \qquad (b)$$
$$33.01\ H_C + 43.60\ V_C + 17\ M_C = 0 \qquad (c)$$

Multiplying (a) by 17 and (c) by 33.01

$$3066.12\ H_C + 7339.41\ V_C + (33.01)(17)\ M_C + 7{,}777{,}296 = 0$$
$$\underline{1089.66\ H_C + 1439.24\ V_C + (33.01)(17)\ M_C = 0}$$
$$1976.46\ H_C + 5900.17\ V_C + 7{,}777{,}296 = 0 \qquad (d)$$

Multiplying (b) by 17 and (c) by 43.60

$$7339.41\ H_C + 50{,}160.2\ V_C + (43.60)(17)\ M_C = 0$$
$$\underline{1439.24\ H_C + 1900.96\ V_C + (43.60)(17)\ M_C = 0}$$
$$5900.17\ H_C + 48{,}259.24\ V_C = 0 \qquad (e)$$

Multiplying (d) by 48,259.24 and (e) by 5900.17

$$95{,}382{,}457.49\ H_C + (48{,}259.24)(5900.17)\ V_C + 375{,}326{,}394{,}215 = 0$$
$$\underline{34{,}812{,}006.03\ H_C + (48{,}259.24)(5900.17)\ V_C = 0}$$
$$60{,}570{,}451\ H_C = -375{,}326{,}394{,}215$$
$$H_C = -6{,}196\ lb$$

Substituting value of H_C in (d)

$$-12{,}246{,}146 + 5900.17\ V_C + 7{,}777{,}296 = 0$$
$$V_C = \frac{4{,}468{,}850}{5900.17} = 757\ lb.$$

Substituting values of H_C and V_C in (c)

$$-204{,}530 + 33{,}005 + 17\ M_C = 0$$
$$M_C = \frac{171{,}525}{17} = 10{,}090\ ft\ lb$$
$$x_0 = -\frac{10{,}090}{6196} = -1.63\ ft.$$

Determination of Moments and Thrusts at Springing – Unsymmetrical Arch
(Unit Load at Definite Points)

Designing Sheet 21·

Pt.	x
L_1	4.38
L_2	13.13
L_3	21.88
L_4	30.63
L_5	39.37
L_6	48.12

Pt.	x	y	xy	x^2	y^2	Unit Load at L_4 m	mx	my
1	4.33	4.50	19.49	18.7	20.25			
2	11.78	10.00	117.80	138.8	100.00			
3	16.33	12.40	202.49	266.7	153.76			
4	19.84	13.82	274.19	393.6	190.99			
5	22.86	14.83	339.01	522.6	219.93			
6	25.54	15.53	396.64	652.3	241.18			
7	[illegible]	[illegible]	[illegible]	[illegible]	256.64			
8	[illegible]	[illegible]	[illegible]	[illegible]	268.63			
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.96	[illegible]	32.52
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	69.39
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	324.5	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Σ	531.38	248.25	8248.06	19407.0	3785.23	96.63	4302.9	1517.64

$$H_1 = \frac{ab - cd}{ae - cf}$$

$$V_1 = \frac{d - H_1 f}{a}$$

$$M_1 = \frac{H_1 \Sigma y - V_1 \Sigma x + \Sigma m}{n}$$

$a = \Sigma x\,\Sigma y - n\,\Sigma xy$

$b = \Sigma mx\,\Sigma y - \Sigma x\,\Sigma my$

$c = \Sigma x^2\,\Sigma y - \Sigma x\,\Sigma xy$

$d = \Sigma m\,\Sigma y - n\,\Sigma my$

$e = \Sigma x\,\Sigma y^2 - \Sigma xy\,\Sigma y$

$f = n\,\Sigma y^2 - (\Sigma y)^2$

$a = (531.38)(248.25) - (17)(8248.06) = -8,301.93$

$b = (4302.9)(248.25) - (531.38)(1517.64) = 261,751.39$

$c = (19,407.0)(248.25) - (531.38)(8248.06) = 434,933.63$

$d = (96.63)(248.25) - (17)(1517.64) = -1,811.48$

$e = (531.38)(3785.23) - (8248.06)(248.25) = -36,185.38$

$f = (17)(3785.23) - (248.25)^2 = 2720.85$

$$H_1 = \frac{(-8,301.93)(261,751.39) - (434,933.63)(-1,811.48)}{(-8,301.93)(-36,185.38) - (434,933.63)(2,720.85)} = \frac{-1,385,168,145}{-882,980,675} = 1.569 \text{ lb}$$

$$V_1 = \frac{(-1,811.48) - (1.569)(2,720.85)}{-8,301.93} = \frac{-6,080.49}{-8,301.93} = 0.732 \text{ lb}$$

$$M_1 = \frac{(1.569)(248.25) - (0.732)(531.38) + 96.63}{17} = \frac{97.16}{17} = 5.715 \text{ ft-lb}$$

$$x_{0\,(\text{horizontally})} = \frac{5.715}{0.732} = 7.81 \text{ ft.} \qquad x_{0\,(\text{vertically})} = \frac{5.715}{1.569} = 3.64 \text{ ft}$$

Determination of Moment and Thrust at Springing – Unsymmetrical Arch

(Temperature fall of 40°F.)

Designing Sheet 21A·

$$H_1 = \frac{a\Sigma x - nc}{ae - cf} \cdot (k) \qquad k = \frac{I}{s} \cdot t_c\, t_D\, \ell\, E_c$$

$$= (.125)(.000006)(-40)(52.4)(2{,}000{,}000)(144)$$

$$= -452{,}736$$

$$V_1 = \frac{k\Sigma x - H_1 e}{c}$$

$$M_1 = \frac{H_1 \Sigma y - V_1 \Sigma x}{n}$$

$$H_1 = \frac{(-8\,301.93)(531.38) - (17)(434{,}933.63)}{-882{,}980{,}675} \cdot (-452{,}736) = -6050 \text{ lb.}$$

$$V_1 = \frac{(-452{,}736)(531.38) - (6050)(-36{,}185.38)}{434{,}933.63} = -1060 \text{ lb.}$$

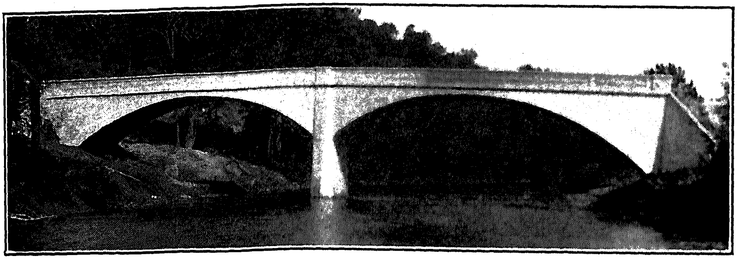
$$M_1 = \frac{(-6050)(248.25) - (-1060)(531.38)}{17} = -55{,}215 \text{ ft. lb.}$$

$$x_0 \text{ (vertically)} = \frac{-55{,}215}{-6050} = 9.12 \text{ ft.}$$

CHAPTER VII

ARCHES WITH ELASTIC PIERS

The necessity for considering relatively thin piers elastic was probably brought to public attention by Mr. Daniel B. Luten of Indianapolis, as early as the year 1904. Undoubtedly the idea came from his experience with the design of the arch bridge across the Wabash River at Peru, Ind., in which the conditions for a satisfactory design required the balancing of thrusts of unequal spans by introducing a moment in the piers.[1] Whether or not the idea was brought forward in this way, it is a fact that Mr. Luten since the Peru bridge experience has built many arch bridges with unusually thin piers, two of the more pronounced structures of this class being shown in Figs. 28 and 29. Mr. A. C. Janni has also made use of the elastic-pier principle in the Kingshighway viaduct, recently constructed at St. Louis, in which the arches and piers were analyzed as a continuous elastic body.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

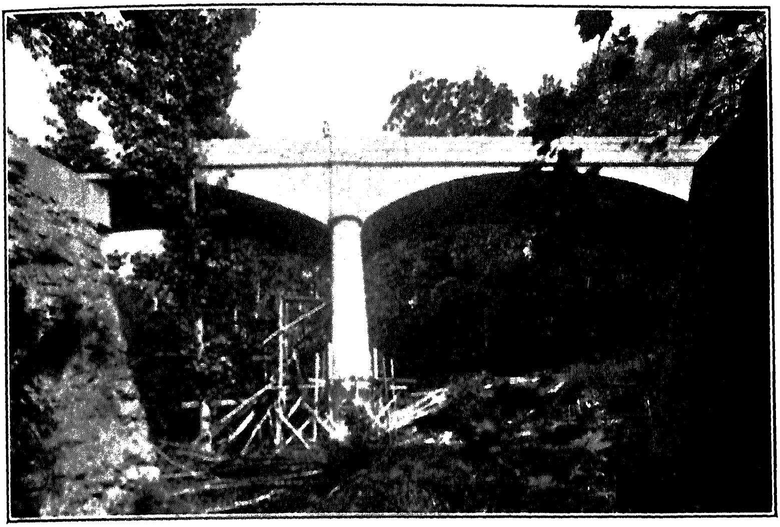
FIG. 28.—Venore bridge over Tellicoe River, Venore, Tenn. (Twin spans of 65 ft. each.)

Arches with elastic piers are undoubtedly somewhat more difficult to analyze than single arches extending only from springing to springing, but this does not change the fact that a great number of arch bridges of multiple spans have piers which should be considered as elastic if a thorough analysis is to be made. In any given structure, good judgment should be exercised in determining whether or not the piers are of such slender proportions that the bending due to unequal arch thrusts should be considered.

---

[1] *Engineering News*, Mar. 29, 1906.

Thin piers not only save concrete in most cases but increase the area of waterway and, on this account, it is quite likely that elastic piers will be much used in the future where architectural appearance is satisfied with this type of construction. In some cases, however, increased thickness of arch rings in elastic-pier construction will offset any advantage gained from diminishing the cross-sectional area of the piers.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 29.—Paint Branch bridge near Washington, D. C.
(Three spans of 60 ft. each.)

37. Method of Analysis.—Very few American engineers have made any attempt to analyze arches having elastic piers, due undoubtedly to a lack of textbook material on the subject. Up to the present time,[1] the method outlined by Mr. A. C. Janni in the Journal of the Western Society of Engineers, May, 1913, and described in the next chapter, is, as far as the writer knows, the only published treatment of arch analysis in this country giving some attention to elastic-pier construction. In the paper referred to, Mr. Janni presents a graphical treatment for the analysis of arches for any and all conditions based on the theory of the ellipse of elasticity. The method to be proposed in this article, however, will not differ in principle from the analysis previously presented in this text for single symmetrical and unsymmetrical arches.

Consider first the two-span arch structure represented in Fig. 30. The points indicated as fixed may be the bottoms of the pier and abutments, or they may be at intermediate sections, depending upon where the designer considers the structure fixed. The method proposed is to take a horizontal section $A - A$ where arches and pier join and study the movement at that section.

In Art. 14 the method employed in deriving formulas for the thrust, shear, and moment at the crown of a symmetrical arch was to cut the arch at the crown and study the horizontal, vertical, and angular moments of the cantilever on the left with respect to similar movements of the cantilever on the right. From the formulas for deflection of curved beams, three independent equations were possible and expressions for the three unknowns (H_c, V_c, and M_c) were then derived.

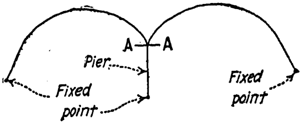


FIG. 30.

A similar method may be followed in finding expressions for the forces acting at the horizontal section $A - A$ in Fig. 30. The thrust from the left arch and the thrust from the right arch combine to produce a resultant thrust on the pier. There are three unknowns with respect to each arch—that is, six unknowns in all, since there is a known relation between the thrust on the pier and the thrusts from the arches. Three independent equations may be written by placing the vertical, horizontal, and angular movements of the pier end of the left arch algebraically equal to similar movements respectively of the pier end of the right arch. The other three independent equations required may be written by placing the vertical, horizontal, and angular movements of the pier end of either arch algebraically equal to similar movements respectively of the top of the pier. These independent equations may be simplified and changed somewhat by placing the vertical movement of the section $A - A$ (due to loading) equal to zero, since this may be done without any appreciable error.

In Fig. 31 the top of the pier is shown in detail. The horizontal arches. What may be called the skewbacks of the arches are shown. The weight of the material between the skewbacks and the section $A - A$ need be considered only in finding the resultant thrusts on the pier sections. The origin of coördinates x and y for each arch may be taken at the middle point C of the section $A - A$ instead of at the center of the skewbacks. This should be obvious from a study of the derivation of the deflection formulas of Chapter II.

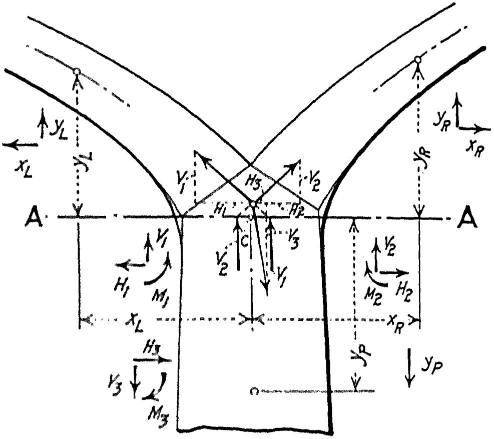


Fig. 31.

The following notation will be employed:

Let

x_L, y_L — coördinates of any point on the axis of the left arch referred to the center of the section $A - A$ as origin. Values of y_L should be considered plus when measured above the axis $A - A$, and as negative when measured below that axis.

x_R, y_R — coördinates of any point on the axis of the right arch referred to the center of the section $A - A$ as origin. Values of y_R should be considered plus when measured above the axis $A - A$, and as negative when measured below that axis.

y_P — depth of any point on the vertical axis of the pier below the section $A - A$.

m_L, m_R = moment at any point on axis of left arch and right arch respectively of all external loads between the point in question and the top of the pier.

M_L, M_R, M_P = moment at any point on axis of left arch, right arch, and pier respectively.

n_L, n_R, n_P = number of $\frac{s}{I}$ divisions in the left arch, right arch, and pier respectively.

c_L, c_R, c_P = values of $\frac{s}{I}$ for left arch, right arch, and pier respectively.

H_1, V_1 = horizontal and vertical components of the thrust from the left arch at the top of pier.

M_1 = moment at section $A - A$ due to thrust from left arch = vertical component of thrust from left arch multiplied by the distance from the point C (the center of the section) to where this thrust produced cuts the section $A - A$.

H_2, V_2 = horizontal and vertical components of the thrust from the right arch at the top of pier.

M_2 = moment at section $A - A$ due to thrust from right arch.

H_3 = resultant shear on section $A - A = H_1 - H_2$.

V_3 = resultant thrust normal on section $A - A = V_1 + V_2$.

M_3 = resultant moment on section $A - A = M_1 - M_2$.

Other notation that will be used is given in Art. 13. The arrows in Fig. 31 indicate what will be considered positive values of the quantities.

Referring to Chapter II on "Deflection of Curved Beams," we have

$$c_L \Sigma M_L y_L = c_R \Sigma M_R y_R$$
$$c_L \Sigma M_L x_L = 0$$
$$c_R \Sigma M_R x_R = 0$$
$$c_L \Sigma M_L = c_R \Sigma M_R$$
$$c_P \Sigma M_P y_P = c_L \Sigma M_L y_L$$
$$c_P \Sigma M_P = c_L \Sigma M_L$$

Designing Sheet 22.

Formulas Required-Arches with Elastic Piers

Loading:

$$c_L\left(M_1\Sigma y_L - H_1\Sigma y_L^2 + V_1\Sigma x_L y_L - \Sigma m_L y_L\right) = -c_R\left(M_2\Sigma y_R - H_2\Sigma y_R^2 + V_2\Sigma x_R y_R - \Sigma m_R y_R\right)$$

$$M_1\Sigma x_L - H_1\Sigma x_L y_L + V_1\Sigma x_L^2 - \Sigma m_L x_L = 0$$

$$M_2\Sigma x_R - H_2\Sigma x_R y_R + V_2\Sigma x_R^2 - \Sigma m_R x_R = 0$$

$$c_L\left(n_L M_1 - H_1\Sigma y_L + V_1\Sigma x_L - \Sigma m_L\right) = -c_R\left(n_R M_2 - H_2\Sigma y_R + V_2\Sigma x_R - \Sigma m_R\right)$$

$$c_p\left[(M_1 - M_2)\Sigma y_p + (H_1 - H_2)\Sigma y_p^2\right] = c_L\left[M_1\Sigma y_L - H_1\Sigma y_L^2 + V_1\Sigma x_L y_L - \Sigma m_L y_L\right]$$

$$c_p\left[n_p(M_1 - M_2) + (H_1 - H_2)\Sigma y_p\right] = -c_L\left[n_L M_1 - H_1\Sigma y_L + V_1\Sigma x_L - \Sigma m_L\right]$$

Bending Moment at any point:

$$M_L = M_1 - H_1 y_L + V_1 x_L - m_L$$

$$M_R = M_2 - H_2 y_R + V_2 x_R - m_R$$

$$M_p = (M_1 - M_2) + (H_1 - H_2) y_p$$

Values of y_L and y_R should be considered plus when measured above the axis A-A, and as negative when measured below that axis. The values of H_3, V_3, and M_3 may be obtained from the following relations:

$H_3 = H_1 - H_2$ $\qquad V_3 = V_1 + V_2$ $\qquad M_3 = M_1 - M_2$

All values of m_L and m_R should be substituted as positive.

The bending moment at any point may be expressed as follows:

$$M_L = M_1 - H_1 y_L + V_1 x_L - m_L$$
$$M_R = M_2 - H_2 y_R + V_2 x_R - m_R$$
$$M_P = M_3 + H_3 y_P = (M_1 - M_2) + (H_1 - H_2) y_P$$

Substituting these expressions in the preceding equations and combining terms, we have the six simultaneous equations given on Designing Sheet No. 22. These six equations may now be solved simultaneously for the values of H_1, V_1, M_1, H_2, V_2, and M_2. The values of H_3, V_3, and M_3 may then be obtained from the following relations:

$$H_3 = H_1 - H_2$$
$$V_3 = V_1 + V_2$$
$$M_3 = M_1 - M_2$$

For arch bridges symmetrical about the center line of pier, the labor involved in solving the simultaneous equations mentioned above will be greatly reduced.

All the simultaneous equations given above pertain to the unknown forces acting at the section $A - A$. With these completely determined, however, the moment and thrust at any section may be found in the manner described for the single symmetrical arch. Each of the three members, of course, must be considered separately and each subjected to exactly the same force that is found to act upon it at the top of pier in the monolithic structure.

Consider now the case of a three-span arch bridge, as represented in Fig. 32. The portion of the structure shown by a solid line should be treated first, solving for the unknowns at section A. The dotted portion should next be considered, solving for the unknowns at section B but placing at A the thrust coming from (or acting upon) the left arch for the given loading. The process may now be repeated by considering the solid portion again and solving for the unknowns at A, but placing at B the thrust coming from (or acting upon) the right arch for the given loading. Likewise in treating the dotted portion the second time, more accurate values of the unknowns at B may be determined by placing at A the second value from (or acting upon) the left arch. It will be only in exceptional cases that the unknowns at sections *A* and *B* will need to be solved more than once in order to obtain sufficient accuracy. In fact, solving for the unknowns at *A* twice and at *B* once is usually all that is desired.

Fig. 33 shows how the method may be applied to four spans and Fig. 34 represents its use in five, or any greater number of spans. It is very seldom that arch bridges of more than six or seven spans are constructed without an intermediate abutment pier. The method for a large number of arches may be simplified by the fact that the effect of a load on any one span extends principally over the span itself and the two spans immediately adjacent.

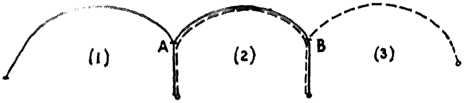


FIG. 32.

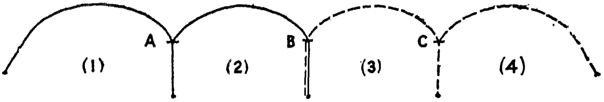


FIG. 33.

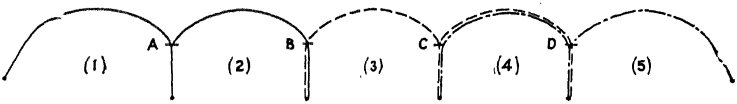


FIG. 34.

In a series of arches with elastic piers, there is seldom, if ever, any radical change in span lengths between successive arches. On this account the unbalanced thrust acting on any pier due to a maximum change in temperature will be relatively small, and each arch of a series when analyzed for temperature stresses may be considered in nearly all cases as having immovable or this method of analysis will give exact results since the piers are not acted upon by any unbalanced thrusts.

Rib shortening causes the same effect as a lowering of the temperature. By referring to Arts. 14 and 34, it should be clear that the following formula may be employed to solve for the equivalent temperature drop:

$$t_D = \frac{c_a}{E_c t_c}$$

38. Analysis of a Two-span Arch Bridge with Pier Elastic.—The earth-filled bridge shown in Designing Sheet No. 23 will be partially analyzed for loading only, assuming a live load of 150 lb. per linear foot over the entire right arch—that is, considering a strip of bridge 1 ft. in width—and dead load only on the left arch. This loading causes the greatest stresses in the pier and also causes the resultant thrust on the base of pier to act at the maximum distance from the center line.

The structure represented is symmetrical about the center line of pier and $x_L = x_R$, $y_L = y_R$. The following values may then be determined in the manner described in previous chapters:

$$\Sigma x = 403.85$$
$$\Sigma y = 100.00$$
$$\Sigma xy = 2258.74$$
$$\Sigma x^2 = 11{,}986.6$$
$$\Sigma y^2 = 671.65$$
$$\Sigma y_P = 64.99$$
$$\Sigma y_P{}^2 = 867.13$$

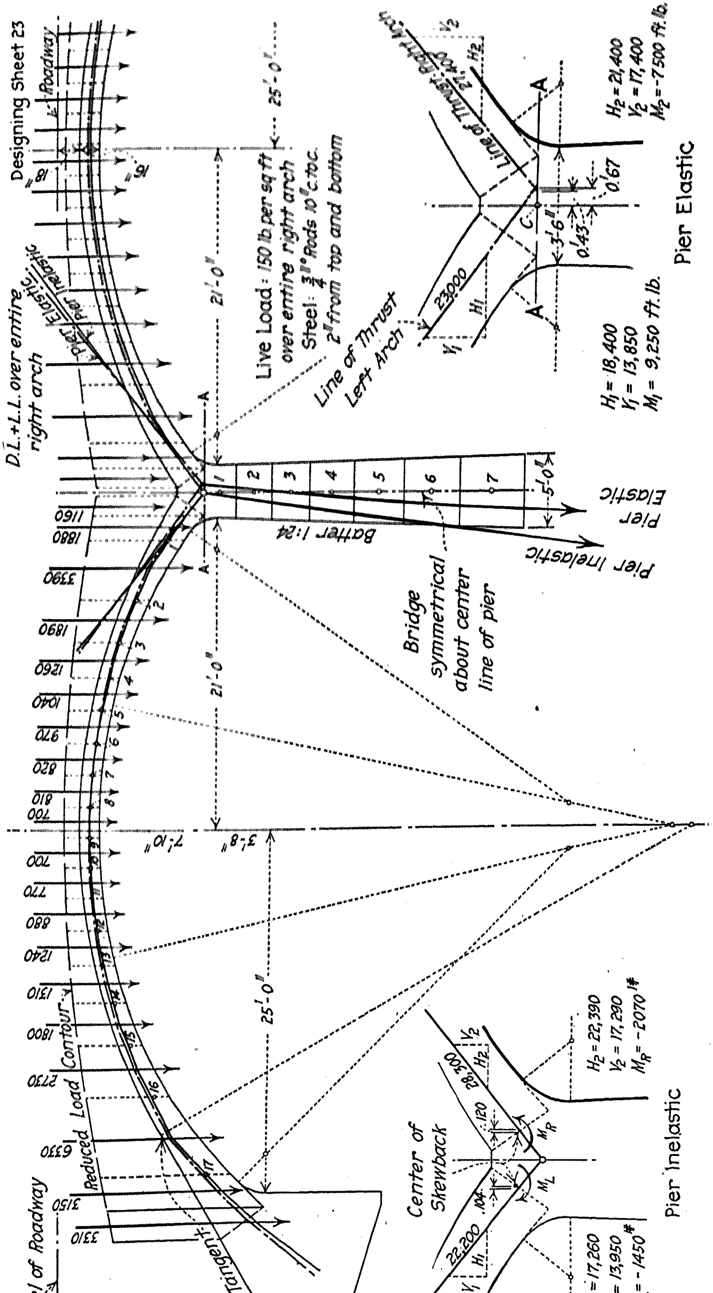
$$\Sigma m_L = 3{,}908{,}000$$
$$\Sigma m_L x = 128{,}241{,}600$$
$$\Sigma m_L y = 19{,}918{,}500$$
$$\Sigma m_R = 4{,}752{,}100$$
$$\Sigma m_R x = 156{,}842{,}600$$
$$\Sigma m_R y = 24{,}056{,}500$$

$$c_L = c_R = 9.40$$
$$c_P = 0.41$$
$$n_L = n_R = 17$$
$$n_P = 7$$

Substituting these values in the six simultaneous equations of Designing Sheet No. 22 and solving, we have

$$H_1 = 18{,}400 \text{ lb.} \qquad H_2 = 21{,}400 \text{ lb.}$$
$$V_1 = 13{,}850 \text{ lb.} \qquad V_2 = 17{,}400 \text{ lb.}$$
$$M_1 = 9250 \text{ ft.-lb.} \qquad M_2 = -7500 \text{ ft.-lb.}$$

Thus both arch thrusts act to the right of the center of the section $A - A$ (Fig. 35). The thrust from the left arch acts through a point $\frac{9250}{13{,}850} = 0.67$ ft. from C and the thrust from the right arch acts through a point $\frac{7500}{17{,}400} = 0.43$ ft. from the same point, both distances measured to the right.


Designing Sheet 23
D.L.+L.L. over entire right arch
Roadway
of Roadway
18"
16"
Elastic pier
Inelastic pier
Live Load: 150 lb per sq ft over entire right arch
Steel: 3/4" Rods 10" c. to c.
2" from top and bottom
Line of Thrust Left Arch
Line of Thrust Right Arch
21'-0"
21'-0"
25'-0"
25'-0"
7'-10"
3'-8"
5'-0"
Batter 1:24
Pier Elastic
Pier Inelastic
Bridge symmetrical about center line of pier
Reduced Load Contour
Tangent
6330
3150
3310
2730
1800
1310
1240
880
770
700
700
810
820
970
1040
1260
1890
3390
1880
1160
1
2
3
4
5
6
7
A
A
C
23000
H_1
V_1
H_2
V_2
3'-6"
0.43
0.67
H_1 = 18,400
V_1 = 13,850
M_1 = 9,250 ft. lb.
H_2 = 21,400
V_2 = 17,400
M_2 = -7,500 ft. lb.
Pier Elastic
Center of Skewback
22,200
28,300
.104
.120
M_L
M_R
H_2 = 22,390
V_2 = 17,290
M_R = -2070#
= 17,260
= 13,950
= -1450#
Pier Inelastic

Designing Sheet No. 23 shows the lines of pressure in the arches and the pier. The resultant thrust on the base of pier is seen to act outside the middle-third point. This should not be allowed in good construction and either the shape of the arches should be changed or a reinforced-concrete footing should be added to the pier to make a more nearly uniform distribution of pressure over the pier base. If the structure should be analyzed assuming the arches to have fixed ends, the resultant thrust would fall even farther from the middle third point, as shown, indicating that the elastic properties of the pier tend to prevent failure in so far as that is possible. It should be noted that if the pier should tip due to unequal intensity of pressure on its foundation, the thrust from the right arch would be somewhat reduced and the thrust from the left arch increased. This would cause a certain righting moment at the top of pier and would tend to prevent failure. Relying on such a possible factor of safety, however, would not be acting according to good practice as unknown stresses might be set up in the arch rings which would otherwise endanger the structure. Of course, it is possible to determine these stresses for any predetermined amount of unequal settling.

If desired, each arch in the above design may be considered as fixed for dead load only. The live-load stresses should then be combined with those due to dead load in order to give the correct maximum stresses. If the arches had been taken unequal, the resultant for dead load should be made to pass through the middle third of the pier base if the arches are to be considered as fixed for this loading. This may be done by overloading the arch having the shorter span.

CHAPTER VIII

ARCH ANALYSIS BY THE METHOD OF THE ELLIPSE OF ELASTICITY[1]

The ellipse-of-elasticity method of arch analysis is almost entirely graphical and is unique in the fact that by its use influence lines may be readily constructed for finding the stresses at any section. Even arches with elastic piers may be rigorously analyzed by this method without any great difficulty. The method is rapid and easily applied, and is likely to be in great favor with American engineers as soon as it becomes well known. A number of underlying principles must be explained before this method can be completely understood.

39. Preliminary Considerations Pertaining to the Method of Treating the Symmetrical Arch.—Let Figs. 13 and 14 in Chapter II represent any given element of an arch, and let k' represent the angle of rotation of the face bc with respect to the face ad which would be caused by a bending moment M. From Art. 10 we know that

$$k' = \frac{Ms}{E_cI} \tag{1}$$

Now if we consider the symmetrical arch shown in Fig. 35A as fixed at the right end and free at the left end, each element will cause a rotation of the left skewback equal to $\frac{Ms}{E_cI}$, where M is the moment acting upon a given element, s its length, and I the moment of inertia of its average cross-section. If the element considered is $abcd$, then s is the distance between the intersections of the neutral axis of the arch with the faces ad and bc, and I is the moment of inertia of the cross-section at the center of gravity of the element. Since the quantity $\frac{s}{E_cI}$ is a characteristic of the given element and shows the amount of its ability to cause rotation if acted upon by a moment, we may treat this quantity as a force acting at the center of gravity of the area *abcd*. We shall call this quantity the *elastic weight* of the element.

---

[1] Method taken by permission from a paper presented before the Western Society of Engineers, January 13, 1913, by Mr. A. C. Janni, C. E. Printed in the Journal of the Western Society of Engineers, May, 1913. Most of the data in this chapter other than that

Suppose the bending moment acting on the element *abcd* is due to a force P having a leverage of x' about the center of gravity of the element. Then the rotation of the left skewback due only to the bending of this element will be $Px'\cdot\frac{s}{E_cI}$. Considering $\frac{s}{E_cI}$ (the elastic weight of *abcd*) as a force, this angle of rotation may also be determined by multiplying the force P by the moment of the elastic weight of *abcd* about an axis through the line of action of P, or

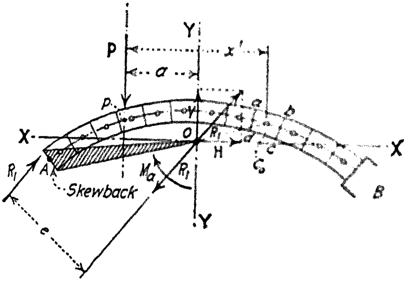
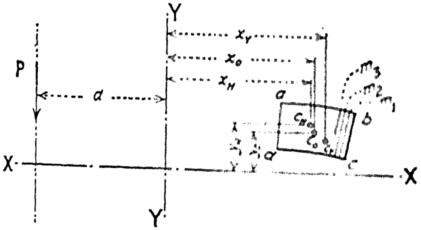


FIG. 35A.



C_0 = *center of gravity of the element abcd, which is also the center of gravity of the* $\frac{s}{E_cI}$ *forces corresponding to the small elements* m_1, m_2, m_3, *etc.*

$\left.\begin{matrix}C_Y\\C_H\end{matrix}\right\}$= *center of gravity of the moments of the small* $\frac{s}{E_cI}$ *forces about the axis* $\left\{\begin{matrix}Y\text{-}Y\\X\text{-}X\end{matrix}\right.$

FIG. 35B.

$$k' = P\cdot\frac{s}{E_cI}x'$$

Designating the elastic weight by the letter G, we have

$$k' = P(Gx') \tag{2}$$

The total angle of rotation of the left skewback due to the force P will equal the sum of all the moments of the elastic weights to the right of the line of action of the force P with respect to this line of action, or

$$k = P\sum_{p}^{B} Gx' \tag{3}$$

In Art. 10 the moment of inertia has been defined as follows: "The moment of inertia of a plane surface with respect to an axis is the sum of the products obtained by multiplying each elementary area by the square of its distance from that axis." If each elementary area is considered a force, then the moment of inertia becomes the sum of the products obtained by multiplying each force by the square of its distance from the given axis. From this interpretation of the definition we may consider the same term to apply in the case of the elastic weights of the arch elements. In other words, whenever we obtain the sum of the products mentioned above for a system of elastic weights with respect to a given axis, we shall designate such sum as the moment of inertia of these weights about the axis in question. Thus if each product Gx' in the expression $\sum_{p}^{B} Gx'$ in the above formula should be multiplied again by x' the result of the expression would be the moment of inertia of all the elastic weights to the right of the line of action of P with respect to this line of action. If any given horizontal axis is also considered, the expression $\Sigma Gx'y$, or in general ΣGxy, will be called the *product of inertia*. If two vertical axes are considered, the expression for the product of inertia takes the form $\Sigma Gxx'$. It should be clear that "moment of inertia" is found with respect to one axis only, while "product of inertia" is found with respect to two axes at any inclination whatever and making any angle with each other.

In Art. 11 it is shown that the vertical and horizontal displacements (Fig. 15) of the free end of the arch due to the moment M acting upon a given element may be expressed respectively by the formulas

$$dy = x\frac{Ms}{EI} \quad \text{and} \quad dx = y\frac{Ms}{EI} \tag{4}$$

where x and y refer to coördinate axes with the origin at the free end. These formulas if applied to the element *abcd* of Fig. 35*A* may be put in the form

$$dy = x \cdot Px' \cdot G \quad \text{and} \quad dx = y \cdot Px' \cdot G$$

Suppose now that a certain point O is assumed to be rigidly connected with the cross-section of the left skewback and let it be desired to find the displacements of this point O due only to the rotation of the element *abcd,* this rotation being caused by the bending moment Px'.

First of all it should be noted that the angle of rotation of the free end (A) of the arch which is determined above by the formula $k' = P(Gx')$ refers to the angle of rotation at any point to the left of the element *abcd,* and also to the point O since this point is in rigid connection with A. If we consider C_o, the center of gravity of the element *abcd,* as the center of rotation (considered so simply for illustration as the student will find later that the center of rotation is not at C_o), then the preceding statement means that the line C_oO turns through the same angle as a line connecting C_o and A. Thus the above formulas for vertical and horizontal displacements can be applied with respect to axes through the point O as well as through A or any other point which is rigidly connected with the arch and is to the left of the given element. We have then the formulas

$$dy = P \cdot Gx' \cdot x \quad \text{and} \quad dx = P \cdot Gx' \cdot y \tag{5}$$

which give the displacements at point O provided x and y are measured with respect to the axes $X - X$ and $Y - Y$.

The total vertical and horizontal displacements of O due to the force P causing bending in all elements to the right of its line of action are

$$\Delta y = -P \sum_p^B Gx \cdot x' \quad \text{and} \quad \Delta x = P \sum_p^B Gy \cdot x' \tag{6}$$

considering positive values of y above the axis $X - X$ and positive values of x to the right of $Y - Y$. The expression $\sum_p^B Gy \cdot x'$ represents the product of inertia of the elastic weights at the right of P with respect to the axis $X - X$ and to the axis through the line of action of P. The expression $\sum_p^B Gx \cdot x'$ represents the product of inertia of the same elastic weights above mentioned with respect to the axis $Y - Y$ and to the axis through the line of action of P.

Suppose now that a force or reaction R_1 acts upon the structure so as to cause equilibrium, and suppose this same reaction R_1 is applied twice at the point O (rigidly connected with left skewback), but acting in opposite directions as shown. It will be possible to resolve one of these two forces at O into two components V and H, and then the opposite force R_1 can be combined with the actual skewback reaction so as to form a couple R_1e. Thus at the point O we have the three unknowns V, H, and $M_a = R_1e$.

The angle of rotation at the point O due to the moment $M_a = R_1e$ is given by the formula

$$k = M_a \sum_A^B G \tag{7}$$

where $\sum_A^B G$ represents the sum of all the elastic weights from A to B. This expression comes directly from Formula (1), taking into consideration the fact that the moment R_1e acts upon the entire arch. Therefore

$$M_a \sum_A^B G = P \sum_p^B Gx' \tag{8}$$

since the angles of rotation at O due to the force P and to the reaction moment $M_a = R_1e$ must be the same.

It will be convenient to take the point O at the center of gravity of all the elastic weights. By center of gravity is meant the point through which the resultant elastic weight will always pass no matter in what parallel direction the weights are supposed to act. This point may be readily determined by finding the lines of action of the vertical and horizontal resultants. The intersection of these two resultants is the center of gravity of the system.

With the point O located as above mentioned, the vertical displacement of this point due to the vertical reaction V is, by Formula (6),

$$\Delta y = V \sum_A^B Gx^2 \tag{9}$$

This means that the vertical displacement of O from this cause is moment of inertia of the whole elastic system with respect to the vertical axis through O. This displacement must be equal to that previously found due to the load P, or

$$V\sum_{A}^{B} Gx^2 = P\sum_{p}^{B} Gx\cdot x' \tag{10}$$

The horizontal reaction H does not cause any vertical displacement at the point O since it acts at right angles to V and through the center of gravity of the system of elastic weights. The expression for the vertical displacement of point O due to force H is as follows: $H\sum_{A}^{B} Gxy$. The arch, however, is symmetrical, so that $\sum_{A}^{B} Gxy$ reduces to zero and no vertical displacement results.

The horizontal displacement of O due to the horizontal reaction H is, by Formula (6),

$$\Delta x = H\sum_{A}^{B} Gy^2 \tag{11}$$

that is, the horizontal displacement of O is equal to the product obtained by multiplying the force H by the moment of inertia of the elastic weights of the whole elastic system with respect to the horizontal axis through O. Placing this equal to the horizontal displacement previously found, we have

$$H\sum_{A}^{B} Gy^2 = -P\sum_{p}^{B} Gy\cdot x' \tag{12}$$

With a known force P (which should be considered as unity in order to plot influence lines), Formulas (8), (10), and (12) may be employed to determine the values of $M_a = R_1e$, V, and H. The moments and products of inertia of the elastic weights may be found readily by graphical methods. The magnitude of the reaction R_1 is equal to the resultant of V and H, and its actual position may be determined by drawing a tangent parallel to this resultant at a distance

$$e = \frac{M_a}{R_1}$$

from the point O. The reaction at the right of the structure may be obtained in the manner shown on Designing Sheet No.

Following is another method of deriving Formulas (8), (10), and (12): Consider the element *abcd* (Fig. 35*A*) with the forces P and R_1 simultaneously applied. The bending moment acting on this element will be

$$M = -Px' + M_a + Vx - Hy$$

If this value of M is inserted in our three fundamental equations for the deflection of curved beams, we have

$$k = \sum_A^B (-Px' + M_a + Vx - Hy)G = 0$$

$$\Delta y = \sum_A^B (-Px' + M_a + Vx - Hy)Gx = 0$$

$$\Delta x = \sum_A^B (-Px' + M_a + Vx - Hy)Gy = 0$$

The arch being symmetrical, and with the coördinate axes at the point O, we know that $\sum_A^B Gx = 0$, $\sum_A^B Gy = 0$, and $\sum_A^B Gxy = 0$. Substituting these values, we obtain

$$\sum_A^B M_aG - P\sum_A^B Gx' = 0$$

$$V\sum_A^B Gx^2 - P\sum_A^B Gx \,.\, x' = 0$$

$$-H\sum_A^B Gy^2 - P\sum_A^B Gy \cdot x' = 0$$

Since Px' becomes zero for all elements to the left of the force P,

$$M_a\sum_A^B G = P\sum_p^B Gx'$$

$$V\sum_A^B Gx^2 = P\sum_p^B Gx \cdot x'$$

$$H\sum_A^B Gy^2 = -P\sum_p^B Gy \cdot x'$$

These equations for k, Δy, and Δx are the same as Formulas (8), (10), and (12) respectively.

It should be remembered that each part of a given element has an elastic weight of its own. In the element *abcd* (Fig. 35*B*), for example, the elastic weights of the smaller elements should be considered at their respective centers of gravity. Let g_1, g_2, and g_3, etc., represent respectively the elastic weights of the small elements m_1, m_2, and m_3, etc., and let C_o be the center of gravity of all these weights. Consider now the moments of these elastic weights about the axis $Y - Y = g_1x_1 + g_2x_2 + g_3x_3 +$

etc. $= Gx_o$, and the moments about the axis $X - X = g_1y_1 + g_2y_2 + g_3y_3 +$ etc. $= Gy_o$. If each small element is now assumed to have an elastic weight acting in a vertical direction equal to the moment of its real elastic weight about the axis $Y - Y$, the center of gravity of these elastic moments will not fall necessarily at the center of gravity (C_o) of the element *abcd*, but at some point as C_V. Likewise the center of gravity of the elastic moments with respect to the axis $X - X$ will fall at some point as C_H. This means that, in finding the moment of inertia of all the elastic weights of these smaller elements, the elastic moments should not be considered to act at the center of gravity C_o of the total elastic weight of the element, but at C_V or C_H as the case may be. If the distance from the point C_V to the axis $Y - Y$ is designated as x_V, then the moment of inertia of the elastic weight of the element *abcd* about this axis is

$$I_Y = Gx_ox_V$$

Similarly, the moment of inertia of the elastic weight of *abcd* about the axis $X - X$ is

$$I_X = Gy_oy_H$$

The product of inertia of the elastic weight of the same element with respect to the axes $X - X$ and the line of action of P is

$$I_{XP} = Gy_o \cdot (x_H + a)$$

and with respect to $Y - Y$ and the line of action of P is

$$I_{YP} = Gx_o \cdot (x_V + a)$$

The points C_V and C_H may readily be determined graphically, as shown later on. These points will be termed *centers* in the articles which follow.

The finding of these centers is a refinement over the method given in the preceding chapters of this book. It should be noted that the elements, as *abcd* (Fig. 35*A*), may be of any given length and the work of dividing the arch into constant $\frac{s}{I}$ divisions is avoided.

40. Properties of an Ellipse.—In the preceding article the reason has been given for finding the points C_V and C_H (Fig. 35*B*) for each element of the arch. A simple graphical construction will suffice to find these points for any given element. Before presenting this method, however, it may be desirable to explain certain properties of an ellipse.

If through a fixed point (C', Figs. 36*A* and 36*B*) any number of secants be drawn to an ellipse, and tangents to the latter be drawn at the points of intersection of each secant with the ellipse, then each pair of tangents will intersect upon a straight line (RS and MN, Figs. 36*A* and 36*B*), called the *polar* of the fixed point (C'). Reciprocally the fixed point is called the *pole* of the straight line. The fixed point symmetrical with the pole with respect to the center of the ellipse is called the *antipole* of the straight line.

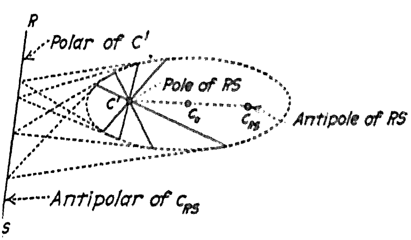


FIG. 36*A*.

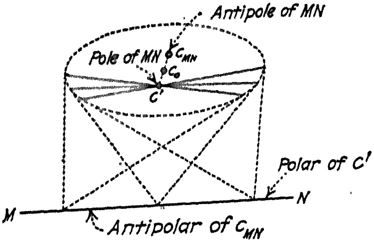


FIG. 36*B*.

In Figs. 36*A* and 36*B* the pole C' is shown inside the ellipse, but the same property holds true for any point outside. The meaning of the term *antipolar* is clearly shown in these figures and no definition is necessary.

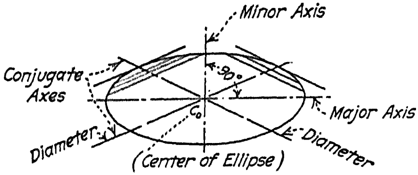


FIG. 36*C*.

If the pole lies on the ellipse, it follows from the above definition that the polar is simply the tangent to the ellipse at the pole. For such a case the antipole is also on the ellipse since it is diametrically opposite the pole and at the same distance from the center.

A diameter is any straight line passing through the center of the ellipse, and is the locus of the middle points of a system of an ellipse are the only diameters perpendicular to the chords they bisect.

Two diameters are said to be *conjugate* when each diameter bisects all chords parallel to the other. The tangent at the extremity of any diameter is parallel to the conjugate diameter.

If the antipole relative to a polar falls upon another polar, the antipole relative to the latter will fall upon the first. Thus in Fig. 37A, the antipole of RS falls upon MN. Reciprocally the antipole of MN falls upon RS. Any line as ZZ (or even $C_{MN}C_o$ which passes through the center of the ellipse) passing through C_{MN}, the antipole of the line MN, will have its antipole somewhere on the line MN. (The above statements hold with respect to poles and polars, as well as antipoles and polars or poles and antipolars.)

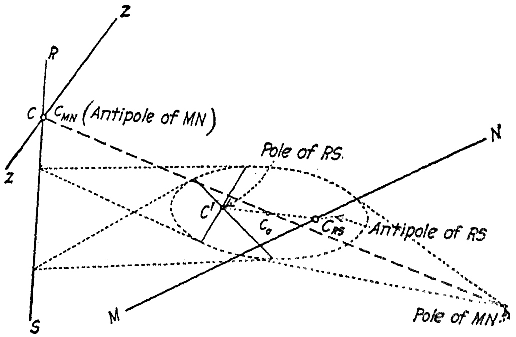


FIG. 37A.

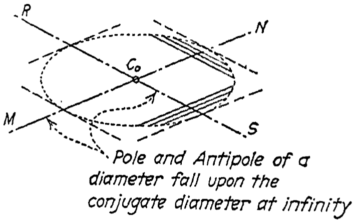

R
N
C_o
M
S
Pole and Antipole of a diameter fall upon the conjugate diameter at infinity

FIG. 37B.

The pole and antipole of a diameter fall upon the conjugate diameter at infinity (Fig. 37B). In Fig. 37A a line through the center C_o parallel to MN and the line $C_{MN}C_o$ are conjugate axes.

If the lengths of the semi-major and semi-minor axes of an ellipse are known, the antipole of any line or axis can be determined by a simple graphical construction. For example, in Fig. 38 the antipole (C_{MN}) of MN may be found in the following manner:

Lay off $C_oB' = C_oB$, and $C_oD' = C_oD$.
Connect B' with F, and D' with G.
Draw $B'B''$ perpendicular to $B'F$, and $D'D''$ perpendicular to $D'G$.
From B'' draw a line parallel to C_oD, and from D'' draw a line parallel to AB.
The intersection of these lines is the antipole C_{MN}.

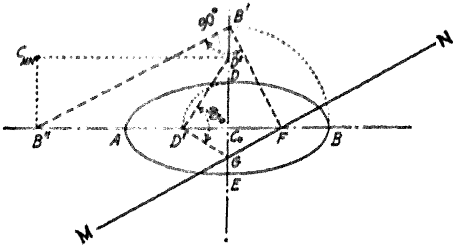


FIG. 38.

41. Properties of the Center of Gravity of Static Moments.—

Let C_o of Fig. 39 be the center of gravity of the given element, which is the center of gravity of all the small elastic weights. Also let C_{RS} be the *center* (see Art. 39) relative to the axis RS, and C_{MN} the *center* relative to the axis MN. Then the product of inertia of all the small elastic weights of the given element about RS and MN is

$$Gx_oy_{RS} = Gy_ox_{MN}$$

This equation shows that if y_{RS} should be zero, then also x_{MN} would be equal to zero. Thus it follows that if the center relative to an axis falls upon another axis, the center relative to the latter will fall upon the first. (It should be noted that this is the same relation which exists between the poles and polars, or antipoles and polars, of an ellipse, Fig. 37.) Also any axis passing through the center relative to another axis will have its center somewhere on that other axis. For such axes, which are *conjugate* axes, the

The moment of inertia of the weights about the axis RS may be expressed as follows:

$$I_{RS} = Gx_o x_{RS}$$

or

$$x_{RS} = \frac{I_{RS}}{Gx_o}$$

If the axis RS should pass through the center of gravity C_o, then x_o would equal zero and x_{RS} would equal infinity, which is the same relation which exists between the poles and diameters, or antipoles and diameters, of an ellipse (Fig. 37B).

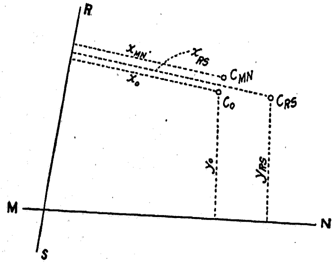


FIG. 39.

It should now be clear that the centers C_V and C_H (Fig. 35B) for the given element with respect to given axes must be either the poles or antipoles of some ellipse which has its center at the center of gravity of the element and one axis lying in the geometrical axis of the arch. The lengths of the major and minor axes of this ellipse, however, we have not yet determined.

42. Determination of the Ellipse of Elasticity.—Points C_V and C_H in Fig. 35B are termed *centers* in Art. 39. They are in reality centers of rotation of the element *abcd* if forces are considered to act on the element along the axes $Y - Y$ and $X - X$ respectively. For example, a force along $Y - Y$ causes a bending moment in each of the small elements m_1, m_2, m_3, etc., and the center of gravity of all these bending moments considered as forces will be the instantaneous center of rotation of the element *abcd*—that is, the point C_V.

In order to show conclusively that this is true, consider the very small elements m_1 and m_2 of *abcd* (Fig. 40), with centers of gravity C_1 and C_2 respectively, and assume that bending is caused in these elements by a force acting along an axis $Y - Y$. The vertical displacement of some point A to the left of *abcd* due to the force acting on the small element $m_1 = M_1x_1g$ (see Formula 4, Art. 39), where M_1 is the moment acting on this given element. Likewise, the vertical displacement of A due to the force acting on the small element $m_2 = M_2x_2g$. Thus the total vertical displacement of A due to the force acting on both m_1 and $m_2 = M_1x_1g + M_2x_2g = g(M_1x_1 + M_2x_2)$. In like manner it may be shown that the total horizontal displacement of A due to the bending in the elements

Suppose we select a point C_{1-2} on the straight line joining C_1 and C_2, such that $\frac{C_1C_{1-2}}{C_{1-2}C_2} = \frac{M_2}{M_1}$. In other words, we will choose a point C_{1-2} which is the center of gravity of forces equal to M_1 and M_2 applied at C_1 and C_2 respectively. If the point A should rotate about this point due to the force acting on the elements m_1 and m_2, the vertical displacement of A would be

$$M_1x_{1-2}\,g + M_2x_{1-2}\,g = (M_1 + M_2)x_{1-2}\,g$$

Since $x_{1-2} = x_2 + \frac{M_1}{M_1 + M_2}(x_1 - x_2)$, we have

$$(M_1 + M_2)x_{1-2}\,g = g(M_1x_1 + M_2x_2)$$

In a similar manner, the horizontal displacement of A under the same conditions would be

$$(M_1 + M_2)y_{1-2}\,g = g(M_1y_1 + M_2y_2)$$

But these values are identical with those found above for the sums of the components of the two displacements. Thus the rotation caused by the force along $Y - Y$ acting on the elements m_1 and m_2 is about a point C_{1-2} which is the center of gravity of the moments M_1 and M_2, when these moments are considered as forces and applied at the centers of gravity of their respective elements. It should be evident that this same proof will hold with respect to any number of small elements and we have then the rule that the center of gravity of the bending moments of the small elements, when these moments are considered as forces, is the center of rotation of the large element *abcd*.

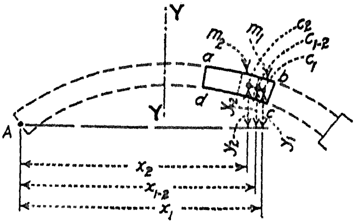


FIG. 40.

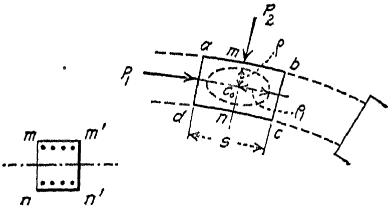


FIG. 41.

Now in Art. 41 it is shown that instantaneous centers C_V or C_H, Fig. 35*B*, are either poles or antipoles of some ellipse with respect, of course, to the lines $X - X$ and $Y - Y$. If the centers were the poles of $X - X$ and $Y - Y$, there would be positions of each axis (polar) for which its corresponding center (pole) would lie on itself. (A polar is tangent to an ellipse at its pole, Art. 40.) This, however, cannot happen since a center of rotation can never lie on the line of action of a force. The centers, therefore, are the antipoles of the ellipse for the given

In order to determine the lengths of the major and minor axes of the ellipse, consider a force P_1 lying in the arch axis and acting on the section *ad* of the element *abcd*, Fig. 41. Since P_1 acts through the center of gravity C_o of the element, the shortening due to this force will be

$$\Delta s = \frac{P_1 s}{E_c A} = P_1 \cdot \frac{s}{E_c I} \cdot \frac{I}{A} = P_1 \cdot G \cdot \frac{I}{A}$$

where A (representing the cross-sectional area) $= a_c + na_s$, and $I = I_c + nI_s$. From Art. 39, we also know that the displacement of the face *ad* is equal to the product of the force by the moment of inertia of the elastic weight G with respect to the line of action of the force P_1. If ρ is the semi-minor axis (radius of gyration) of the ellipse of elasticity in question, then $G\rho^2$ will be the moment of inertia we desire, and $P_1 G\rho^2$ will be the displacement of the section *ad* with respect to the section *bc*, so that

$$\Delta s = P_1 \cdot G \cdot \rho^2$$

It should be clear that this displacement, computed by two different methods, must be the same. Hence

$$P_1 \cdot G \cdot \frac{I}{A} = P_1 \cdot G \cdot \rho^2$$

or

$$\rho = \sqrt{\frac{I(\text{of } mm'n'n)}{A(\text{of } mm'n'n)}}$$

which is the semi-axis of the ellipse of elasticity normal to the geometrical axis of the arch, and which also turns out to be the radius of gyration of the central cross-section of the element *abcd*. In a somewhat similar manner, using force P_2 rigidly connected with face *ad*, the semi-axis of the ellipse lying on the arch axis may be shown to be

$$\rho_1 = \sqrt{\frac{I\,(\text{of } abcd)}{A\,(\text{of } abcd)}} = s\sqrt{\frac{1}{12}} = 0.289s$$

The graphical construction in Fig. 38 (Art. 39) may now be employed to determine centers or antipoles relative to any given axis.*

It should be clear from the above that we may treat the elastic weights of the elements (as *abcd*) for the whole arch in the same manner as we have the small elastic

Then it follows that an ellipse with its center at O, Fig. 35A (major and minor axes horizontal and vertical respectively for a symmetrical arch) may be employed to find the "centers" of the *entire* system of elastic weights with respect to given axes—that is, the antipole of such an ellipse, with respect to an axis through the line of action of a given force, will be the center of rotation of the free end of the arch (A, Fig. 35A) due to this given force. The length of the semi-minor axis would be equal to

$$\sqrt{\frac{I}{\Sigma_A^B G}}$$

where I represents the moment of inertia of all the elastic weights about the axis $X - X$, and $\overset{B}{\underset{A}{\Sigma}} G$ represents the *sum* of all these weights. The length of the semi-major axis may be found by the same general expression, but I would be taken about the axis $Y - Y$. Of course, the ellipse for an unsymmetrical arch would not have vertical and horizontal axes as above described. The method of determining the position of the ellipse for such arches will be given in Arts. 45 and 46.

For a symmetrical arch (Fig. 35A), it is shown in Art. 39 that any vertical force V acting through the point O would cause only vertical displacement of this point. This same conclusion is reached from the theory of the ellipse of elasticity. For example, the force V acts along the minor axis of the ellipse and the antipole, or instantaneous center of rotation, falls upon the major axis at infinity (Art. 40); which means that this force causes a rotation about a point at infinity—that is, a displacement in a straight line perpendicular to the major axis.

43. Graphical Constructions for Finding Moments and Products of Inertia of the Elastic Weights.—In Art. 39 the following formulas are derived:

$$M_a \overset{B}{\underset{A}{\Sigma}} G = P \overset{B}{\underset{p}{\Sigma}} Gx'$$

$$V \overset{B}{\underset{A}{\Sigma}} Gx^2 = P \overset{B}{\underset{p}{\Sigma}} Gx \cdot x'$$

$$H \overset{B}{\underset{A}{\Sigma}} Gy^2 = - P \overset{B}{\underset{p}{\Sigma}} Gy \cdot x'$$

by which it is possible to determine the moment and the horizontal and vertical components of the reaction acting at the point

O, Fig. 35*A*. In order to find M_a, V, and H, it is first necessary to determine the values of the following expressions:

$$\sum_A^B G \qquad \sum_p^B Gx'$$

$$\sum_A^B Gx^2 \qquad \sum_p^B Gx \cdot x'$$

$$\sum_A^B Gy^2 \qquad \sum_p^B Gy \cdot x'$$

This may be done graphically. In fact it will be shown later that the graphical constructions for finding moments and products of inertia of the elastic weights will give directly the values of M_a, V, and H, if the constructions are made with this result in view.

For the present it will suffice to show the general method to be followed in finding the above expressions graphically. In order to gain clearness in the explanation of this method, the weights assumed—namely: G_1, G_2, G_3, G_4, and G_5 (Fig. 42)—will be considered as the weights of heavy particles concentrated at the points 1, 2, 3, 4, and 5 respectively. In other words, five concentrated weights will be assumed and these will be applied at any five selected points, thus eliminating any consideration of "centers" or antipoles in this preliminary discussion. The method will be shown as perfectly general by choosing coördinate axes oblique to each other.

The first consideration is to find the center of gravity O of the system and this may be accomplished by means of equilibrium polygons f_a and f_b.[1] For example, to determine the axis $X - X$ (the inclination of which is previously decided upon), the weights are assumed to act parallel to this axis, next the force polygon F_a is constructed, and then the corresponding equilibrium polygon f_a. The line $X - X$ through the intersection p' of the first and last sides of the equilibrium polygon f_a will contain the center of gravity of the system. Also, the sides of the polygon f_a will determine, on this line, segments proportional to the moments of the corresponding weights with respect to this same axis through the center of the system. The axis $Y - Y$ may be determined in a like manner employing the force polygon F_b and the equilibrium polygon f_b. The intersection of the axes $X - X$ and $Y - Y$ determines the center of gravity O of the system.

---

[1] See "Elements of Structures," by the same author.

If verticals are drawn through the points of application of the weights, the segments cut off on any given vertical by the

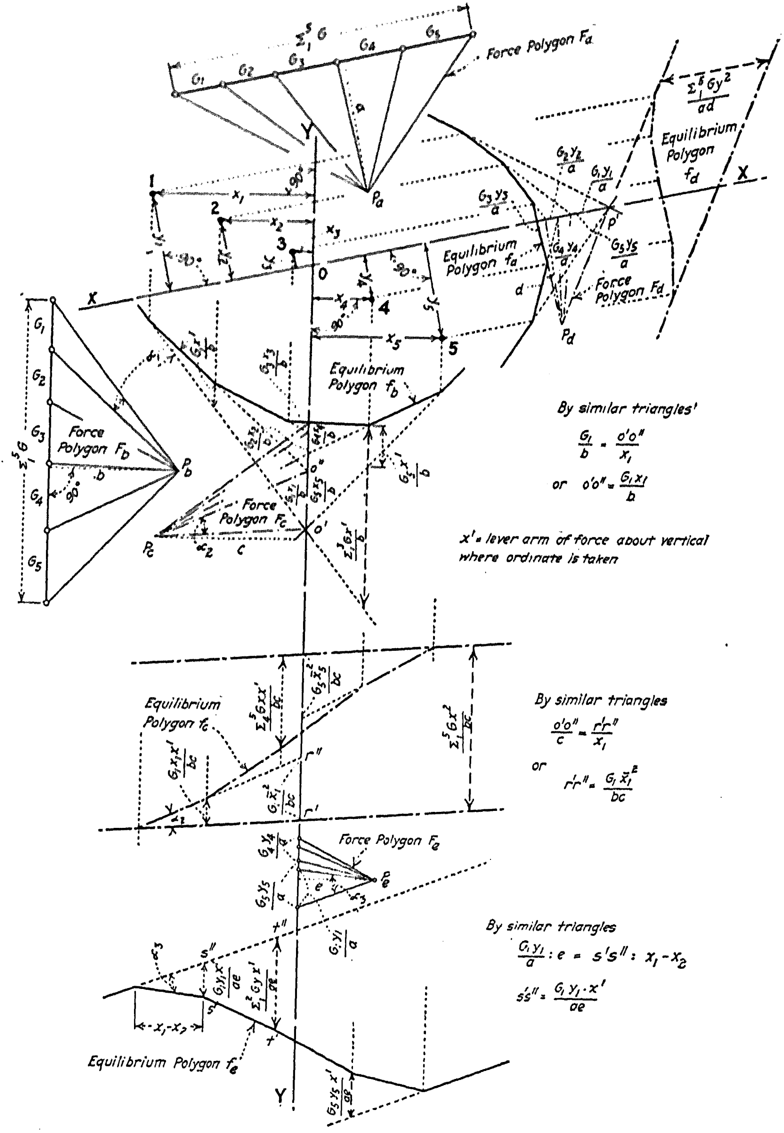

Force Polygon F_a
$\Sigma_1^5 G$
G_1 G_2 G_3 G_4 G_5
P_a
Y
X
1 2 3 4 5
O
x_1 x_2 x_3 x_4 x_5
y_1 y_2 y_3 y_4 y_5
90°
Equilibrium Polygon f_a
$\frac{G_2 y_2}{a}$ $\frac{G_1 y_1}{a}$ $\frac{G_3 y_3}{a}$ $\frac{G_4 y_4}{a}$ $\frac{G_5 y_5}{a}$
p'
d
Force Polygon F_d
P_d
$\frac{\Sigma_1^5 G y^2}{ad}$
Equilibrium Polygon f_d
Force Polygon F_b
P_b
b
Equilibrium Polygon f_b
Force Polygon F_c
P_c
c
o' o''
$\frac{\Sigma_1^5 G x'}{b}$
By similar triangles
$\frac{G_1}{b} = \frac{o'o''}{x_1}$
or $o'o'' = \frac{G_1 x_1}{b}$
x' = lever arm of force about vertical where ordinate is taken
Equilibrium Polygon f_c
$\frac{\Sigma_1^5 G x^2}{bc}$
$\frac{\Sigma_4^5 G x x'}{bc}$
$\frac{G_1 x_1 x'}{bc}$
$\frac{G_1 \bar{x}_1^2}{bc}$
$\frac{G_5 \bar{x}_5^2}{bc}$
r' r''
By similar triangles
$\frac{o'o''}{c} = \frac{r'r''}{x_1}$
or $r'r'' = \frac{G_1 \bar{x}_1^2}{bc}$
Force Polygon F_e
P_e
e
$\frac{G_4 y_4}{a}$ $\frac{G_5 y_5}{a}$ $\frac{G_1 y_1}{a}$
s' s'' t' t''
$\frac{G_1 y_1 x'}{ae}$
$\frac{\Sigma_1^2 G y x'}{ae}$
$\frac{G_5 y_5 x'}{ae}$
$x_1 - x_2$
Equilibrium Polygon f_e
By similar triangles
$\frac{G_1 y_1}{a} : e = s's'' : x_1 - x_2$
$s's'' = \frac{G_1 y_1 \cdot x'}{ae}$
Y

FIG. 42.

equilibrium polygon f_b will be proportional to the moment of all

Fig. 42 shows what intercepts should be considered to give desired moments.

As stated above for polygon f_a, the sides of the equilibrium polygon f_b intercept, on the axis $Y - Y$, segments proportional to the moments of the weights with respect to this axis. If these segments are treated as vertical forces applied at the points of application of the original weights, the equilibrium polygon f_c (constructed by means of the force polygon F_c) will intercept, on the axis $Y - Y$, a segment which will be proportional to the moment of inertia of the weights with respect to this axis. This segment is determined by the first and last sides of the polygon f_c. As shown in Fig. 42 the intercepts on verticals through the points of application of the weights correspond to the general expression $\Sigma Gx \cdot x'$. If equilibrium polygon f_d is constructed in a similar manner to polygon f_c (only with respect to the axis $X - X$ instead of $Y - Y$), then the segment cut off on the axis $X - X$, between the first and last sides of the polygon f_d, is the moment of inertia of the weights about this coördinate axis.

The equilibrium polygon f_a determines the moments of the weights with respect to the axis $X - X$. If these moments are considered as forces and applied at the points of application of the original weights, but this time acting vertically, the equilibrium polygon f_e (corresponding to force polygon F_e) will intercept on a vertical through any weight, a segment proportional to the product of inertia of the weights to one side of the given vertical with respect to this vertical and the $X - X$ axis. Fig. 42 shows what intercepts to consider for moments of the weights to the right and those for moments of the weights to the left of any given vertical.

44. Method of Analyzing Symmetrical Arches for Loading Only.—The graphical constructions in Fig. 43 are similar to those given in Fig. 42 except that G_1, G_2, etc., apply to the elastic weights of arch elements (made of equal lengths for convenience) instead of to the weights of concentrated loads. Each part of a given element has an elastic weight of its own (Art. 39), consequently the *centers* of each element with respect to axes $X - X$ and $Y - Y$ must be determined. These may be found by means of the ellipse of elasticity, as explained in Art. 40. Since the arch is symmetrical about a center line, only one-half of the structure need be considered. The arch is divided into a small number of elements in order to show the method without any considerable duplication of work.

In finding the moment of inertia of the elastic weights about

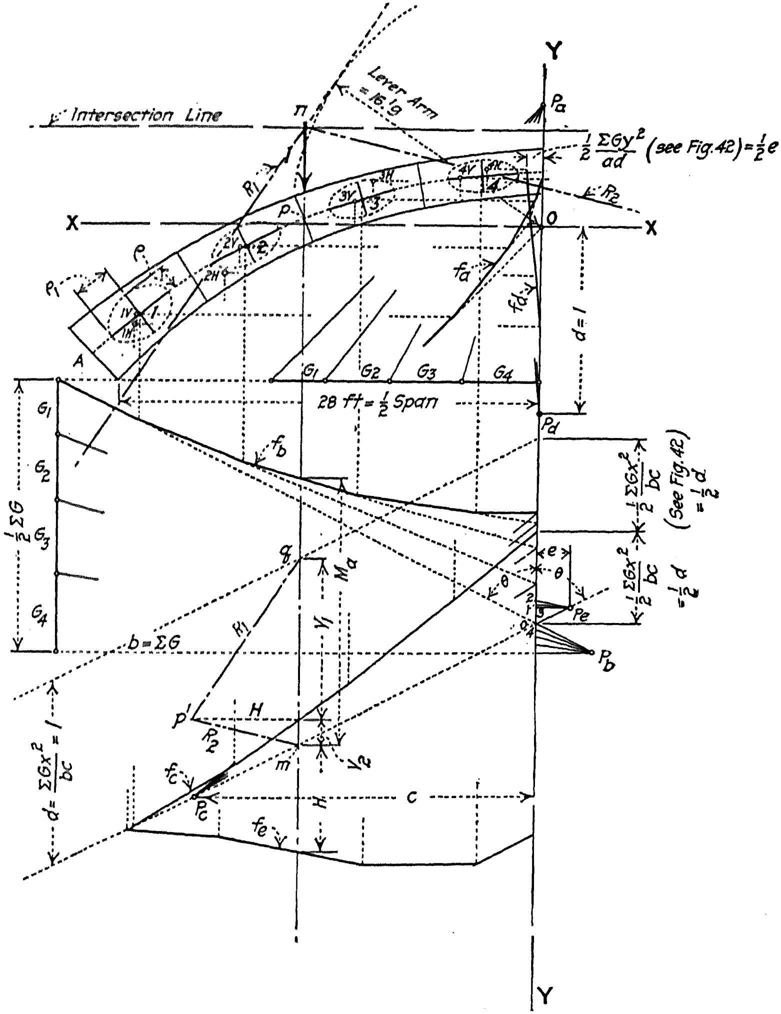


FIG. 43.

the axis $Y - Y$, by means of the equilibrium polygon f_c, the moments of the elastic weights determined by equilibrium polygon f_b should be considered as vertical forces acting at the

(1V, 2V, etc.). Similarly, the moment of inertia of the elastic weights about the axis $X - X$ (see equilibrium polygon f_d) should be determined by assuming the moments of the elastic weights, given by equilibrium polygon f_a, to act as horizontal forces applied at the *centers* of the arch elements relative to the axis $X - X$. The equilibrium polygon f_e should be drawn by assuming the forces corresponding to the moments with respect to the axis $X - X$ to be placed at the *centers* relative to the same axis, but acting vertically. (The results from equilibrium polygons f_a and f_d should be checked analytically in many cases as the intersections are not advantageous and may lead to cumulative errors. The force polygon with P_e as pole may be drawn to a larger scale for convenience in drawing the corresponding equilibrium polygon f_e.)

It should be noted in Fig. 43 that the pole distance b is taken equal to ΣG; that d is made equal to $\frac{\Sigma Gx^2}{bc}$ and considered as unity in scaling; that e is given a value equal to $\frac{\Sigma Gy^2}{ad}$; and that the pole P_c is taken on the end string of the equilibrium polygon f_b which occurs at the right of the entire arch, this string being produced backward to the left side of the axis $Y - Y$. By so doing the values of the reactions and the moment M_a for a load unity at any given point may be scaled directly.

The truth of the preceding statement may be proved as follows: From the preceding article we know that

$$M_a = \frac{P\sum_{P}^{B} Gx'}{\sum_{A}^{B} G}$$

With $P = 1$ and $b = \Sigma G$, we have

$$M_a = \frac{\sum_{P}^{B} Gx'}{b}$$

which may be scaled from Fig. 43 in the manner shown for a load unity at p. (See also Fig. 42.) Likewise

$$V_1 = \frac{P\sum_{P}^{B} Gx \cdot x'}{\sum_{A}^{B} Gx^2} = \frac{\sum_{P}^{B} Gx \cdot x'}{bc} \times \frac{bc}{\sum^{B} Gx^2} \quad \frac{\sum_{P}^{B} Gx \cdot x'}{bc}$$

$$H = \frac{P\sum_{p}^{B} Gy \cdot x'}{\sum_{A}^{B} Gy^2} = \frac{\sum_{p}^{B} Gy \cdot x'}{ae} \times \frac{ae}{\sum_{A}^{B} Gy^2}$$

$$= \frac{\sum_{p}^{B} Gy \cdot x'}{ae} \quad (\text{since } \frac{\sum_{A}^{B} Gy^2}{ad} = e, \text{ or } \sum_{A}^{B} Gy^2 = ae)$$

The student should note that the first and last sides of the equilibrium polygon f_e in Fig. 43 would coincide if the right-hand half of the polygon were completed. This occurrence is due to the fact that the directions of the coördinate axes are conjugated and the product of inertia of all the elastic weights with respect to these axes must be zero (Art. 41). Thus the values of M_a, V_1, V_2, and H due to a load unity at any given point p may be determined by drawing a vertical line through the point in question and, in the manner shown in Fig. 43, scaling the intercepts which are cut off on this line by the sides of the equilibrium polygons f_b, f_c, and f_e. By laying off H horizontally as in the drawing and joining the point p' to the ends of V_1 and V_2, the magnitude and directions of the reactions may be determined. The actual position of R_1 may be found by drawing a tangent parallel to $p'q$ at a distance $\frac{M_a}{R_1}$ from the point O. The right reaction R_2 may then be drawn from the intersection of R_1 with the vertical through p parallel to $p'm$.

Considering d equal to 1 lb. in scaling, we have, for a 1-lb. load at p in Fig. 43, $R_1 = 1.05$ lb. and $M = 17.7$ ft.-lb. The lever arm of R_1 about $O = \frac{17.7}{1.05} = 16.9$ ft. In a similar manner the reactions may be found for a 1-lb. load at other points on the arch. The intersection line shown passing through n (Fig. 43) is intended to represent the line of intersection of all reactions with their corresponding loads. It is shown as horizontal in this case in order to save the labor of determining its exact location.

Influence lines may now be drawn in the manner described in Art. 34 and the maximum stresses computed. Another and more graphical method, however, of determining values of ordinates to influence lines to represent maximum stress is shown in Fig. 44 with reference to a given section designated as S. It is convenient influence line for horizontal thrust (H) which, of course, may be plotted directly from the data given in Fig. 43. Seven positions of the unit load are shown.

Assuming no tension at a given section greater than the tensile strength of the concrete, we have, from Art. 74, Volume I, that the stress on the extreme fiber

$$f_c = \frac{N}{A} \pm \frac{N x_0 x_1}{I}$$

where N is the thrust normal to the section, and x_0 and x_1 are the distances from the gravity axis to the point of application of the thrust and to the extreme fiber respectively. Let ρ be the letter used for the radius of gyration of the section, then

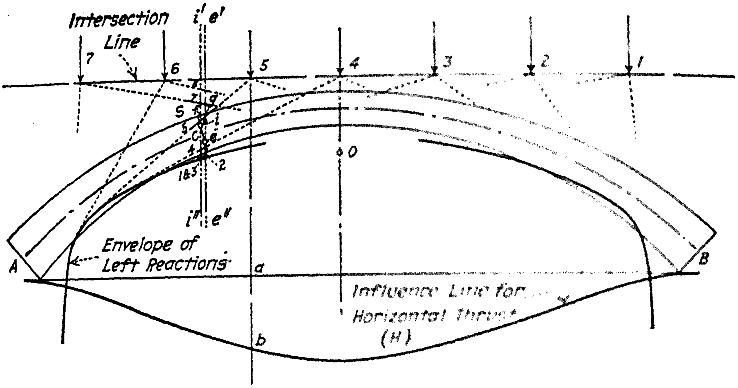

Intersection Line
i' e'
7
6
5
4
3
2
1
S
7
9
1
5
C
e
4
2
1&3
O
i'' e''
Envelope of Left Reactions
A
B
a
b
Influence Line for Horizontal Thrust (H)

FIG. 44.

$$f_c = \frac{N\left(\frac{\rho^2}{x_1} \pm x_0\right)x_1}{A\rho^2} = \frac{N\left(\frac{\rho^2}{x_1} \pm x_0\right)}{A\,\frac{\rho^2}{x_1}}$$

If the stress in the *upper* fiber is desired, the numerator in the above expression represents the moment about a point a distance $\frac{\rho^2}{x_1}$ *below* the gravity axis of the section, where x_1 is the distance from the gravity axis to the *upper* fiber. Similarly the moment for the *lower* fiber would be taken about a point a distance $\frac{\rho^2}{x_1}$ *above* the gravity axis where x_1 is the distance from the gravity axis to the *lower* fiber. If we denote these moments by M_a

and M_i, and the maximum fiber stresses by f_e and f_i respectively, then

$$f_e = \frac{M_e}{A \cdot \frac{\rho^2}{x_1}} = \frac{M_e}{A(ce)} \qquad \text{(See Fig. 45)}$$

$$f_i = \frac{M_i}{A \cdot \frac{\rho^2}{x_1}} = \frac{M_i}{A(ci)} \qquad \text{(See Fig. 45)}$$

M_e and M_i are given graphically in Fig. 45 as the products $R(ae)$ and $R(bi)$ respectively. It will be more convenient, however, to draw vertical lines through the points e and i, and to determine the products $H(ge)$ and $H(f_i)$.

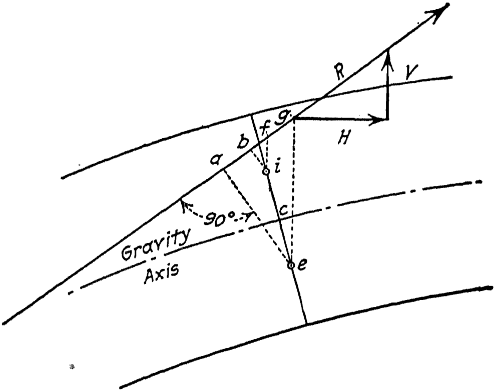


FIG. 45.

With the above discussion in mind, let us consider the influence on section S, Fig. 44, of a 1-lb. load at point 5. A vertical through this point shows the horizontal thrust to have a value $ab = 0.98$ lb., and a line drawn from the same point on the intersection line, tangent to the envelope of the left reaction, cuts off the distances ge and fi on the verticals through points e and i of the given section S. $ge = 2.00$ ft. $fi = 0.35$ ft. Therefore $M_e = 1.96$ ft.-lb. and $M_i = 0.34$ ft.-lb., and these moments may be plotted as the influence lines for fiber stress since the denominators in the preceding equations are constants.

45. Method of Analyzing Unsymmetrical Arches for Loading Only.—In deriving equations (8), (10), and (12) of Art. 39, the

= 0. The first two assumptions express the condition that the origin of coördinates is taken at the center of gravity of the elastic weights, while the third assumption is satisfied when the product of inertia of these weights referred to the two coördinate axes is zero. In Arts. 41 and 42 it is shown that the latter is the case when the two axes are conjugate diameters with reference to an ellipse for the entire arch with its center at the center of gravity

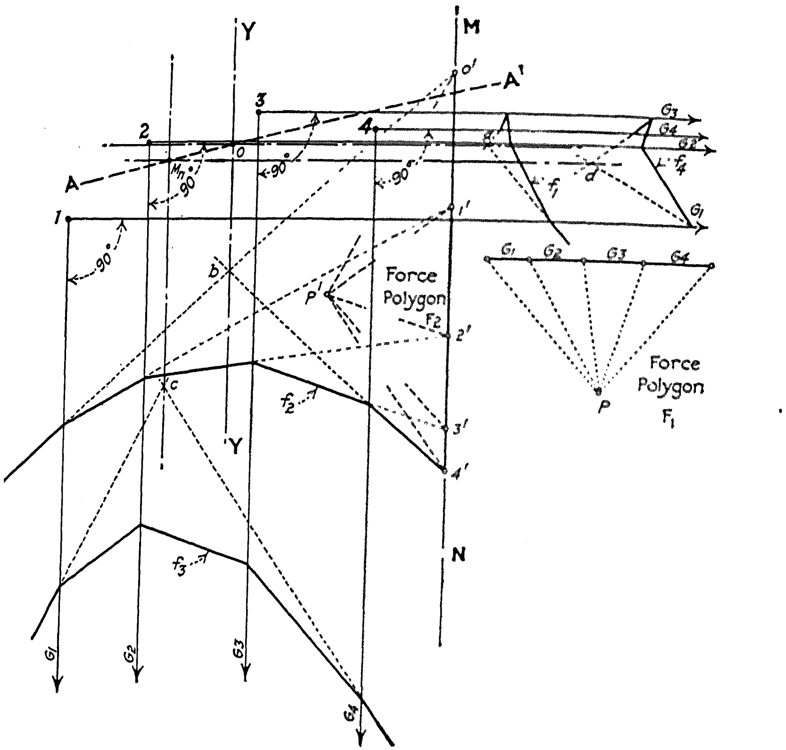


FIG. 46.

of the elastic weights. It will be convenient to assume a vertical direction for one of the coördinate axes. In a symmetrical arch it is then recognized at once that the horizontal axis is conjugate to the vertical, but this is not so for an arch that is unsymmetrical. For such an arch the additional problem is presented of finding a coördinate axis which will be conjugate to the assumed vertical axis. This may readily be accomplished graphically.

In Fig. 46 let points 1, 2, 3, and 4 be

of elements with elastic weights G_1, G_2, G_3, and G_4, respectively. O is the center of gravity of all the elastic weights and may be found as usual by assuming the elastic weights to act, first horizontally, and then vertically, and finding the line of action of the resultant in each case. The intersection of these two lines locates the center of gravity of all the weights. The construction to find O is all shown by means of force polygon F_1 and equilibrium polygons f_1 and f_2.

Now in order to find a conjugate diameter AA' to a vertical through O, take any vertical axis MN and extend all sides of the equilibrium polygon f_2 to intersect this axis at $0'$, $1'$, $2'$, $3'$, and $4'$. The segments, as we know, will be proportional to the moments of the elastic weights with respect to MN. Consider these segments $0'1'$, $1'2'$, $2'3'$, and $3'4'$ as forces and find their center of gravity M_n assuming that they are applied at the same points 1, 2, 3, and 4. The construction is the same as for finding O except that the acting forces are $0'1'$, $1'2'$, etc., instead of G_1, G_2, etc. The force polygon F_2 and the equilibrium polygons f_3 and f_4 furnish the solution. The vertical passing through the intersection c of the first and last sides of f_3 will contain the point M_n. Rotating by 90° the forces $0'1'$, $1'2'$, etc., about points 1, 2, 3, and 4, the horizontal through the intersection d of the first and last side of f_4 will also contain M_n. Thus the meeting point of the vertical through c and the horizontal through d will definitely determine the point M_n which is the antipole of the ellipse of the entire arch with respect to the vertical axis MN. As shown in Fig. 37A, a line through this point M_n and the center O of the ellipse is a conjugate diameter to the line MN or to a parallel or vertical line through O. Thus AA' is conjugate to a vertical through the center of gravity of the elastic weights—that is, AA' is the proper coördinate axis $X - X$ corresponding to a vertical axis $Y - Y$.

Once the conjugate axes for an unsymmetrical arch have been determined, the remainder of the arch analysis is similar in every way to the analysis of a symmetrical arch described in the preceding article with the exception, of course, that the entire arch must be considered. The method of analysis should be clear from a study of Figs. 42, 43, and 44.

46. Temperature and Rib Shortening.—If the left end of the unsymmetrical arch AB, Fig. 47, is assumed as free, a variation to a length $1 + t_c t_D$, t_c being the coefficient of linear temperature expansion. The length of arch chord, l', will become $l'(1 + t_c t_D)$ and this will be accomplished without any rotation. The reaction at the free end of the arch to return the arch axis to its original position must be such that no rotation is produced, which means that the line of action of such reaction due to temperature must pass through the elastic center O, and lie in the diameter conjugate to the normal to the chord of the arch (see Arts. 40 and 41). The method of finding a conjugate axis to the normal is the same in principle as the method explained in the preceding article for finding a conjugate axis to the vertical. By

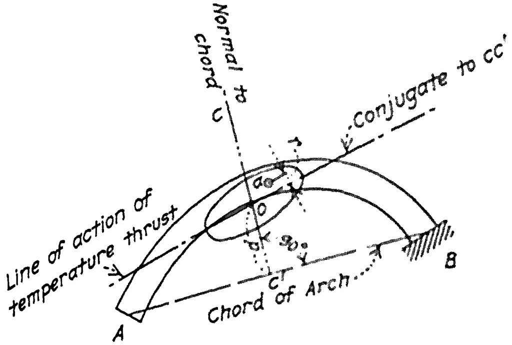


FIG. 47.

formula (6) Art. 39, the displacement of O (rigidly fastened to A) is as follows:

$$H\sum_A^B Gy\cdot x' = l't_ct_D$$

in which H is the reaction and $\sum_A^B Gy\cdot x'$ is the product of inertia of the entire elastic system with respect to the line of action of the force H and to the arch chord. If a, Fig. 47, is the antipole of the ellipse with respect to the chord AB, then the displacement along AB is equal to $H(OC')(r)\sum_A^B G$, or

$$H\cdot d\cdot r\Sigma G = l't_ct_D$$

$$H = \frac{l't_ct_D}{d\cdot r\cdot \Sigma G}$$

In order to determine the antipole a of the ellipse with respect to the chord AB using the graphical construction of Fig. 38, the location and lengths of the principal axes of the ellipse need to be found. The quantities already determined by the graphical constructions for loading, admit drawing gents to the ellipse of elasticity of the system parallel to the conjugate axes X and Y. For example, in Fig. 48 the normal distance to axis $X - X$ is

$$ab = \sqrt{\frac{I_x}{\sum_A^B G}}$$

where I_x represents the moment of inertia of all the elastic weights about the axis $X - X$ and $\sum_A^B G$ represents the *sum* of all these weights (see Art. 42). Likewise

$$cd = \sqrt{\frac{I_Y}{\sum_A^B G}}$$

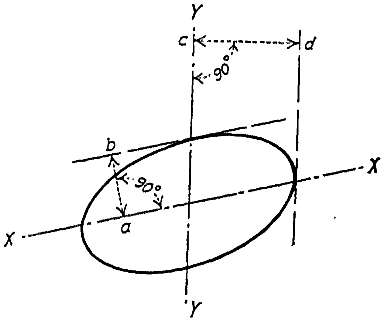


FIG. 48.

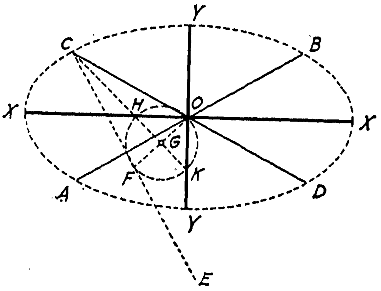


FIG. 49.

The lengths of the conjugate diameters being determined in the above manner, the graphical construction shown in Fig. 49 may be employed to determine the directions and lengths of the principal axes.

Let AB and CD be two conjugate diameters of an ellipse. From C draw CE normal to AB and upon it, starting from C, lay off $CF = OA$. Join F with O and on FO, as diameter, describe the circle G. Join C with the center G obtaining the two intersections H and K. Then OK and OH will determine the direction of the principal axes. CH will be the length of OY, and CK that of OX. (In order to obtain sufficient accuracy it will be necessary in many cases to assume an ellipse with the lengths of its conjugate diameters some suitable multiple of the lengths of the conjugate diameters of the true ellipse.)

If the arch is symmetrical, then the force H acts along the horizontal or major axis of the ellipse of elasticity and

$$H \sum_A^B Gy^2 = lt_ct_D$$

or

$$H = \frac{lt_ct_D}{\sum_A^B Gy^2}$$

The value of $\sum_{A}^{B} Gy^2$ is easily found from the graphical construction of Fig. 43—that is, from the equilibrium polygon f_d. (If E_c has been considered as unity in calculating the elastic weights for Fig. 43, then the above value of H should be multiplied by the actual value of E_c. It is permissible to assume E_c as unity in analyzing for loading since equations 8, 10, and 12, Art. 39, contain the value of E_c on each side of the equality sign.)

Rib shortening causes the same effect as a lowering of the temperature. By referring to Arts. 14 and 34, it should be clear that the following formula may be employed to solve for the equivalent temperature drop:

$$t_D = \frac{c_a}{E_c t_c}$$

where c_a is the average unit compression in concrete of arch ring due to thrust.

47. Analysis of a Symmetrical Arch—By A. C. JANNI, Consulting Engineer, New York City, Designer of the Carondelet Park Bridge, St. Louis, Mo.—The arch of the Carondelet Park bridge (see Plates XII, XIII, and XIV of Chapter XIII) is a reinforced-concrete hingeless arch of the solid barrel type, with segmental intrados and extrados. Its principal dimensions are as follows:

Span	95.00 ft.
Rise	7.50 ft.
Thickness at key	1.50 ft.
Thickness at springing	2.50 ft.
Intrados radius	154.00 ft.
Extrados radius	179.00 ft.

A longitudinal section of the barrel of the arch 1 ft. in width was considered, and the graphical constructions were confined to one-half the arch on account of its symmetry.

Since the length of the geometrical axis of one-half the arch was found to be 48.80 ft., the dividing of the arch into eight equal parts (made equal for convenience) gave each element a length of 6.10 ft., and the semi-axes of all the ellipses of elasticity of the various elements lying in the arch axis had the same value, namely:

$$\rho_1 = 6.10 \times \sqrt{\frac{1}{12}} = 6.10 \times 0.289 = 1.76 \text{ ft.}$$

Taking into account the steel reinforcement in the various cross-sections of the elements, the area, A, and moments of inertia, I, were found to be as follows:

	A	I
1	2.48 sq. ft.	0.46
2	2.48 sq. ft.	0.46
3	1.93 sq. ft.	0.46
4	2.03 sq. ft.	0.55
5	2.13 sq. ft.	0.64
6	2.33 sq. ft.	0.87
7	3.18 sq. ft.	1.48
8	3.38 sq. ft.	1.92

The elastic weights of the elements $\left(\frac{\Delta s}{E_c I} = G\right)$, assuming $E_c = 1$ for the sake of simplicity, were found to have the following values:

	G		G
1	13.26	5	9.53
2	13.26	6	7.01
3	13.26	7	4.12
4	11.09	8	3.18

The semi-axes of the ellipses of elasticity $\left(\sqrt{\frac{I}{A}}\right)$, normal to the axis of the arch, were as follows:

1	0.43 ft.	5	0.55 ft.
2	0.43 ft.	6	0.61 ft.
3	0.49 ft.	7	0.68 ft.
4	0.52 ft.	8	0.75 ft.

The graphical constructions on Designing Sheet No. 24 were made in the following manner:

Polygon p_1.—The elastic weights given above were considered as vertical forces applied at the corresponding centers of gravity of the elements. The static moments of these forces with respect to center line Y were then found by means of the force polygon with the pole distance ΣG, and the corresponding equilibrium polygon p_1. The sides of P_1, produced, cut on the vertical through O the static moments required.

Polygon p_2.—The same elastic weights given above were next assumed as applied *horizontally* at the centers of the elements, and the polygon p_2 was constructed to find the position of the center of gravity O of the elastic weights, as well as the static moments of these weights with respect to the horizontal passing through O.

Polygon p_3.—Knowing the semi-axes of the ellipses of elasticity of the various elements of the arch, the next step was to locate the antipoles O'_1. . . . O'_8 of the vertical through O with respect to each one of these ellipses. The static moments obtained by polygon p_1 were then regarded as vertical forces applied at the corresponding antipoles of the elements, and the equilibrium polygon p_3 constructed. It should be clear that this polygon gave the moment of inertia of the whole arch with respect to the vertical Y. (The pole P_3 was chosen arbitrarily.)

Polygon p_4.—The antipoles of the horizontal OX were next located with respect to the various ellipses, and these points designated as O''_1. . . . . O''_8. The static moments, obtained by polygon p_2, were regarded as horizontal forces applied respectively at the points O''_1. . . . . O''_8. (The pole or center P_4 was selected at a distance below OX equal to the vertical distance between the first and the last sides of the polygon p_3.) The corresponding equilibrium polygon p_4 gave the moment of inertia of the elastic system with respect to OX.

Polygon p_5.—The polygon p_5 was constructed by considering the same static moments as were used in determining the polygon p_4, assuming also these moments to act as forces at the same points O''_1. . . . . O''_8, but this time in a vertical direction. (The pole distance of the force polygon P_5 was made equal to e. In Designing Sheet No. 24 this distance is shown as $2e$, but this is because the entire force polygon was made twice the usual size for convenience in drawing the corresponding equilibrium polygon p_5.)

Polygons p_1, p_3, and p_5 are the only polygons necessary for the study of the arch. They are, so to speak, characteristics peculiar to the arch, by means of which moments, vertical reactions, and horizontal thrusts can be determined immediately for any hypothesis of loading.

Dead Load.—For 1-ft. width the following loads were computed:

	Weight of elements	Weight of fill over elements
1.	0.686 ton	0.457 ton
2.	0.686 ton	0.457 ton
3.	0.732 ton	0.518 ton
4.	0.778 ton	0.670 ton
5.	0.823 ton	0.854 ton
6.	0.915 ton	1.110 tons
7.	1.006 tons	1.386 tons
8.	1.098 tons	1.590 tons

The verticals of these loads, applied at the respective centers of gravity, cut the polygons p_1, p_3, and p_5. The intercepts on these verticals with polygon p_1, measured to the scale of the drawing and multiplied each by its corresponding load, gave in each instance the amount of the moment at the point O for the load in question. Adding together all these partial moments, the total moment at O was obtained due to the dead load of the entire arch.

Similarly, the intercepts on these verticals with polygon p_5 (measured on a scale having as a unit the vertical distance between the first and last side of polygon p_3) multiplied each by its corresponding load, gave in each instance the amount of horizontal thrust due to the load under consideration. Adding together all these partial horizontal thrusts, the total horizontal thrust was obtained.

The results were as follows:

$$\Sigma M = 442.816 \text{ ft.-tons}$$
$$\Sigma H = 33.458 \text{ tons}$$

Knowing the total dead load of the arch = 13.766 tons, the points where the resultant reaction crossed the axes X and Y were found in the following manner, the values of x and y being given with respect to the center O:

$$x = \frac{442.816}{13.766} = 32.16 \text{ ft.} \qquad y = \frac{442.816}{33.458} = 13.20 \text{ ft.}$$

The exact position of the polygon of pressure, therefore, was located, and the maximum stresses (tons per square foot) in the four sections S_1, S_2, S_3, and S_4 due to dead load were found to be as follows:

Section	Intrados	Extrados
S_1	11.354	17.034
S_2	8.663	26.447
S_3	11.686	19.481
S_4	33.400	−105.000 (steel)

A short graphical method of analyzing an arch for dead load (including temperature and rib shortening) is shown in Designing Sheet No. 25. Since dead load usually controls in the shape of an arch ring, this method should be used for making preliminary investigations with a view to determine the probable adequacy of the arch dimensions assumed. Proof of the correctness of the graphical construction may be found in the Journal of the Western Society of Engineers, issue of May, 1913. The method in detail is as follows: Construct equilibrium polygons p_2 and p_4 as in Designing Sheet No. 24. On an arbitrary vertical ab lay off the dead loads and assume the pole P_0 on the horizontal through a. Draw the corresponding equilibrium polygon p_0 intersecting with the verticals through the points of application of the loads. Now load the polygon p_0 horizontally with the same elastic weights as determined for the arch, at the points where the verticals through the centers of gravity of the elements meet the polygon p_0. Then intersecting these horizontals with an equilibrium polygon p'_2, corresponding to the force polygon P_2, the center of gravity O' of this new elastic system is determined. Again load the polygon p_0 with the static moments found for the construction of polygon p_4, but this time assume them applied at the meeting point of the verticals through the antipoles with the sides of the polygon p_0. The equilibrium polygon p'_4, corresponding to the force polygon P_4, determines a distance $\frac{1}{2}e'$ between its first and last sides. This distance is proportional to the moment of inertia of the elastic weights acting on this new elastic system, in the same manner that $\frac{1}{2}e$ is proportional to the moment of inertia of the system of elastic weights with respect to the horizontal passing through O.

Between the polygon p_0 and the true pressure polygon p there is a geometrical relation. For example, the distance of any point on the true pressure polygon p from the horizontal through O can be obtained by multiplying by the ratio $\frac{e}{e'}$ the distance from the horizontal through O' to the point of the polygon p_0 on the same vertical with the point in question. Thus any distance as kk equals $k'k'$ multiplied by $\frac{e}{e'}$. The horizontal thrust aP, likewise, may be found by multiplying the assumed thrust aP_0 by the in-

In the arch in question $\frac{e'}{e} = \frac{0.59}{0.44} = 1.34$ and the assumed horizontal thrust = 25 tons. The true thrust, therefore, was $1.34 \times 25 = 33.5$ tons. The ratio $\frac{e}{e'} = \frac{0.44}{0.59} = 0.745$, and the vertical distance between O' and the polygon $p_0 = 0.70$. Then $0.745 \times 0.70 = 0.52$ determined the exact point where the true horizontal thrust intersected the key of the arch.

Note that the coördinates of the resultant obtained by the method of Designing Sheet No. 24 checked exactly with those obtained by this method. The horizontal thrust also checked by the two methods.

Temperature.—The horizontal thrust, due to temperature changes, passed through O and its intensity was given by the formula:

$$H = \frac{lt_cE_c}{\Sigma Gy^2}$$

where

t_c = coefficient of expansion = 0.0000066.

l = span of the geometrical axis = 95.7 ft.

E_c = modulus of elasticity of concrete = 144,000 tons per sq. ft.

ΣGy^2 = moment of inertia of the arch with respect to the horizontal passing through $O = e \times 14.16 \times 11.76 = 2.64 \times 14.16 \times 11.76 = 439.61$.
(*e* was measured to the same scale as the elastic weights—that is, 1 in. = 6 units. The polar distances were measured to the same scale as the drawing, namely: 1 in. = 3 ft.)

Substituting these values

$$H = \frac{0.0000066 \times 95.7 \times 144{,}000}{439.61} = 0.20$$

for 1° change of temperature. For a change of 30°F.

$$H = 6.00 \text{ tons.}$$

For a drop of temperature of 30°, which was the only case needing consideration, the following stresses were obtained in tons per square foot:

Intrados Extrados

Shortening of Arch Due to Dead Load and Shrinkage.—The shortening of the geometrical axis of the arch due to dead load, at the time when the arch is allowed to bear upon its supports, may be considered as a shortening due to a drop in temperature.

Knowing, by means of the pressure polygon, the amount of compression on each cross-section, and knowing also the modulus E_c, the shortening of the arch was calculated and found to correspond to a drop in temperature of 23.4°F.

The arch was concreted by the alternate block method (see Art. 53) and the shrinkage of the arch was confined to the shrinkage of its keys only. The total key space was 6 ft., and $\frac{1}{2000}$ was considered the ratio of shrinkage in a 1:2:4 concrete. The shortening of the arch axis, using these values, was found to correspond to a drop of 4.6°F. in the arch.

The total drop of temperature due to the above two causes was assumed at 28°F. The corresponding horizontal thrust which, of course, passed through O, induced the following stresses (tons per square foot) in sections S_1 and S_4 of the arch.

	Intrados	Extrados
At spring (section S_1).	8.30 (concrete)	−315.00 (steel)
At key (section S_4).	−42.00 (steel)	5.80 (concrete)

It might be interesting to note that had the arch been concreted by longitudinal sections from springing to springing in one operation, the shrinkage in the concrete would have induced in the arch, stresses equivalent to a drop of temperature of 75°F.

Stresses Due to Live Load.—From Designing Sheet No. 24 it should be clear how the envelope lines and the intersection line can be constructed, the construction being fully indicated for position 4 of the load.

In Designing Sheet No. 26, which represents the whole arch, the envelope lines, the intersection line, and the polygon p_6 have been redrawn.

By means of the above lines, the various reactions corresponding to the several positions of the unit live load, together with the corresponding horizontal thrust in each case, have been determined. Moments have been determined, likewise, for the kern points i and e of each section and the results obtained are given in the following table:

Influence lines for kern moments have been plotted in Design-

Positions	H	S_1				S_2				S_3				S_4			
		ci	ce	M_i	M_e	ci	ce	M_i	M_e	ci	ce	M_i	M_e	ci	ce	M_i	M_e
1	0.09	2.08	3.05	0.18	0.27	−0.52	0.25	−0.05	0.02	−1.96	−1.30	−0.17	−0.12	−1.60	−1.10	−0.14	−0.10
2	0.50	2.02	3.00	1.01	1.50	−0.53	0.25	−0.26	0.12	−1.90	−1.25	−0.95	−0.62	−1.43	−0.93	−0.71	−0.47
3	1.10	1.90	2.86	2.09	3.15	−0.55	0.22	−0.60	0.24	−1.80	−1.20	−1.98	−1.32	−1.27	−0.77	−1.39	−0.85
4	1.88	1.65	2.62	3.10	4.92	−0.60	0.18	−1.13	0.34	−1.60	−1.00	−3.00	−1.88	−0.90	−0.40	−1.69	−0.75
5	2.65	1.40	2.38	3.71	6.31	−0.66	0.12	−1.70	0.32	−1.40	−0.78	−3.71	−2.07	−0.48	0.02	−1.27	0.05
6	3.16	1.02	2.00	3.22	6.32	−0.73	0.08	−2.31	0.25	−1.10	−0.45	−3.47	−1.42	0.13	0.63	0.41	1.99
7	3.30	0.50	1.50	1.65	4.95	−0.82	−0.05	−2.70	−0.16	−0.70	0.0	−2.31	0.0	1.04	1.54	4.51	5.08
8	3.16	−0.35	0.68	−1.11	2.15	−0.98	−0.19	−3.09	−0.60	0.02	0.68	0.06	2.15	......	......	......	......
9	2.65	−1.70	−0.62	−4.50	−1.64	−1.15	−0.30	−3.05	−0.79	1.32	2.00	3.49	5.30	......	......	......	......
......	2.53	......	......	......	......	......	......	......	......	1.65	......	4.17	......	......	......	......	......
......	2.54	......	......	......	......	......	......	......	......	......	2.30	......	5.84	......	......	......	......
10	1.88	−4.10	−3.00	−7.71	−5.64	−1.17	−0.29	−2.20	−0.54	1.12	1.78	2.10	3.34	......	......	......	......
11	1.10	−8.50	−7.30	−9.35	−8.03	0.0	0.91	0.0	1.00	0.57	1.20	0.63	1.32	......	......	......	......
......	0.63	......	......	......	......	4.30	......	2.71	......	......	......	......	......	......	......	......	......
......	0.65	......	......	......	......	......	5.05	......	3.28	......	......	......	......	......	......	......	......
12	0.50	−17.45	−15.90	−8.72	−7.95	4.20	4.93	2.10	2.47	0.30	0.92	0.15	0.46	......	......	......	......
13	0.09	−46.85	−42.90	−4.22	−3.86	4.02	4.75	0.36	0.43	0.15	0.80	0.01	0.07	......	......	......	......

ing Sheet No. 26. These influence lines have no direct importance in a design of a reinforced-concrete arch section where the tension in concrete is omitted, and the intention in showing them in this case is to demonstrate how careful a designer should be in applying certain empirical rules of loading in order, as he thinks, to find the maximum stresses in the arch. These lines show clearly that the maximum stresses at intrados and extrados for the same section do not happen with the same hypothesis of loading.

The stresses at section S_1 due to the live load of 100 lb. per square foot were found to be as follows in tons per square foot:

Intrados	Extrados
10.60 (concrete)	−243.0 (steel)

Total stresses in section S_1 of the arch were finally determined and found to be:

f_e = 12,750 lb. per square inch (steel)
f_i = 850 lb. per square inch (concrete)

The computations of the stresses in the sections S_2, S_3, and S_4 have been omitted, as they would have shown smaller stresses than those found above for section S_1.

Design of Abutment.—The resultant due to the dead load of the arch was found to be 36.2 tons (per 1-ft. length of abutment) and the vertical dead load on the abutment itself, 45.00 tons. The resultant of these two forces was 67.500 tons, and its vertical component 58.7 tons. (See Designing Sheet No. 27.) The resultant dead load intersected the base at point E which is not the center of the base. Using formulas for bending and direct stress for compression over the whole section, there resulted a pressure of 2.5 tons per square foot at C and a pressure of 0.38 ton per square foot at D.

K_2 and K_1 are shown as the middle third points of the rectangular base $CD \times 1$ ft. If from K_1 we draw the tangent K_1B to the left envelope line, all positions of the live load to the right of B will cause compression at C. Also if we draw from K_1 the vertical K_1F, and from C the vertical CG, all positions of the live load between G and F will affect the point C in the same manner, so that the most prejudicial hypothesis of loading, as far as pressure at C is concerned, is when it is assumed that all arch and abutment is loaded except the space FB.

Assuming, as for the arch, a live load of 100 lb. per square foot, the maximum compression at *C* was found to be 0.50 ton per square foot.

On account of a rise in temperature, an additional pressure at *C* was taken into account. Assuming that there would be a rise of 30°F., which corresponds to a horizontal thrust of 6.0 tons, the corresponding pressure at *C* was found to be 0.48 ton per square foot.

The total maximum pressure at *C* was found to be as follows:

$$2.50 + 0.50 + 0.48 = 3.48 \text{ tons per square foot}$$

The part *PD* of the footing slab was designed as a slab supported on two sides (counterforts) and loaded as shown by the pressure diagram.

The part *CP* was reinforced to stand a deflection indicated by the arrow.

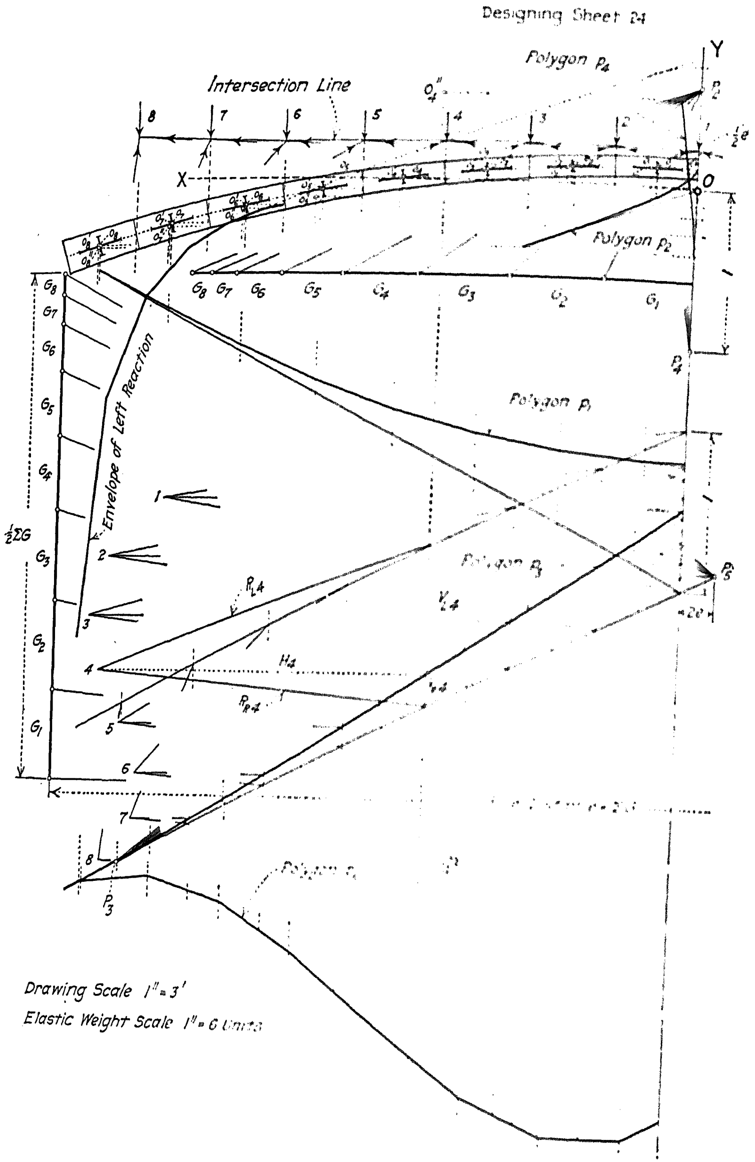
For shear, it was necessary only to ascertain the intensity at *P*, where it was 22.5 tons per 1-ft. depth of slab, safely absorbed by the sections of concrete, longitudinal upper reinforcement, and stirrups.

Of course the shear (likewise the moment) increased going from *P* to *E*, where it reached the maximum; but it is not necessary to go into details for the sections on the right of the section *PQ*, since the shape and area of these sections preclude any possibility of maximum conditions. The maximum compression at *Q* was found to be 601 lb. per square inch.

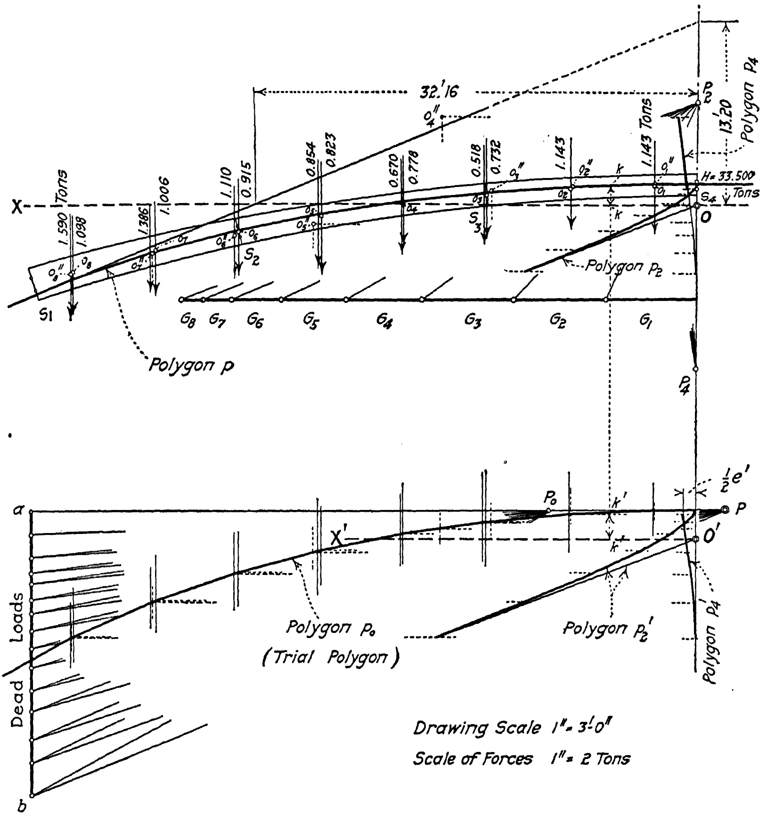
The front wall of the pier (2 ft. thick) was found to safely support the vertical component of the total reaction, or 13.76 tons + 4.70 tons.

Concerning now the horizontal component of the above reaction, two sections were considered for shear—namely: sections *HL* and *MN*. The maximum shear in concrete at section *HL* was found to be 52 lb. per square inch (considering the steel reinforcement to be 12.5 sq. in. per linear foot depth), and at section *MN*, 64 lb. per square inch. Considering that a good vertical compression acted on these two sections, it will be seen that the above stresses were quite legitimate.

Finally, the counterforts were designed as solids under combined action of direct compression and bending with regard to the thrust of the arch. Fig. 50 on Designing Sheet No. 27 gives an idea of how the problem was solved by means of the formula for bending and direct stress. *F* is the resultant acting upon the counter-


Designing Sheet 24
Polygon P_4
Y
Intersection Line
O_4''
8
7
6
5
4
3
2
1
$\frac{1}{2}e$
X
O
Polygon P_2
G_8
G_7
G_6
G_5
G_4
G_3
G_2
G_1
P_4
Envelope of Left Reaction
Polygon P_1
$\frac{1}{2}\Sigma G$
Polygon P_3
P_5
R_L4
V_L4
H_4
R_R4
2e
P_3
Polygon P_3
Drawing Scale 1" = 3'
Elastic Weight Scale 1" = 6 Units

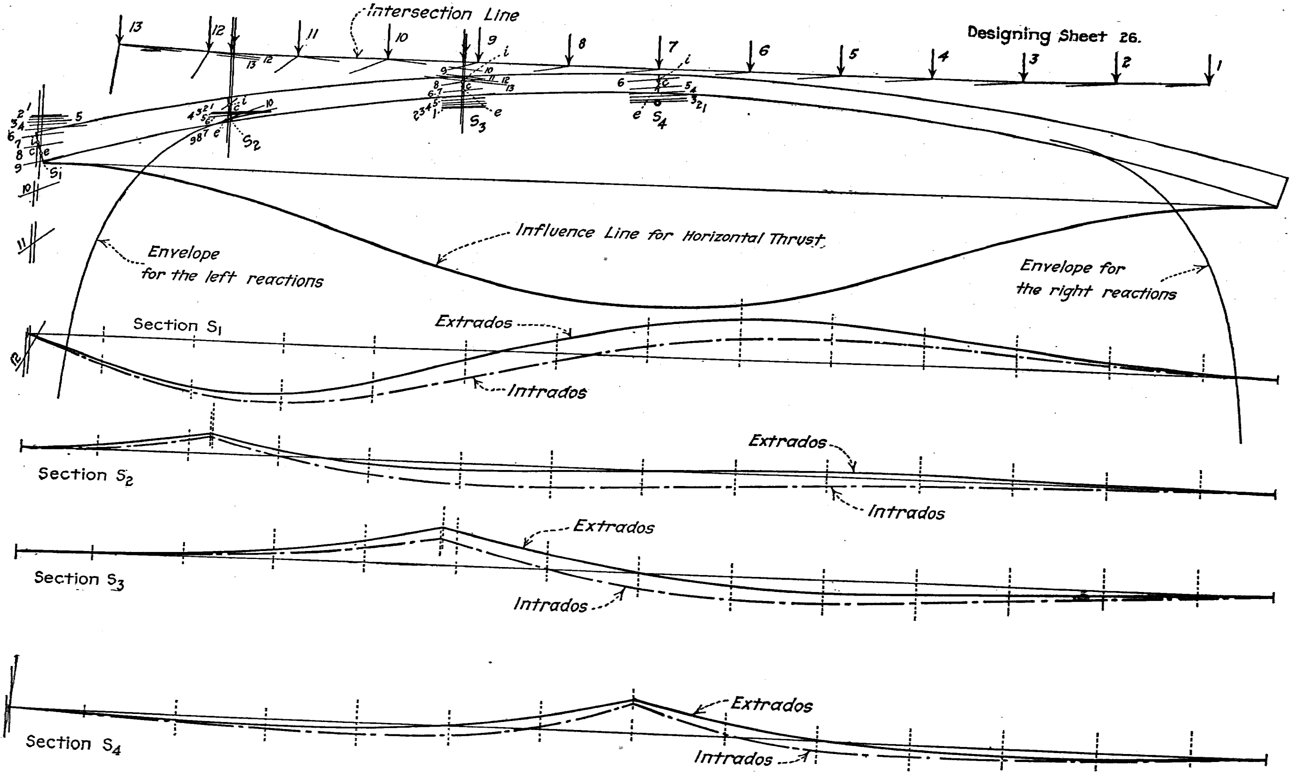
Designing Sheet 25.



Drawing Scale 1" = 3'-0"

Scale of Forces 1" = 2 Tons

Designing Sheet 26.



Designing Sheet 27.

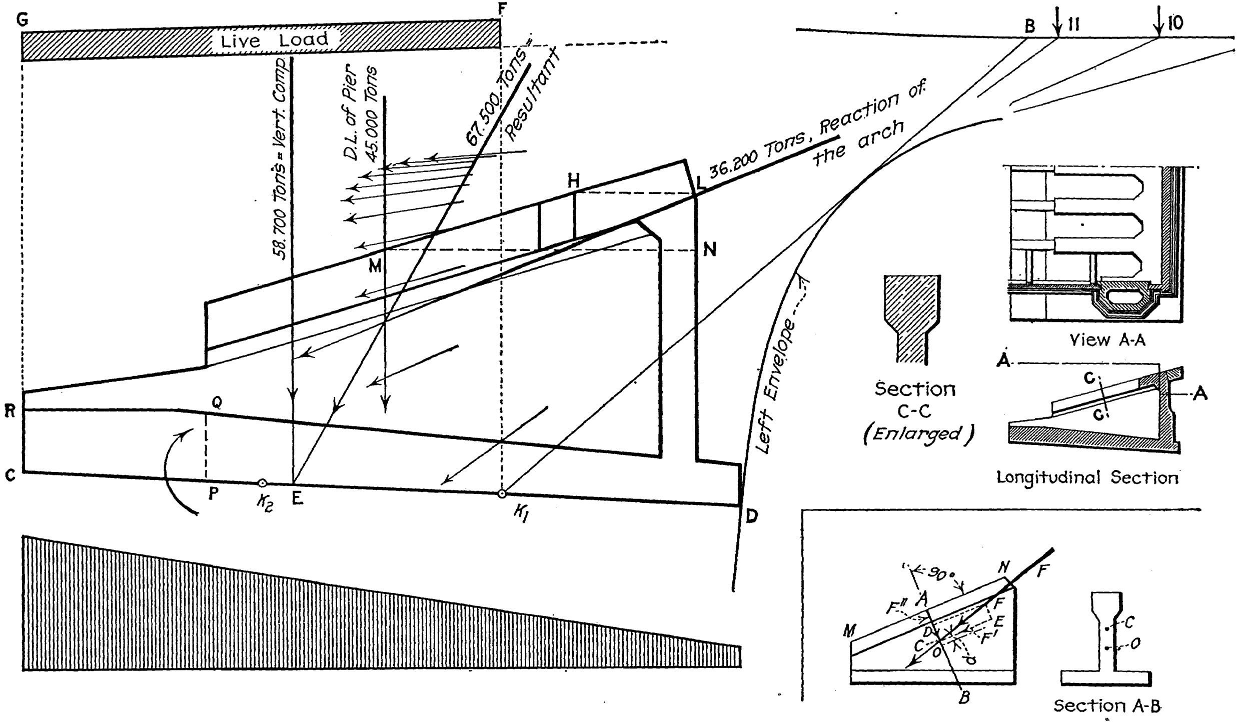


Fig. 50

fort, and AB is a section taken normally to MN. F' is the compressive force on the section and F'' the shear. O is the center of gravity of the cross-section AB.

48. Method of Analyzing a Series of Arches with Deformable Supports.—Consider first a three-span symmetrical arch structure as shown in Fig. 51, whose piers P_1 and P_2 are of such dimensions that they can be regarded as deformable. If the arches A_1, A_2, and A_3 were equal, then, as far as the dead load is concerned, each of them could be regarded as a fixed arch. If they are unequal as shown, then, with the overloading of the short spans, it is usually possible to drive the resultant to the center or near the center of the base of the pier. This result being accomplished, it is then permissible to consider each arch as fixed.

The live load, in every case, requires a totally different treatment from that mentioned above. It is evident that the middle arch A_2 is supported by two elastic systems—A_1P_1 on the left and A_3P_2 on the right. The method is to construct the ellipse of elasticity for the arch A_1 and for the pier P_1 (Fig. 52)—that is to say, to construct the ellipse O_a (ellipse of elasticity of the arch with respect to its terminal section $A - A$) and the ellipse O_p (ellipse of elasticity of the pier with respect to its terminal section $A - A$)—and then to construct the ellipse of elasticity with respect to the same section $A - A$ when this section is regarded as a section of the entire system A_1P_1.

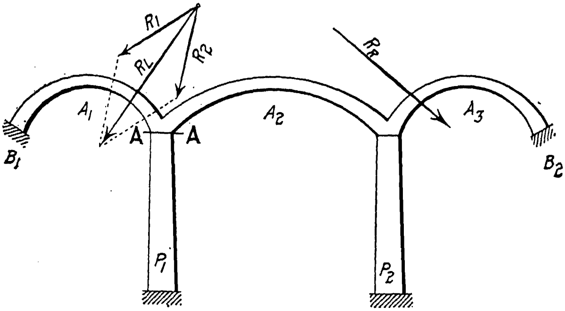


FIG. 51.

A horizontal displacement without rotation of section $A - A$, regarded as the terminal section of P_1, cannot happen but for the action of a horizontal force F_p passing through O_p, and if ΣG_p is the elastic weight of the pier P_1, this displacement of $A - A$ will be given by (see Art. 42)

$$F_p \cdot \Sigma G_p \cdot \rho_p^2$$

This same displacement of $A - A$, when $A - A$ is regarded as a terminal section of A_1, is given by (see Fig. 52)

$$F_a \cdot \Sigma G_a \cdot \rho_a^2$$

where ΣG_a is the elastic weight of the arch A_1.

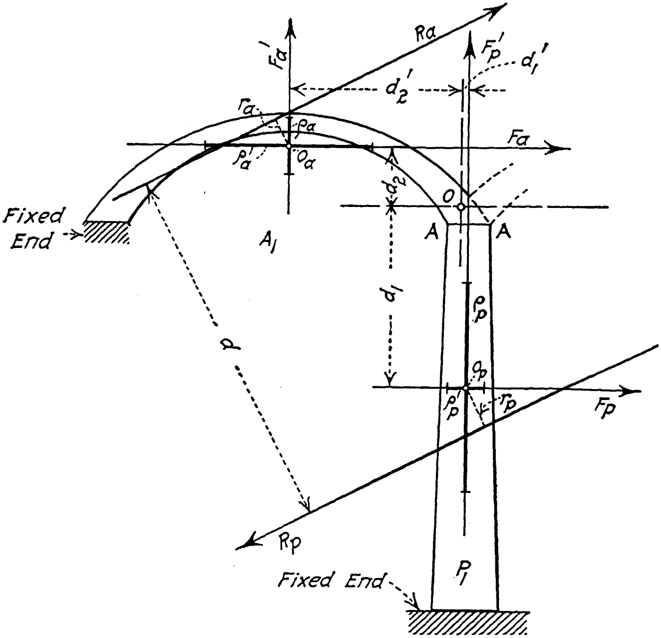


FIG. 52.

We have, therefore:

$$F_p \cdot \Sigma G_p \cdot \rho_p^2 = F_a \cdot \Sigma G_a \cdot \rho_a^2$$

or

$$\frac{F_a}{F_p} = \frac{\Sigma G_p \cdot \rho_p^2}{\Sigma G_a \cdot \rho_a^2} \tag{1}$$

which gives the position of the horizontal passing through the center O. In fact, if we regard $A - A$ as a section of the elastic system A_1P_1, its horizontal displacement must be caused by a horizontal force $F_a + F_p$ passing through O. If we designate

d_1 and d_2 as the perpendicular distances from O to F_p and F_a respectively, then

$$\frac{d_1}{d_2} = \frac{\Sigma G_p \cdot \rho_p{}^2}{\Sigma G_a \cdot \rho_a{}^2}$$

or

$$d_1 = \frac{d_1 + d_2}{1 + \dfrac{\Sigma G_a \cdot \rho_a{}^2}{\Sigma G_p \cdot \rho_p{}^2}} \tag{2}$$

In exactly the same way, we obtain

$$\frac{F'_a}{F'_p} = \frac{\Sigma G_p \cdot (\rho'_p)^2}{\Sigma G_a \cdot (\rho'_a)^2} = \frac{d'_1}{d'_2}$$

or

$$d'_1 = \frac{d'_1 + d'_2}{1 + \dfrac{\Sigma G_a \cdot (\rho'_a)^2}{\Sigma G_p \cdot (\rho'_p)^2}} \tag{3}$$

This expression gives the position of the vertical through O.

The point O being located, the elastic weight of the combined arch and pier must now be determined. Since O is the center of the ellipse, any rotation of section $A - A$ (as belonging to A_1P_1) about O cannot be caused but by a couple. (A force of which O is the antipole must lie at infinity—that is, will act along the antipolar of O—which is the same thing as saying that the rotation is caused by that of a couple.) On the other hand, the same rotation of $A - A$, regarded as terminal section of A_1, is caused by a force R_a acting along the antipolar of O with respect to the ellipse O_a; also the same rotation of $A - A$, regarded as terminal of P_1, is caused by a force R_p acting along the antipolar of O with respect to the ellipse O_p. Hence the forces R_a and R_p will constitute a couple, or $R_a = R_p = R$. The angle of rotation is given by the formula (Art. 39)

$$k = M\Sigma G = Rd\Sigma G$$

and considering the section $A - A$ as belonging first to A_1 and then to P_1, we have

$$k = Rr_a\Sigma G_a = Rr_p\Sigma G_p$$

or

$$\Sigma G = \frac{r_a\Sigma G_a}{d} = \frac{r_p\Sigma G_p}{d} \tag{4}$$

which determines the total elastic weight of arch and pier.

The horizontal displacement of $A - A$ as belonging to A_1P_1 is

$$(F_a + F_p)\Sigma G \cdot \rho^2$$

This displacement must be the same as that due to the force F_p, or

$$(F_a + F_p)\Sigma G \cdot \rho^2 = F_p \cdot \Sigma G_p \cdot \rho_p{}^2$$

$$\rho^2 = \frac{\Sigma G_p \cdot \rho_p{}^2}{\Sigma G\left(1 + \dfrac{F_a}{F_p}\right)}$$

Substituting the value of $\dfrac{F_a}{F_p}$ from Equation (1)

$$\rho^2 = \frac{\Sigma G_p \cdot \rho_p{}^2}{\Sigma G\left(1 + \dfrac{\Sigma G_p \cdot \rho_p{}^2}{\Sigma G_a \cdot \rho_a{}^2}\right)}$$

or

$$\rho^2 = \frac{\Sigma G_p \cdot \rho_p{}^2}{\Sigma G\left(1 + \dfrac{d_1}{d_2}\right)} \tag{5}$$

Similarly

$$(\rho')^2 = \frac{\Sigma G_p(\rho'_p)^2}{\Sigma G\left(1 + \dfrac{d'_1}{d'_2}\right)} \tag{6}$$

Equations (5) and (6) determine the lengths of the semi-minor and semi-major axes of the ellipse at O.

The ellipse of elasticity for the system A_1P_1 (Fig. 51) is now fully determined. In a like manner the ellipse may be drawn for the system $A_1P_1A_2$ by combining the ellipse for A_1P_1 with the ellipse for A_2, and so on. The same operation can be carried out starting from the right abutment.

Suppose the effect of loads on the arch A_2 is to be determined. The method of doing this is to find the ellipse of elasticity for the system A_1P_1 and for the system A_3P_2. (If the entire arch system is symmetrical about the center line between piers P_1 and P_2, there is a great deal of work saved.) The arch A_2 is then studied in the usual way, with the only variation that it is assumed that the arch is constituted by its elements plus two *ideal* elements, one on each side of the arch A_2 (Fig. 53), each having the ellipse of elasticity as above determined. These ellipses of the ideal elements are considered as any other of the ellipses of the arch elements.

Having found by the above method the reactions R_L and R_R due to the actions of the elastic systems A_1P_1 and P_2A_3 respectively against the arch A_2, it becomes necessary to determine which component of R_L belongs to the arch A_1 and which belongs to the pier P_1. (In Fig. 51 consider R_1 the component acting upon A_1, and R_2 the component acting upon the pier.) This is most easily accomplished by means of the three ellipses O_a, O_p, and O, already found (Fig. 52). Consider the section $A - A$ as a section of the system A_1P_1 under the action of the force R_L. A rotation of this section will be performed about a certain point D (not shown), which is the antipole of R_L with respect to the ellipse O. The same rotation of $A - A$, if we regard $A - A$ as the terminal section of P_1, will be caused by a force R_2 acting along the antipolar line of the same point D with respect to the ellipse O_p. Similarly, if $A - A$ is regarded as the terminal section of A_1, then the same rotation of this section will be caused by a force R_1 acting along the antipolar of D with respect to O_a. With the usual construction for finding polar and antipole, or, what is the same thing, for finding antipolar and pole, the lines of action of R_1 and R_2 can be located. Of course, as a check, the two components as found above must meet on R_L. A similar method of finding components can be applied to the system P_2A_3 for finding the forces acting on P_2 and A_3.

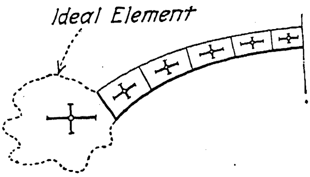


Fig. 53.

In the case of a bridge of two spans, each arch could be regarded as fixed at one end and supported at the other end by the elastic system, including arch and pier. The end arch of a three-span structure could be regarded in a similar manner except that the elastic system would consist of two arches and two piers.

It can be readily appreciated that the analysis of a large series of arches in the manner above suggested would be exceedingly laborious on account of the large number of ellipses of elasticity which would need to be determined and the consequent resolving of reactions into components. The method may be greatly simplified by the fact that the effect of a load on any one span extends principally over the span itself and the two spans immediately adjacent. For example, if A_2 in Fig. 51 is assumed as an arch of a large series of arches, it is permissible for all practical purposes to consider the arches A_1 and A_3 as fixed at B_1 and B_2 and to analyze arch A_2 with its supports A_1P_1 and A_3P_2 without regard to any of the remaining arches of the system. If the arches are all equal, then the same calculations will serve for any three arches of the series.

It is possible to take into account the stresses in an arch resulting from the yielding of the soil beneath the piers due to an eccentricity of the resultant thrust on the base. Knowing E of the ground, which is the same thing as knowing the load necessary to have the soil yield 1 in., and neglecting the even or horizontal displacements of the soil, an ellipse of elasticity may be determined due to yielding. If we call I the moment of inertia of the supporting surface in contact with the footing of the pier, and g_p the amount of yielding found as above, then the expression

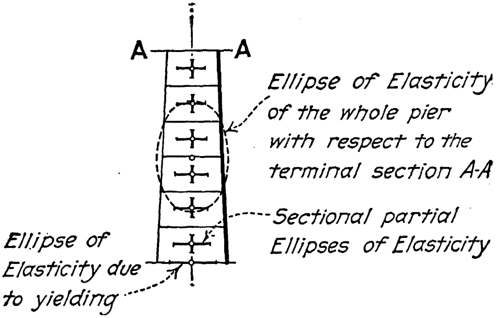

A
A
Ellipse of Elasticity of the whole pier with respect to the terminal section A-A
Sectional partial Ellipses of Elasticity
Ellipse of Elasticity due to yielding

FIG. 54.

$$G_g = \frac{1}{g_p I}$$

will be the elastic weight to be applied at the center of gravity of that section. The ellipse of elasticity of this point (Fig. 54) will have only the horizontal axis, since the vertical one will be zero on account of the assumption that the horizontal displacements may be neglected.

The method of analyzing a series of unsymmetrical arches with elastic piers is the same in principle as the method already explained for symmetrical arches. A graphical construction is convenient, however, to find the lengths and directions of the principal axes of an ellipse when two pairs of conjugate axes are known.

These principal axes are required in order to use the construction of Fig. 38 to find antipoles from polars or antipolars from poles. A brief description of the method of combining ellipses for unsymmetrical arches will be given.

Referring to Fig. 55, consider first the arch AB, and let us determine a force F_a which will cause the point A to undergo a horizontal displacement, without rotation. The line of action of this force F_a will be along the conjugate diameter of the vertical through the center of the ellipse O_a. The horizontal displacement is equal to

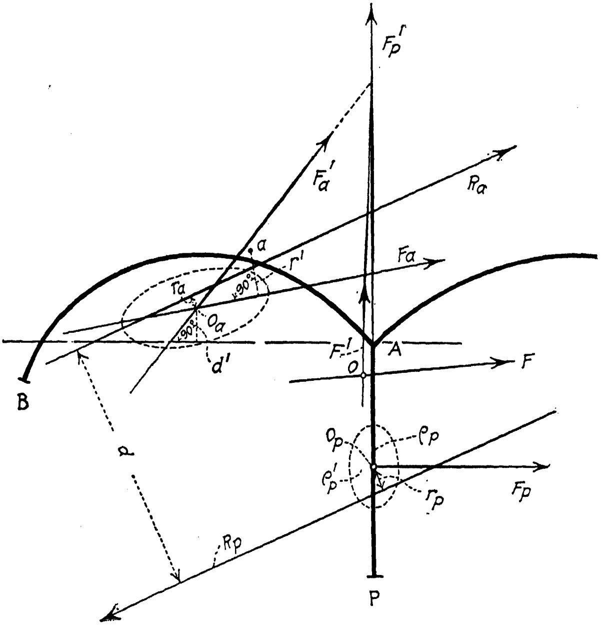


FIG. 55.

$$F_a \cdot \Sigma G_a \cdot r' \cdot d'$$

considering a the antipole of the horizontal through the point A. (See Art. 46.) The horizontal displacement of the point A due to force F_p is, as before,

$$F_p \cdot \Sigma G_p \cdot \rho_p{}^2$$

Then

$$F_p \cdot \Sigma G_p \cdot \rho_p{}^2 = F_a \cdot \Sigma G_a \cdot r' \cdot d'$$

or

$$\frac{F_a}{F_p} = \frac{\Sigma G_p \cdot \rho_p{}^2}{\Sigma G_a \cdot r' \cdot d'}$$

If F_a and F_p are prolonged until they intersect, then the resultant F of these two forces, the direction of which may be determined by means of the above equation, will cause A of the system BAP to move, without rotation, through the same horizontal distance as the force F_a acting on the system AB or the force F_p acting on the system AP.

Considering the vertical displacement of A, we obtain a similar equation to the above. The resultant F' of the forces F'_a and F'_p will cause point A of the system BAP to move through the same vertical displacement without rotation.

The two forces F and F' will cause point A to be displaced along a line passing through the center of the ellipse of elasticity of the system. Consequently the center of the ellipse is at the intersection of the forces. The elastic weight ΣG of the system BAP may be found in the identical manner previously explained for the symmetrical arch and pier.

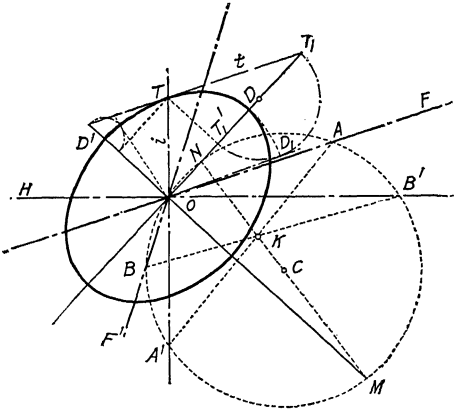


FIG. 55A.

The graphical construction of Fig. 55A[1] serves to determine the principal axes of the ellipse O. Considering ellipse O, the line of action of the force F and a vertical (V) through O on the one hand, and the line of action of the force F' and a horizontal (H) through O on the other hand, constitute two pairs of conjugate elements of the ellipse by means of which the principal axes may be determined.

To find the major and minor axes, describe an arbitrary circle

---

[1] Taken from an article by Prof. H. Lossier of the University of Lausanne in the "Genie Civil," volume of 1903. Translation by Mr. Samuel Morecll, Jr., Bridge Designing Engineer for City of Chicago.

(with center C) passing through O. This circle will intersect the straight lines F, V, F', and H in the points A, A', B, and B' respectively. The point of intersection K of the straight lines AA' and BB' will be the center of the figure $AA'BB'$. Connect points C and K and prolong to intersections M and N on the circumference of the circle. The two conjugate diameters OM and ON will be perpendicular and determine the direction of the principal axes of the ellipse. The length of the axes required may be found by drawing the tangent t to the ellipse O, parallel to F. The point of contact T of t lies on the vertical V at a distance OT from O equal to $i = \sqrt{\frac{1}{\Sigma G \cdot H_o}}$, where H_o is the horizontal projection of the force F which causes a horizontal displacement of the point O equal to unity without rotation. Let T_1 be the point of intersection of the tangent t with the axis ON and let T'_1 be the projection of T on this axis. Then the length of the semi-axis is

$$OD = \sqrt{(OT'_1)(OT_1)}$$

that is, OD may be measured by the length OD_1 of the tangent passing through O to the circle described on $T_1T'_1$ as a diameter. The length of the other principal axis may be found in a like manner.

The statements made in the preceding chapter in regard to temperature and rib shortening in a series of arches with elastic piers should be noted.

CHAPTER IX

DETAILS OF ARCH BRIDGES

49. Spandrel Details in Earth-filled Bridges.—As stated in Art. 3, the filling material in solid-spandrel bridges is held in place laterally by retaining walls which rest upon the arch ring. These retaining walls may be of either the gravity or the reinforced type, or they may consist of thin vertical slabs tied together by reinforced-concrete cross walls as employed in the design of the arch bridge of Chapter IV. In the usual type of solid-spandrel construction the sidewalk rests upon the earth filling, which is the type shown in Fig. 56. Where the counterforted type of spandrel wall is employed, sidewalks are sometimes cantilevered beyond the faces of the arch ring, as illustrated in Figs. 57, 58, and 59. The faces of spandrel walls may be entirely plain (Fig. 57), or panels of approximately a triangular shape may be formed either by indenting the portion above the arch ring or by nailing beveled strips to the form work with the result shown in Fig. 60. Brick and stone are used in some cases as a facing for arch rings and spandrel walls.

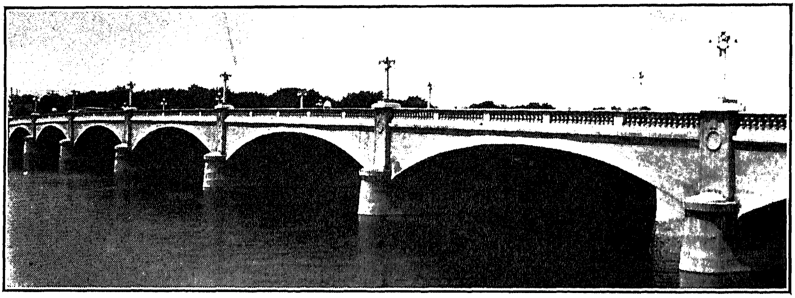
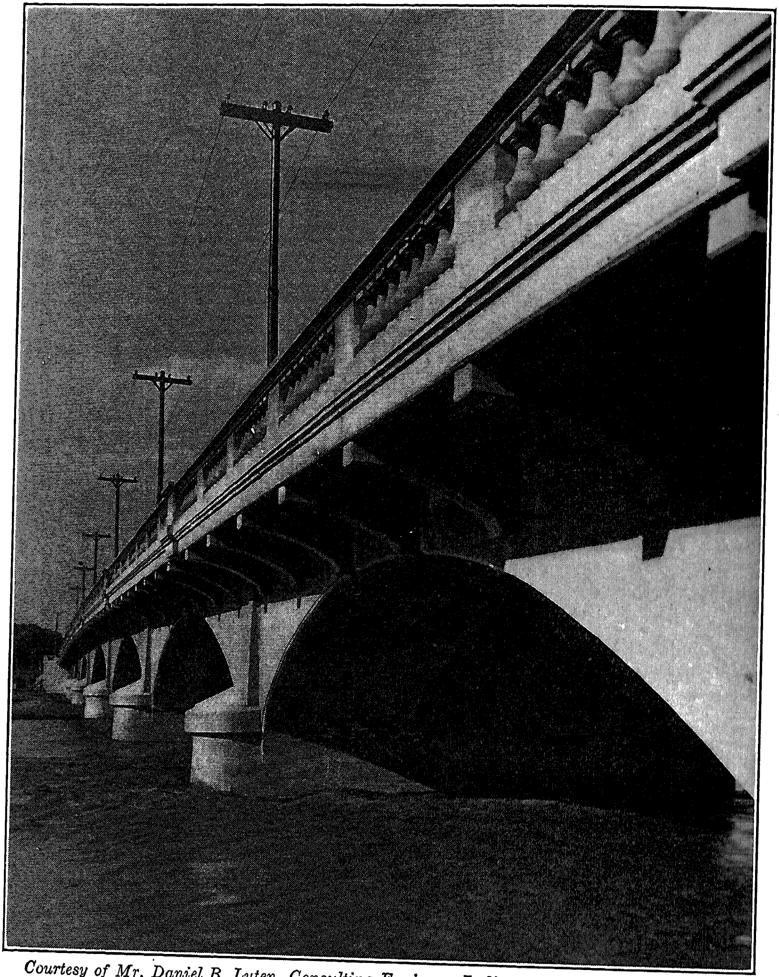


FIG. 56.—Leonard Street bridge over Grand River, Grand Rapids, Michigan.

Figs. 61*A* and 61*B* show a portion of a flat arch bridge designed for the city of Lima, Ohio. The spandrel walls are of the reinforced cantilever type. Other examples of this form of spandrel wall and sidewalk construction may be found in Chapter XIII.

A bridge with gravity spandrel walls is shown in Fig. 62. The brick facing for the arch ring and the cast concrete and brick belt courses should be noted. The spandrel walls rest partly on the brick facing and partly on the concrete portion of the arch,



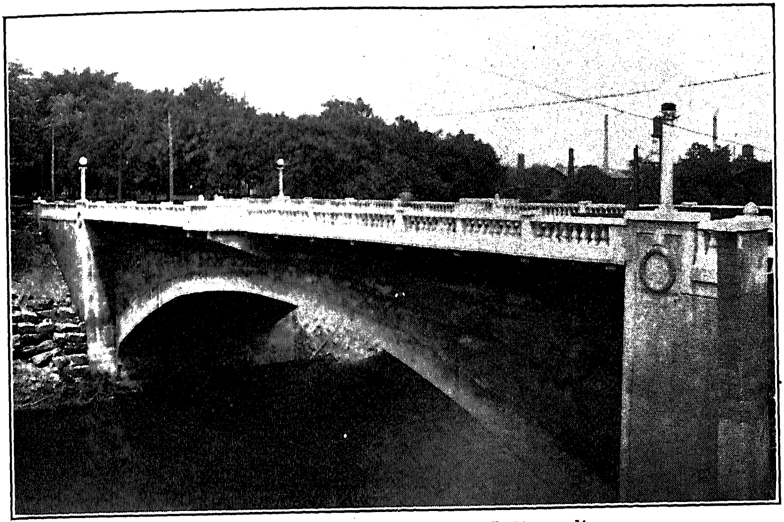
Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 57.—Bridge at La Junta, Colorado.

and are keyed into the concrete portion by means of a projection which fits into a 6 in. by 12-in. groove in the arch.

Engineering and Contracting, issue of Oct. 7, 1914, describes the character of the cast concrete and brick masonry as follows:

"The cast concrete blocks for the belt courses and copings are composed of 1 part cement and 3 parts crushed stone chips, the chips being of such sizes as to pass through a $\frac{3}{8}$-in. mesh screen. Waterproofing material is incorporated in the concrete of the cast stone. After being cast from 40 to 60 days, all exposed surfaces of the stones were dressed with six-cut bush hammers. The cast concrete blocks are laid in 1:2



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 58.—Washington Avenue bridge, Elyria, Ohio.

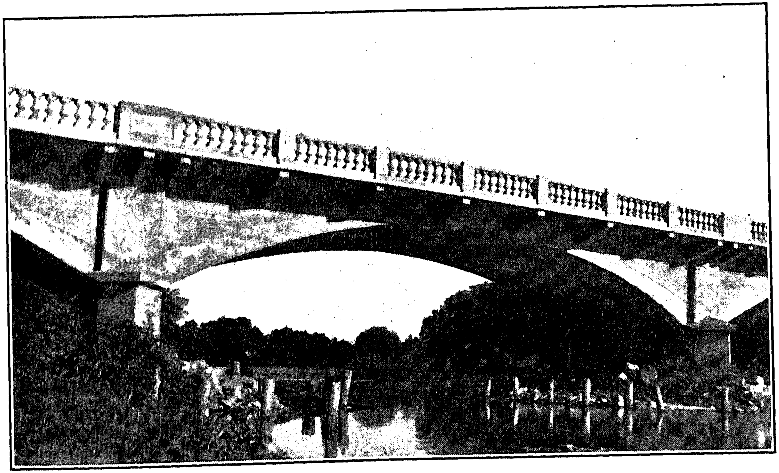
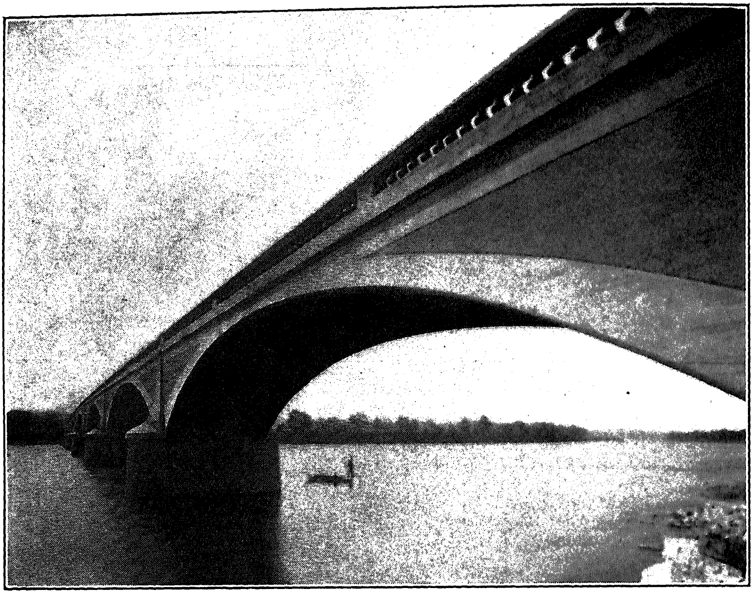


FIG. 59.—Adams Street bridge, Troy, Ohio.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 60.—Georgetown bridge over Wabash River, Georgetown, Indiana.

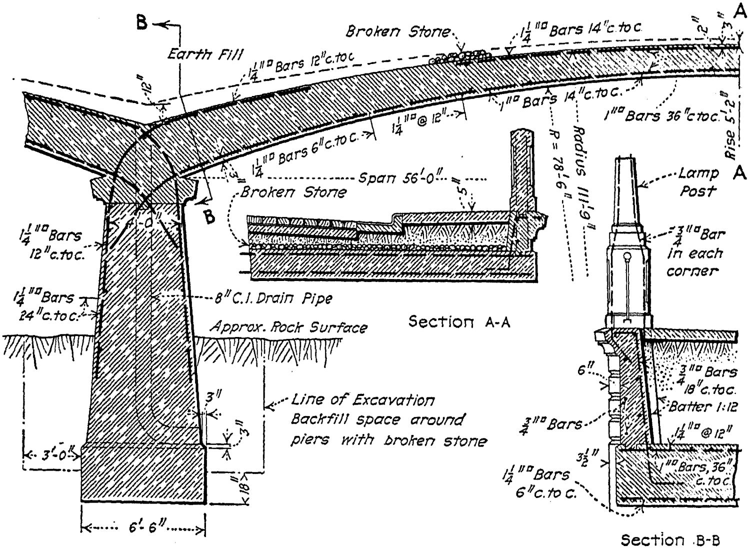
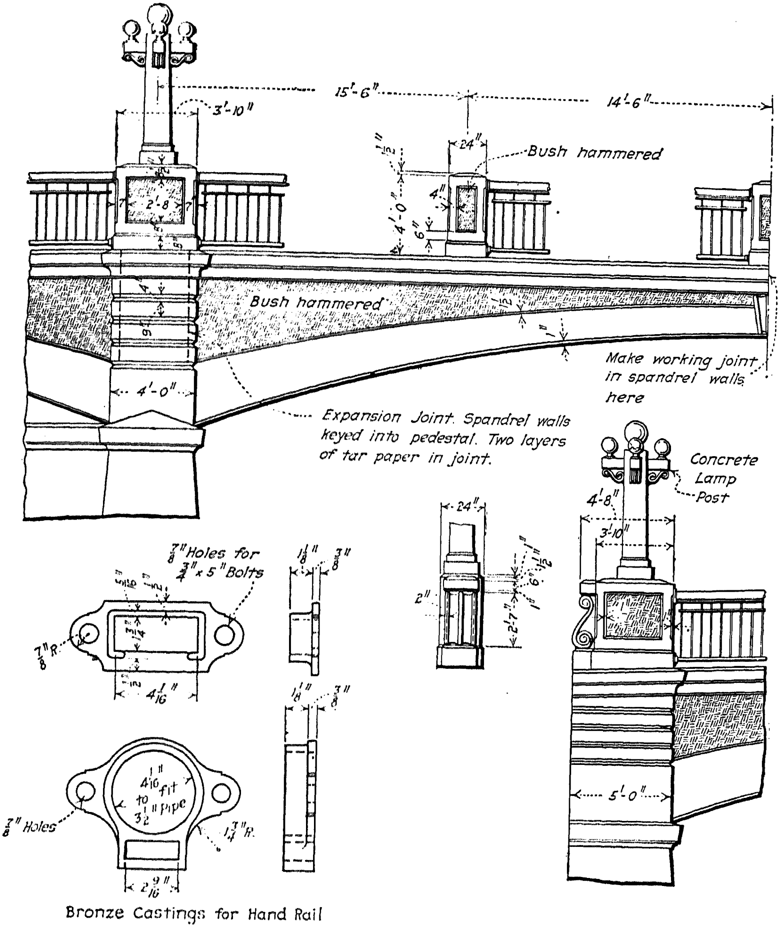
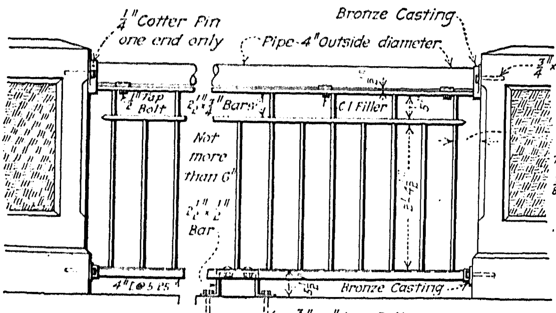


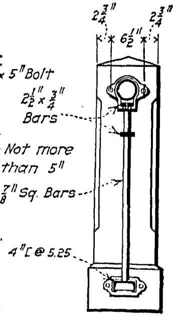
FIG. 61*A*.—Details of Pine Street bridge, Lima, Ohio.


15'-6"
14'-6"
3'-10"
24"
Bush hammered
1/2"
4'-0"
4"
6"
2'-8
Bush hammered
4'-0"
1"
Make working joint in spandrel walls here
Expansion Joint. Spandrel walls keyed into pedestal. Two layers of tar paper in joint.
Concrete Lamp Post
4'-8"
3'-10"
5'-0"
24"
1"
6"
1/2"
2"
2'-7"
7/8" Holes for 3/4"×5" Bolts
7/8"R
4 1/16"
1 1/8"
3/8"
4 1/16" fit to 3 1/2" Pipe
7/8" Holes
1 3/4"R
2 9/16"

Bronze Castings for Hand Rail



Details of Railing

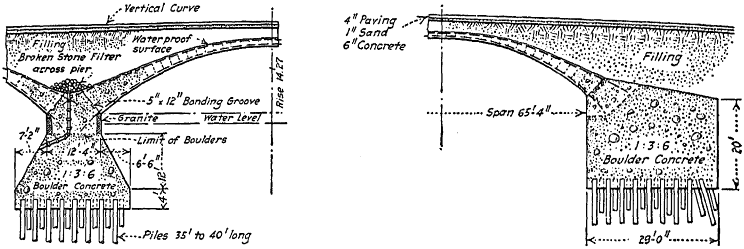


Section

Portland cement mortar, with joints of not more than $\frac{1}{4}$ in. Before the mortar had set the joints were raked out for a depth of 3 in. and pointed with 1:1 Portland cement mortar.

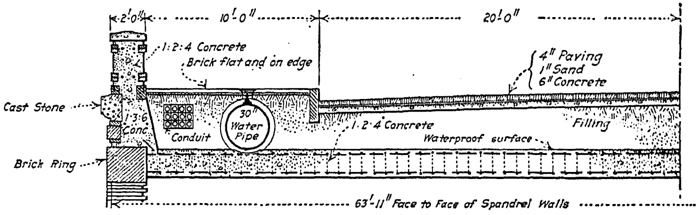
"The bricks used for facing the arch rings and for the ornamental courses are up-and-down, hard-burned, dark, water-struck bricks. They are laid in 1:3 Portland cement mortar, except the outer 4-in. faces where "Puzzolan" cement is used. One pail of lime putty was allowed to be added for each barrel of cement."

A counterforted type of spandrel wall is shown in Fig. 63. These walls are 12 in. thick and are reinforced on both faces with a double system of rods. The counterforts occur at about 9-ft. intervals, and cantilever brackets are placed at these counterforts to support the sidewalks. The following description is taken from Engineering Record, issue of February 22, 1913.



Cross Section of Pier and Half Longitudinal Section of 76'-8" Central Span

Cross Section of Abutment and Half-Longitudinal Section

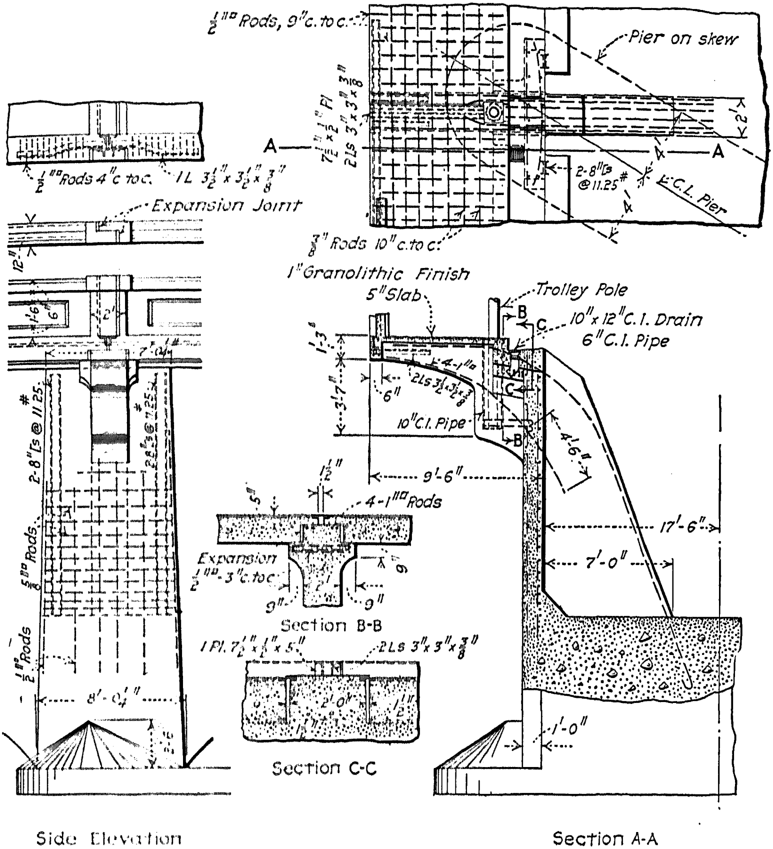


Half Cross Section at Crown of 76'-8" Central Span

Fig. 62.—Details of Larz Anderson bridge over the Charles River, Cambridge and Boston, Mass.

"The entire width of the arch ring between outside faces of spandrel walls is 35 ft., and the roadway above is 39 ft. wide, thus giving an overhang of 2 ft. on each side of the bridge between the curb lines. This 2-ft. overhang constitutes the concrete gutter of the roadway and, as such, will be subject to heavy concentrated wheel loads coming upon the cantilever section. It was, therefore, built as a heavily-reinforced concrete beam. This beam is 2 ft. 9 in. wide, having a depth of 15 in.

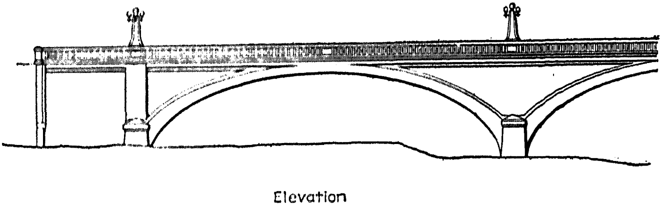
at the spandrel wall and 10 in. at the curb, and is reinforced with fourteen $\frac{3}{4}$-in. rods, with additional reinforcement at the brackets.



Side Elevation

Section A-A

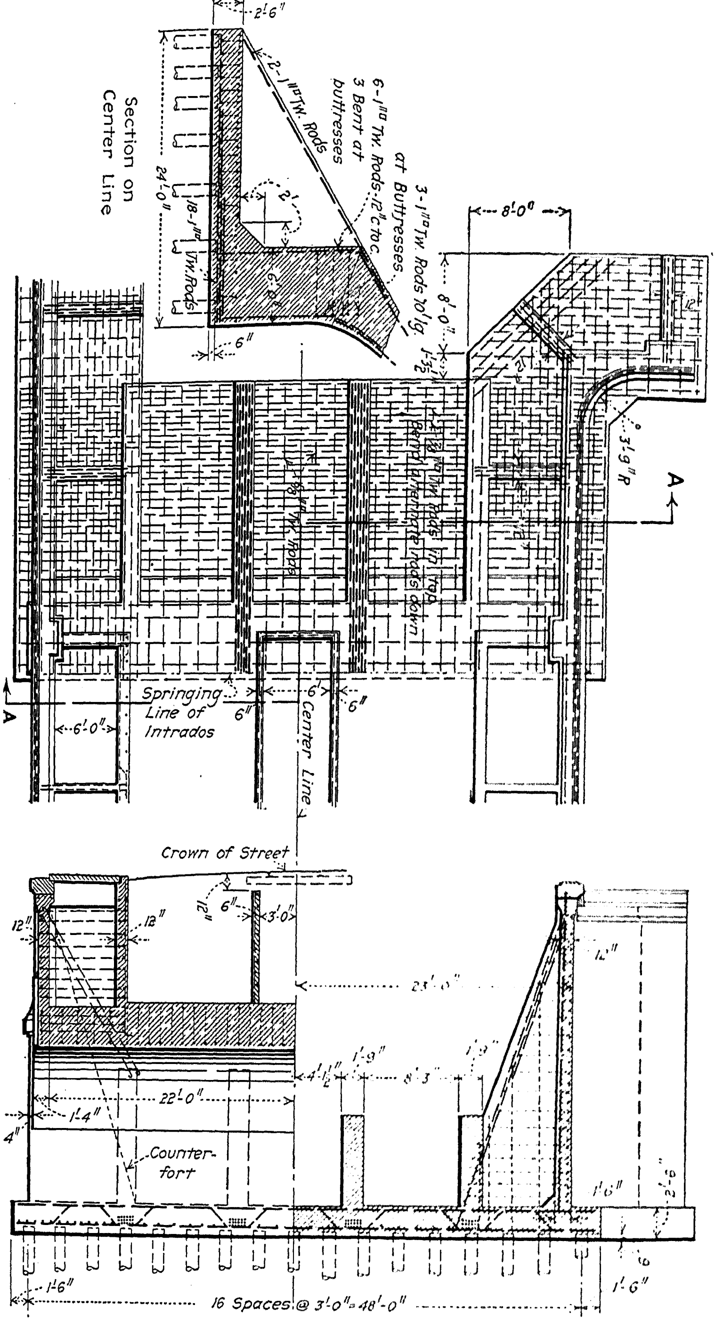
FIG. 63.—Counterforted spandrel wall, highway bridge at Ansonia, Conn.



Elevation

FIG. 64A.—Elevation of west end of Olentangy River bridge on King Avenue, Columbus, Ohio.

"The bridge is designed for trolley traffic, and provision is made for the trolley poles by anchoring sections of 10-in. cast-iron waterpipe in the


2'-6"
2-1" Tw. Rods
6-1" Tw. Rods-12" c to c. 3 Bent at buttresses
3-1" Tw. Rods 10' lg. at Buttresses
Section on Center Line
24'-0"
18-1" Tw. Rods
2'
6'-0"
6"
8'-0"
8'-0"
1'-3½"
12"
12"
3'-9" R
A
1-1⅛" Tw. Rods in top Bend alternate rods down
1-⅞" Tw. Rods
Springing Line of Intrados
6'-0"
6"
6'
6"
A
Center Line
Crown of Street
12"
6"
3'-0"
12"
12"
23'-0"
12"
4'-1½"
1'-9"
8'-3"
1'-9"
22'-0"
1'-4"
4"
Counter-fort
1'-6"
2'-6"
1'-6"
1'-6"
16 Spaces @ 3'-0" = 48'-0"

Section A-A

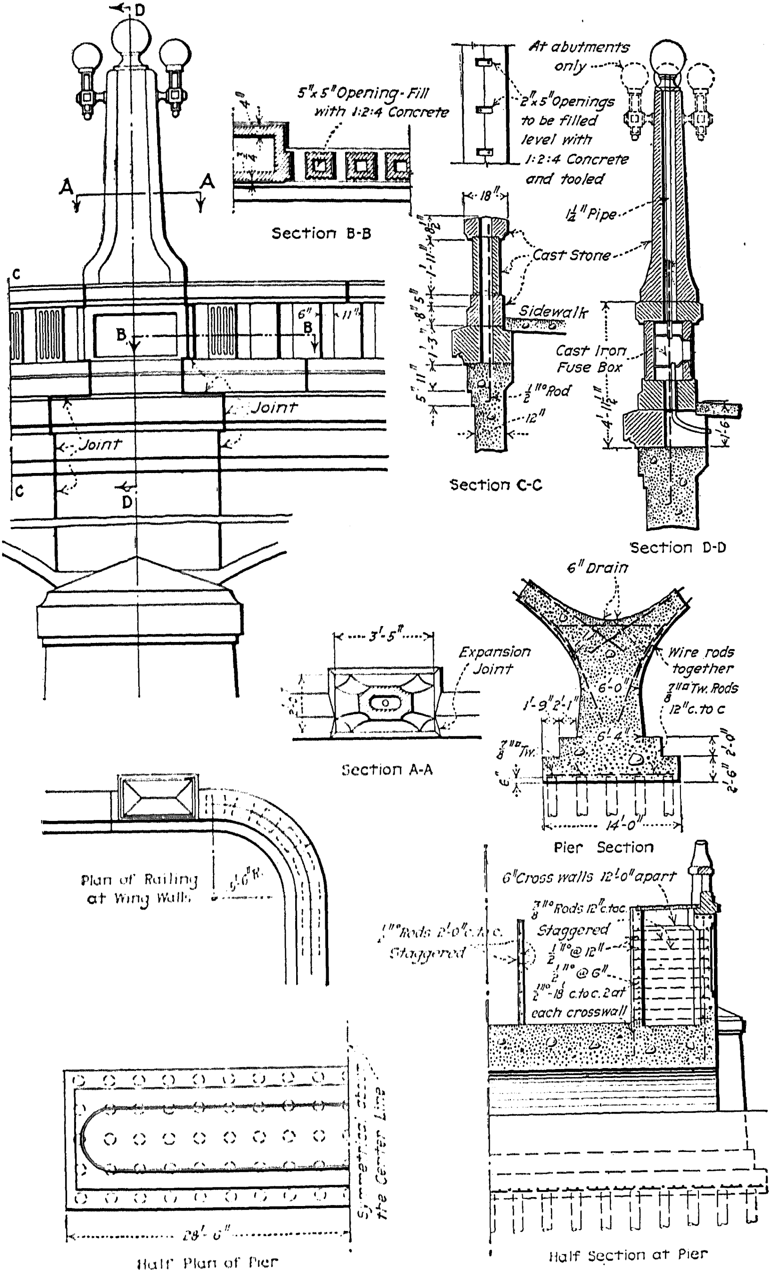


FIG. 64C.—Details of Olentangy River bridge on King Avenue, Columbus, Ohio.

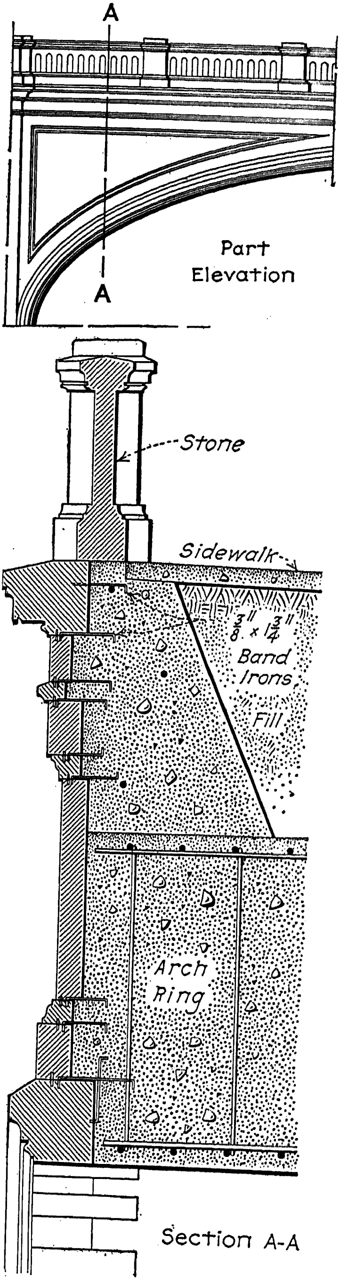
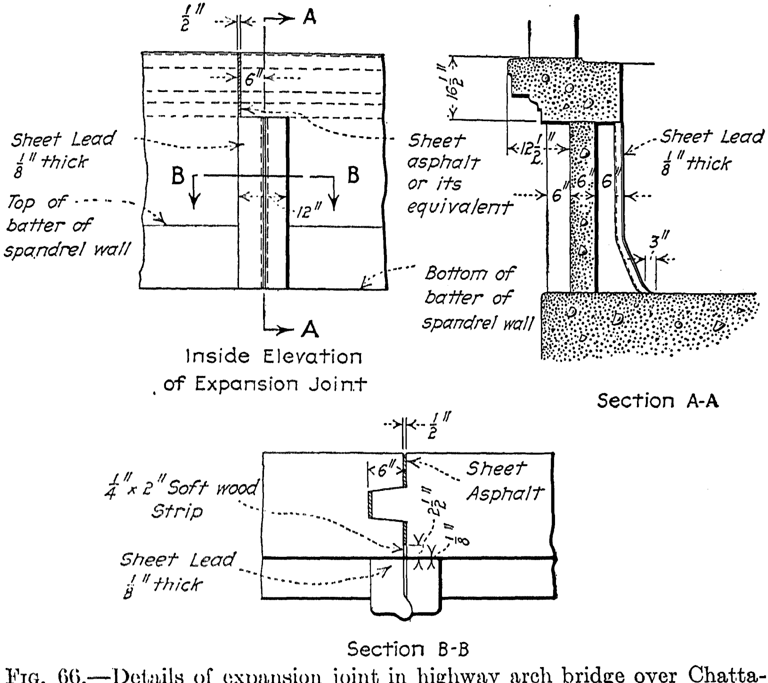


FIG. 65.—Details of stone facing of Michigan Street bridge over St. Joseph River, South Bend, Ind.

brackets of the piers and abutments. Drainage is provided by means of 6-in. cast-iron drains in the gutters over the piers."

Details of the arch bridge over the Olentangy River on King Avenue, Columbus, Ohio, is shown in Figs. 64*A*, 64*B*, and 64*C*. The type of spandrel walls without pilasters over piers should be noted. Since the space beneath each sidewalk is hollow, the inner wall was designed as a slab with the principal steel placed horizontally between cross walls. The longitudinal walls under the car track were employed to prevent the usual settlement of the track when laid on a new fill. The ties were laid directly on top of these walls and earth filling was dumped both sides of, and also between, the longitudinal walls.

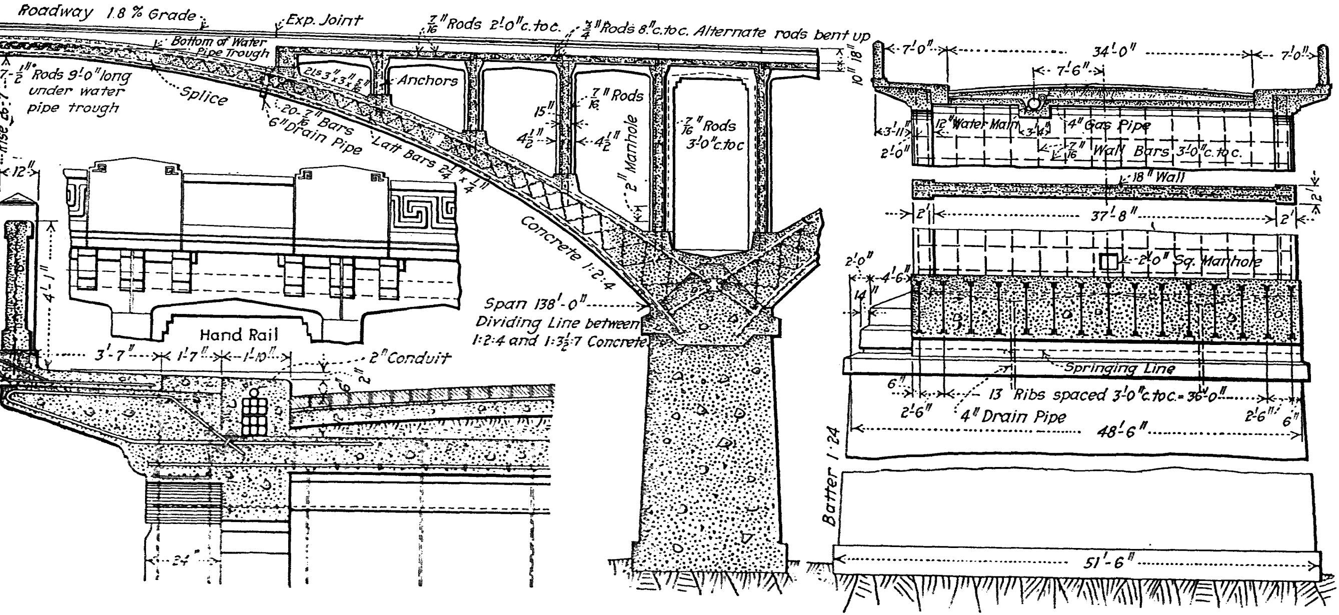


1/2"
A
6"
Sheet Lead 1/8" thick
B
B
12"
Top of batter of spandrel wall
A
Inside Elevation of Expansion Joint
16 1/2"
12 1/2"
Sheet asphalt or its equivalent
6"
6"
6"
Sheet Lead 1/8" thick
3"
Bottom of batter of spandrel wall
Section A-A
1/2"
6"
Sheet Asphalt
1/4" x 2" Soft wood Strip
2 1/2"
8"
Sheet Lead 1/8" thick
Section B-B

FIG. 66.—Details of expansion joint in highway arch bridge over Chattahoochee River at Columbus, Georgia.

An earth-filled arch faced entirely with stone is shown in Fig. 65. The bonding irons should be noted.

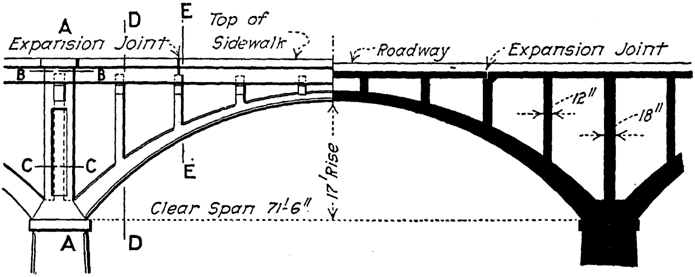
Drains should be placed on each side of the roadway of a concrete bridge at intervals of 30 to 40 ft. when the roadway is level and about every 100 ft. when on a grade. These drains should

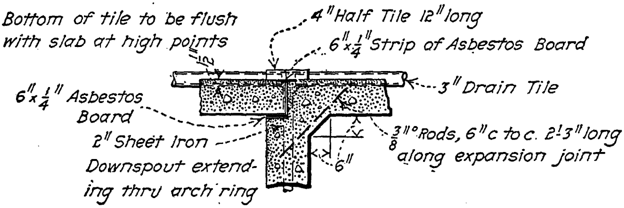

Roadway 1.8 % Grade
Exp. Joint
$\frac{7}{16}$" Rods 2'-0" c. to c.
$\frac{3}{4}$" Rods 8" c. to c. Alternate rods bent up
Bottom of Water Pipe Trough
7-$\frac{1}{2}$" Rods 9'-0" long under water pipe trough
Splice
2 Ls 3" x 3" x $\frac{5}{16}$"
Anchors
20-$\frac{7}{16}$" Bars
6" Drain Pipe
Latt Bars $2\frac{1}{4}$" x $\frac{1}{4}$"
15"
$4\frac{1}{2}$"
$\frac{7}{16}$" Rods
2" Manhole
$\frac{7}{16}$" Rods 3'-0" c. to c.
10"
18"
Concrete 1:2:4
Span 138'-0"
Dividing Line between 1:2:4 and 1:$3\frac{1}{2}$:7 Concrete
12"
4'-1"
Hand Rail
3'-7"
1'-7"
1'-10"
2" Conduit
6"
2"
24"
7'-0"
34'-0"
7'-6"
12" Water Main
3'-4"
4" Gas Pipe
3'-11"
2'-0"
$\frac{7}{16}$" Wall Bars 3'-0" c. to c.
18" Wall
2'
37'-8"
2'-0" Sq. Manhole
4'-6"
14"
Springing Line
13 Ribs spaced 3'-0" c. to c. = 36'-0"
2'-6"
4" Drain Pipe
48'-6"
Batter 1:24
51'-6"

Detail of Sidewalk

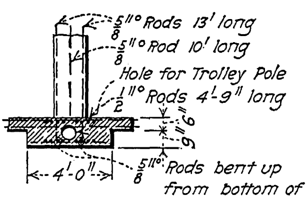
Section of Bridge at Pier

FIG. 67.—Details of Broadway bridge across the Oswego River at Fulton, N. Y.

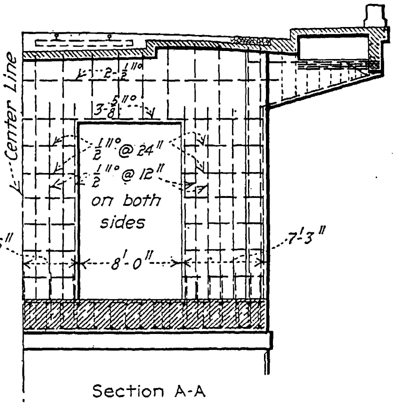



Bottom of tile to be flush with slab at high points
4" Half Tile 12" long
6"x1/4" Strip of Asbestos Board
1/2"
3" Drain Tile
6"x1/4" Asbestos Board
2" Sheet Iron Downspout extending thru arch ring
3/8"° Rods, 6" c to c. 2'-3" long along expansion joint
6"

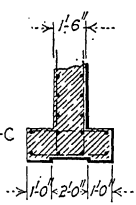
Section through Expansion Joint at Center Drain



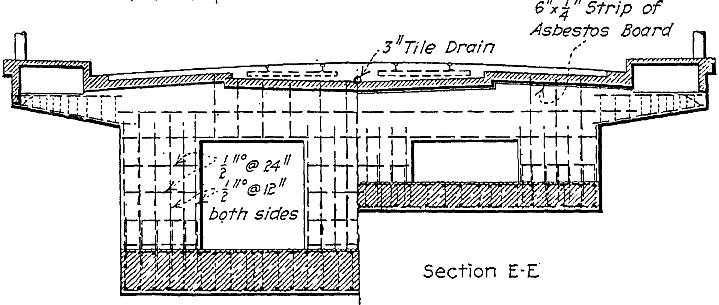
Section B-B



Section A-A



Section C-C



Section E-E

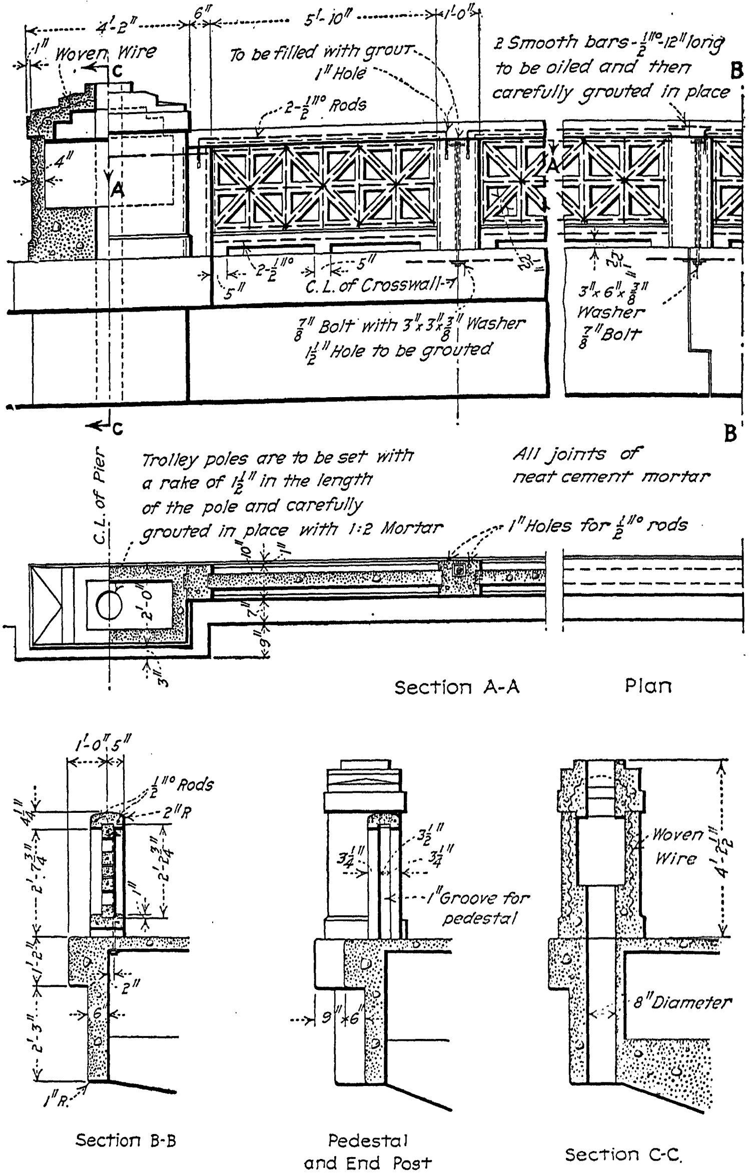
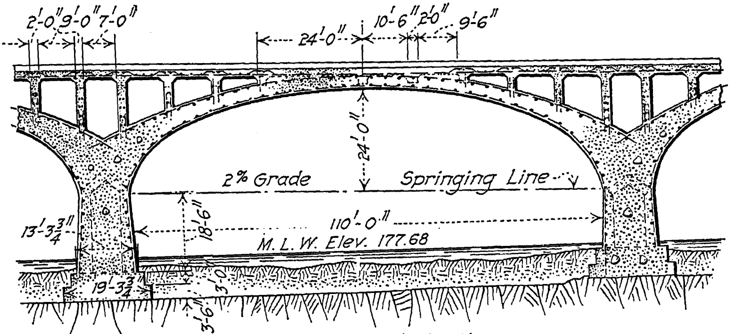


FIG. 68*B*.—Details of the Dallas-Oak Cliff viaduct, Dallas, Texas.

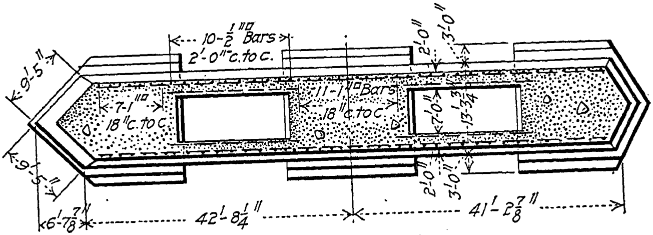
have a diameter of not less than 3 in. The minimum area of a drain in square inches may be computed by the formula

$$a = \frac{A}{200}$$

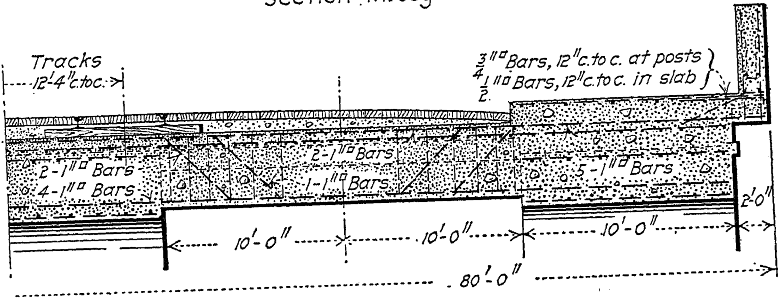
where A = area of the surface drained in square feet.



Longitudinal Section



Section through Pier



Half Section through Crown of 110-foot Arch

FIG. 69.—Details of Penn Street viaduct, Reading, Pa.

For remarks concerning expansion joints, see Art. 32. One type of expansion joint in a simple cantilever wall is shown

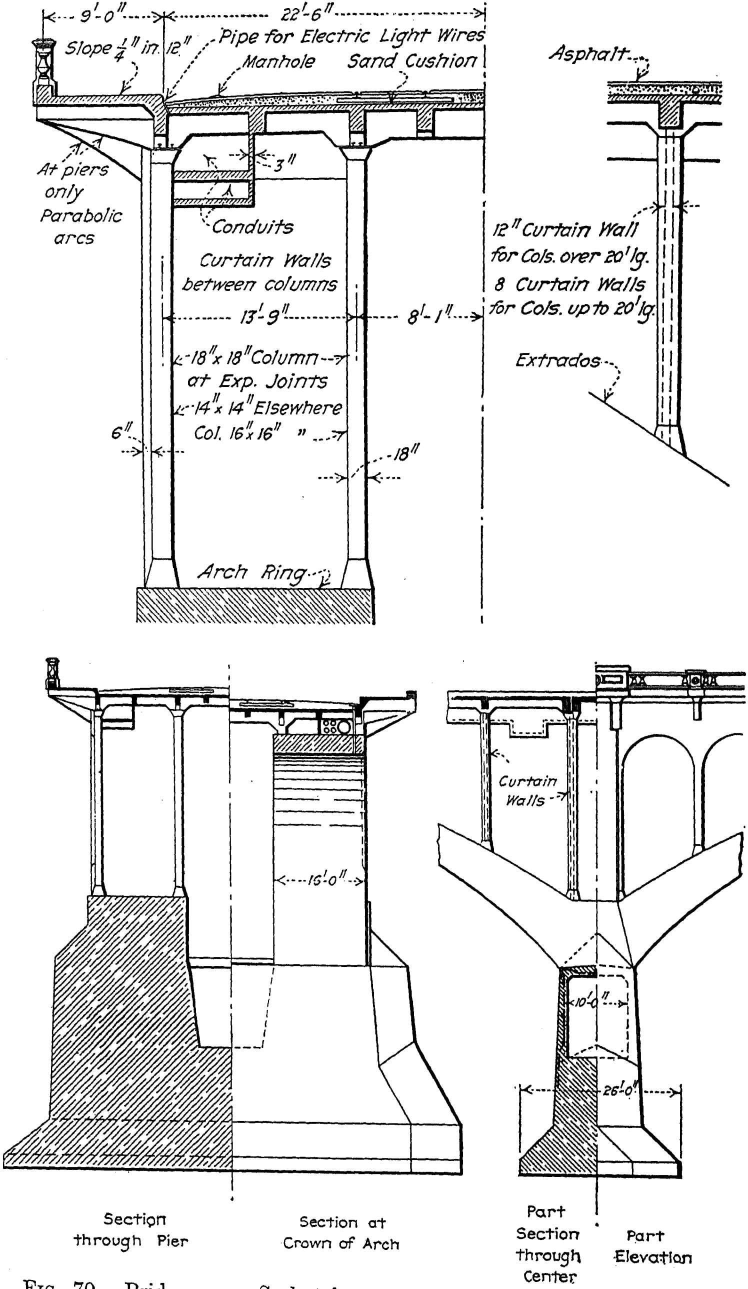

9'-0"
22'-6"
Slope 1/4" in 12"
Pipe for Electric Light Wires
Manhole
Sand Cushion
Asphalt
At piers only Parabolic arcs
3"
Conduits
Curtain Walls between columns
12" Curtain Wall for Cols. over 20' lg.
8 Curtain Walls for Cols. up to 20' lg.
13'-9"
8'-1"
18"x 18" Column at Exp. Joints
14"x 14" Elsewhere
Col. 16"x 16" "
Extrados
6"
18"
Arch Ring
Curtain Walls
15'-0"
10'-0"
26'-0"
Section through Pier
Section at Crown of Arch
Part Section through Center
Part Elevation

FIG. 70.—Bridge over Saskatchewan River at Saskatoon, Canada.

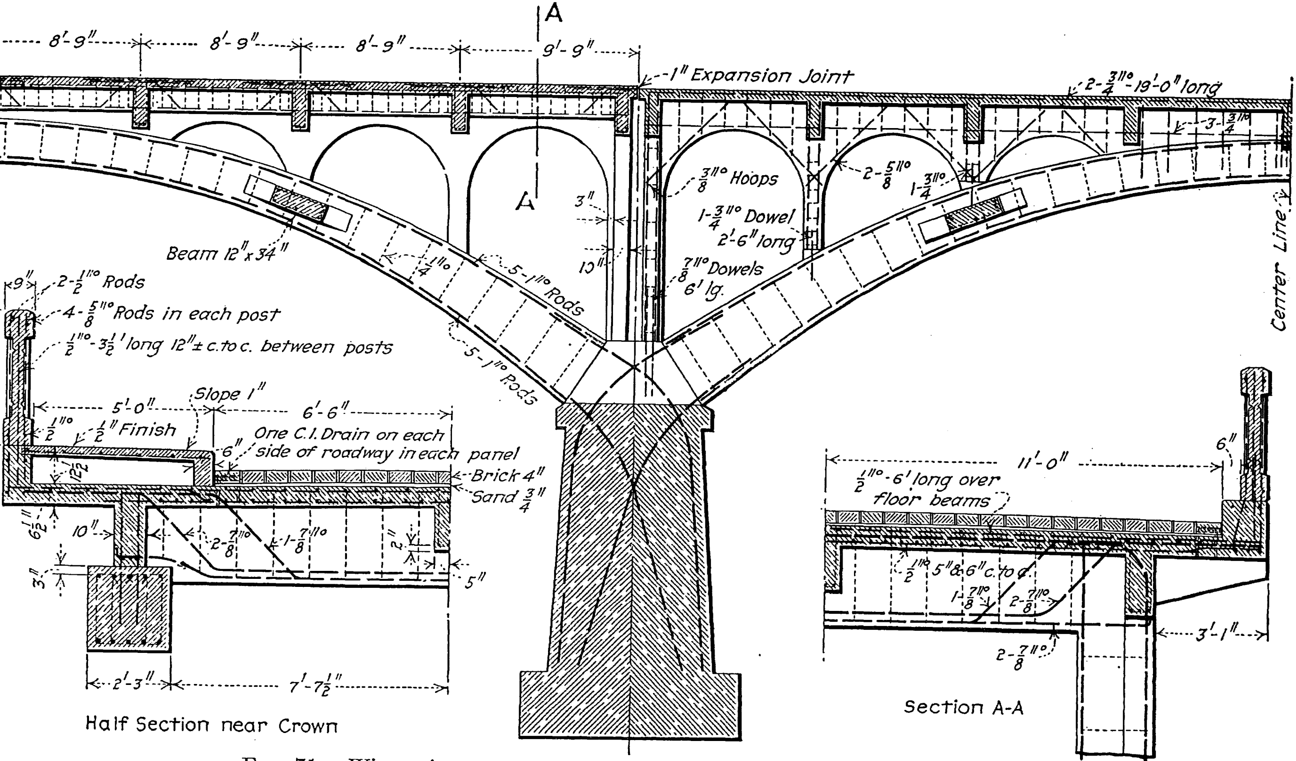
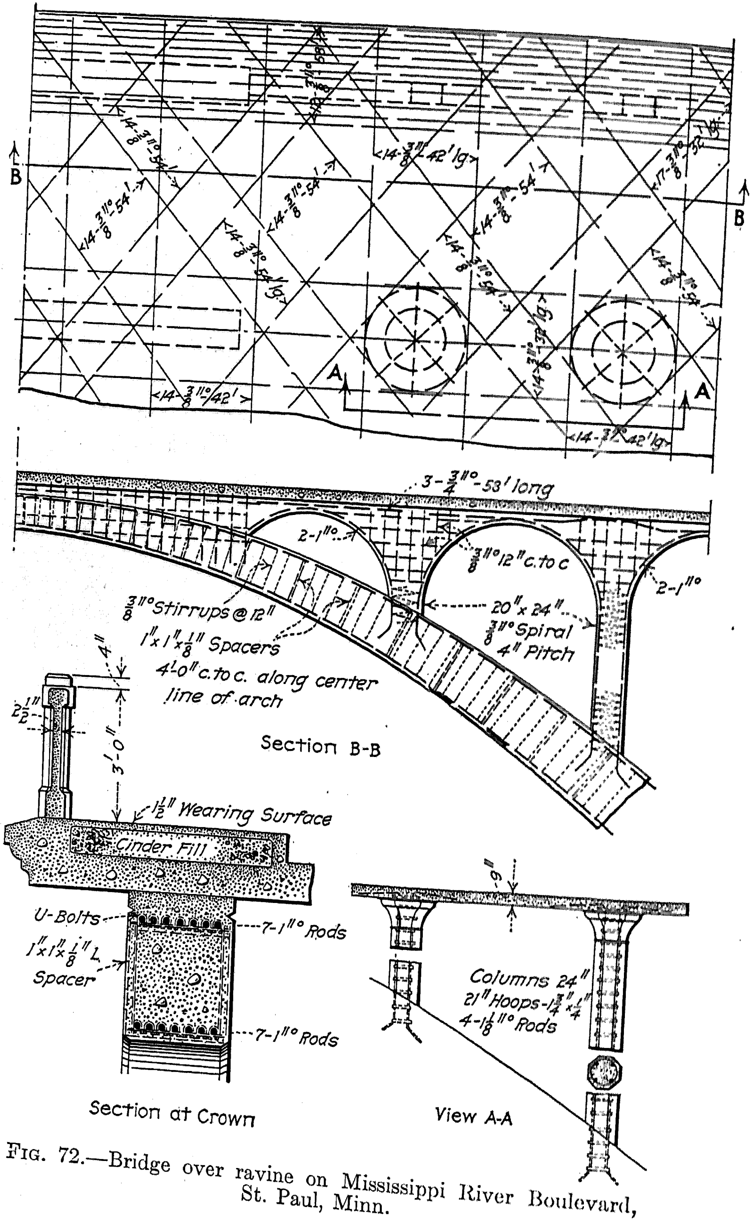


FIG. 71.—Wisconsin approach to high wagon bridge at Winona, Minn.



Section B-B

Section at Crown

View A-A

FIG. 72.—Bridge over ravine on Mississippi River Boulevard, St. Paul, Minn.

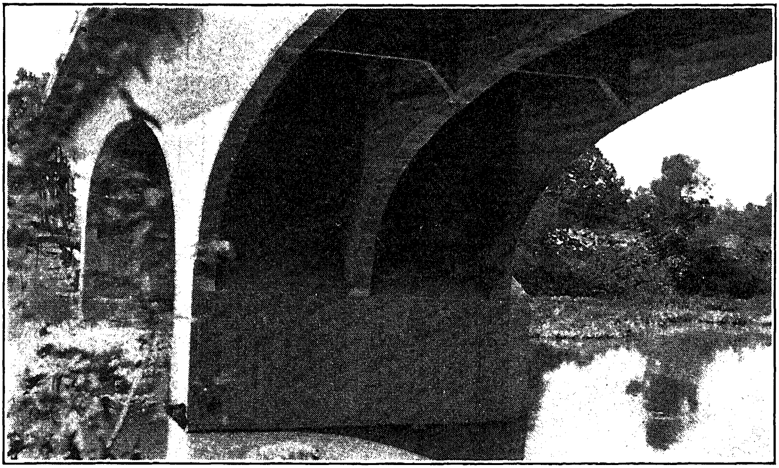


FIG. 73.—Highway bridge at Covington, Ohio.

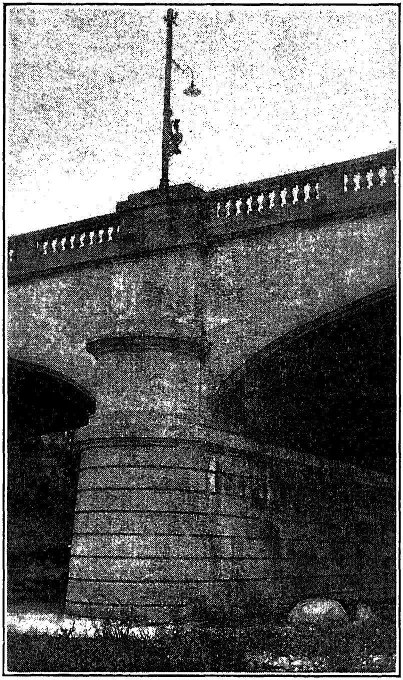


FIG. 74.—Third Street bridge, Dayton, Ohio.

Other types of expansion joints may be found in Chapter XIII.

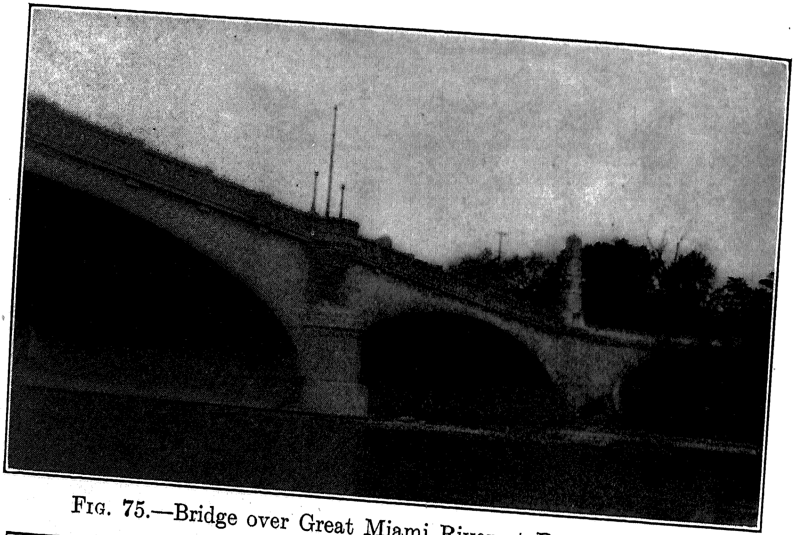
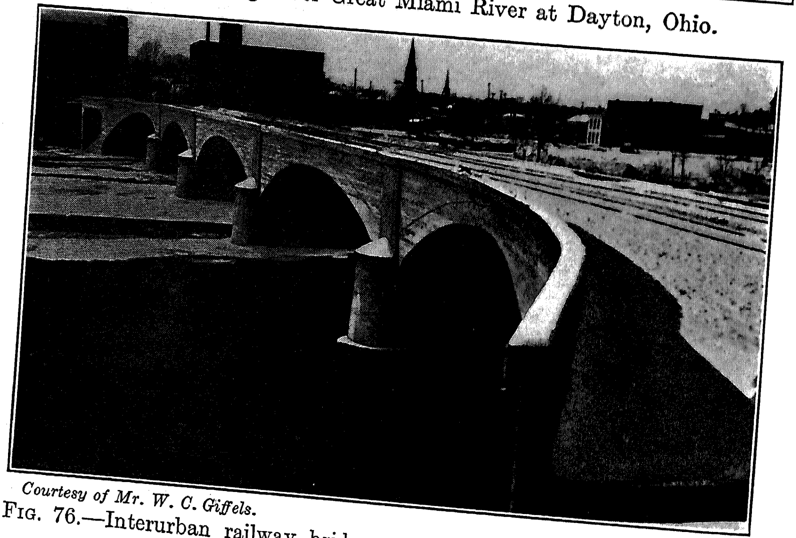


FIG. 75.—Bridge over Great Miami River at Dayton, Ohio.



Courtesy of Mr. W. C. Giffels.
FIG. 76.—Interurban railway bridge over Grand River, Grand Rapids, Michigan.

50. Spandrel Details in Open-spandrel Bridges.—The general types of open-spandrel bridges have been described in Art. 3. Figs. 67 to 72 inclusive will serve to illustrate details of some of these types. Other details may be found in Chapter XIII.

Fig. 67 shows a full-barrelled arch reinforced with typical Melan trusses made up of 3 in. by 3 in. by $\frac{5}{16}$-in. angles and $2\frac{1}{4}$ in. by $\frac{1}{4}$-in. lattice bars. The floor system is carried on a series of transverse spandrel walls and the floor slab is provided with expansion joints as shown. The sidewalks leave an overhang of about 3 ft. and are supported on cantilever brackets. The cantilever section of the sidewalk is cast in units 5 ft. long and laid in place. Tile conduits are provided under each sidewalk for the necessary wires, and a 4-in. gas pipe and 12-in. water main are laid in specially-designed reinforced-concrete troughs beneath the roadway.

Transverse spandrel walls with openings to save material are shown in Figs. 68*A* and 68*B*. The method of carrying the sidewalk should be noted.

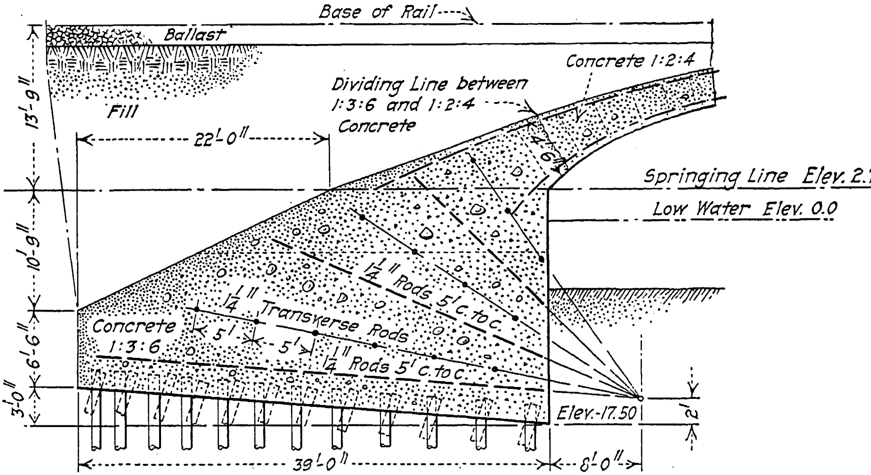


Fig. 77.—Abutment of causeway arch construction, Galveston, Texas.

The details of the ribbed arches in Figs. 69, 70, 71, and 72 should need no explanation. Fig. 73 shows a ribbed arch structure in which the ribs extend up to the floor level throughout each entire span.

The curtain walls between columns in Fig. 70 were considered simply as a bracing system. The columns were designed to carry all loads, but doubtless the curtain walls help to distribute the total loading. In taking the loading for the arch rings, it was assumed that the loading from columns was equally distributed over 12 ft. of arch ring instead of having two loads concentrated on an arch ring 16 ft. wide.

The resistance offered by piers to the passage of water varies with the type of starling. Experiments show that the value in this respect of the different shapes of piers is in the following order: first, elliptical horizontal sections; second, rectangular body with starlings formed by two circular arcs, tangent to the sides and described on the sides of an equilateral triangle; third, rectangular body with semicircular starlings; fourth, rectangular body with triangular starlings, the angle at the nose being 90°; and fifth, rectangular body without starlings. Piers of the third type mentioned are shown in Figs. 74, 75, and 76. Other types may be found in the many illustrations scattered throughout the book.

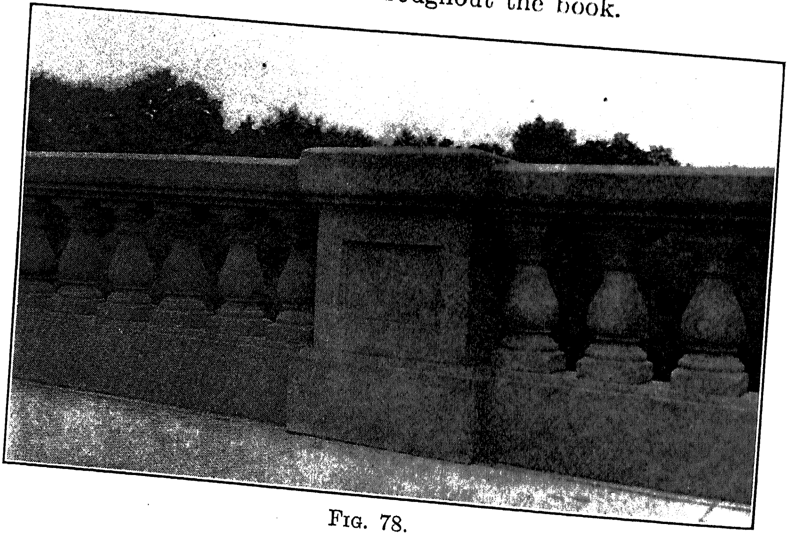
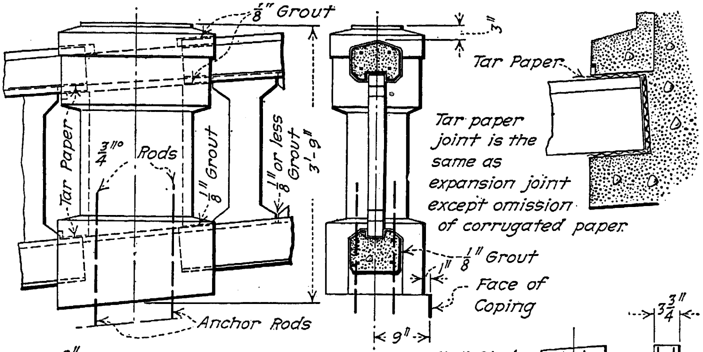
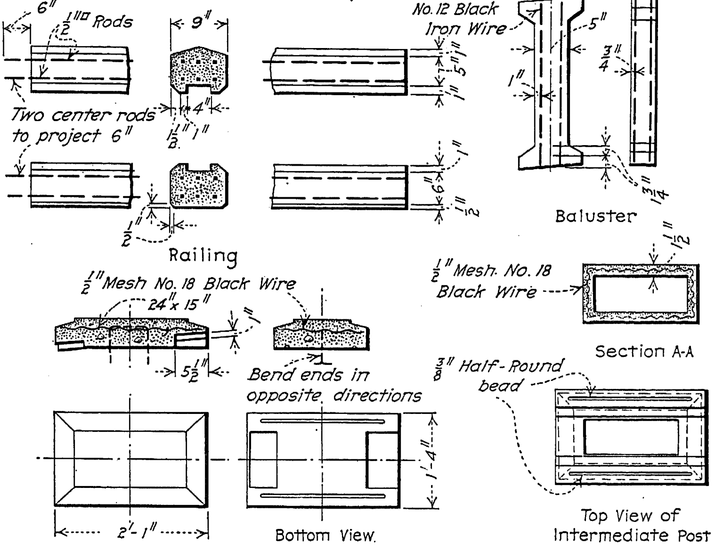


Fig. 78.

The ordinary type of abutment in earth-filled arches is shown in Plate I, Chapter IV. What might be called a buttressed abutment is shown in detail in Fig. 64*B*. Fig. 77 represents a very wide abutment in which a network of rods has been employed to make the entire abutment act as a unit. Hollow piers for ribbed-arch structures are shown in Figs. 69 and 70. See Chapter XIII for other examples of cellular construction.

52. Railing and Ornamental Details.—A common type of spindle balustrade is shown in Fig. 78. In such railings the spindles or balusters are usually the only members which are not cast in place. Expansion joints should be provided each side of the posts and also over the spandrel joints. Railing and ornamental details





Railing

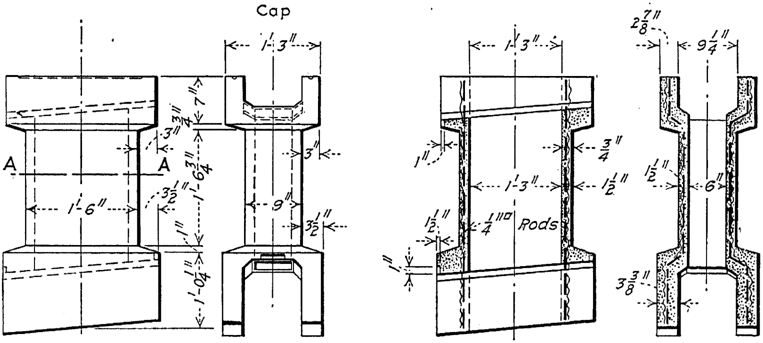
Baluster

Section A-A

Cap

Bottom View.

Top View of Intermediate Post

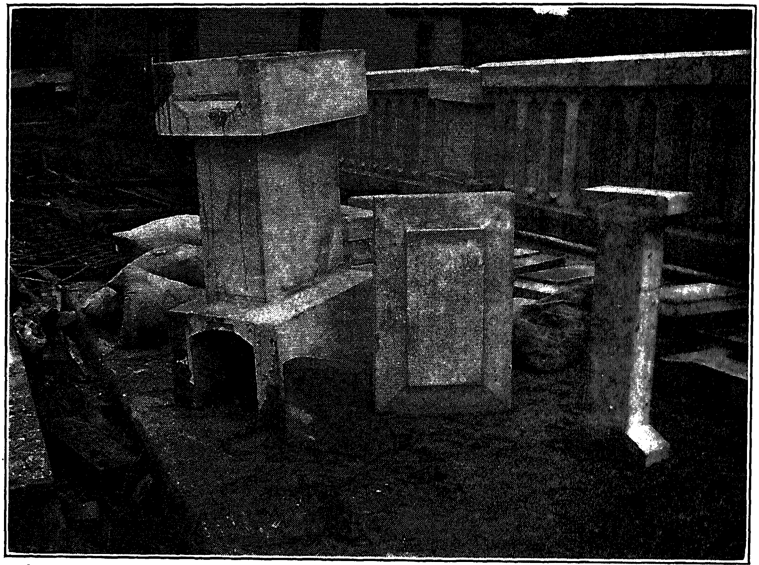


Intermediate Post

of various kinds are shown in Figs. 61*B*, 64*C*, 67, 68*B*, and in Chapter XIII.

A typical unit-built handrail is shown in Figs. 79 and 80. This particular design was used on the viaducts of the Kansas City Terminal Railway Company. The following description of the design and construction of this railing is taken from the Engineering Record, issue of Dec. 19, 1914:

"The railing consists of a top and bottom rail set parallel to the pavement and grooved to receive the 3¾ in. by 5-in. spindles, which are set truly vertical, regardless of the grade of the rails. For this purpose the tops and bottoms of the spindles are shaped to the required angle.



Courtesy of Horton Concrete Construction Co., Kansas City.

FIG. 80.—Unit-built handrail on Kansas City viaduct.

"The panels between the hollow posts are usually 20 ft. long, but vary from 10 to 22 ft. The bottom rail rests on blocks 3 in. above the floor to give a detached appearance. Both rails extend into the post beyond the post shell. Projecting reinforcing rods in the rails entering from the high side, bond with the green concrete subsequently filled into the posts, but an expansion joint has been secured on the opposite and lower side of the post by wrapping the rail ends in tar paper.

"A basket reinforcement of four ¼-in. rods wrapped with No. 2 wire held down by a wire embedded in the cap during its construction. This wire was forced down into the soft concrete after the shell was filled.

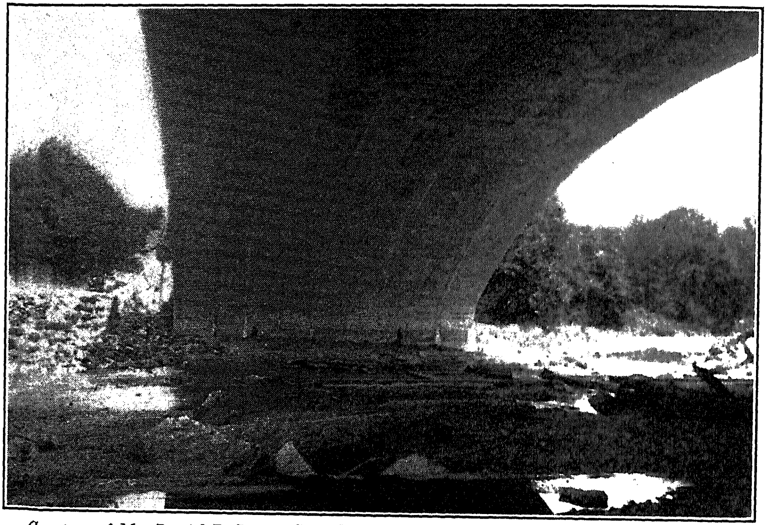
"Separate members were cast near the slab plant used to construct the large units for the subways, as described in the Engineering Record of Aug. 30, 1913, page 288. A pug mill, steam house, and gantry crane comprised the equipment. The pug mill was used to obtain uniformity of mixture, as mortar in it could be observed and the foreman was relied upon to obtain the proper consistency. Wood forms were employed to prevent craze and sand marks. In cold weather the steam house facilitated setting, while in summer 45 minutes in the steam retarded setting a sufficient amount to avoid crazing. The aggregate consisted of 1 part cement, 2½ parts Kaw River sand, considered exceptionally good, and 1 part of crusher-run soft native limestone passing a ¾-in. mesh. All measurements were made by hand in a 1-cu. ft. box.

"The rails were turned upon a sand cushion 24 hours after being poured. The posts were cast at three separate pourings so that shrinkage would not crack them at the points of top and bottom enlargement."

CHAPTER X

CONSTRUCTION OF ARCH BRIDGES

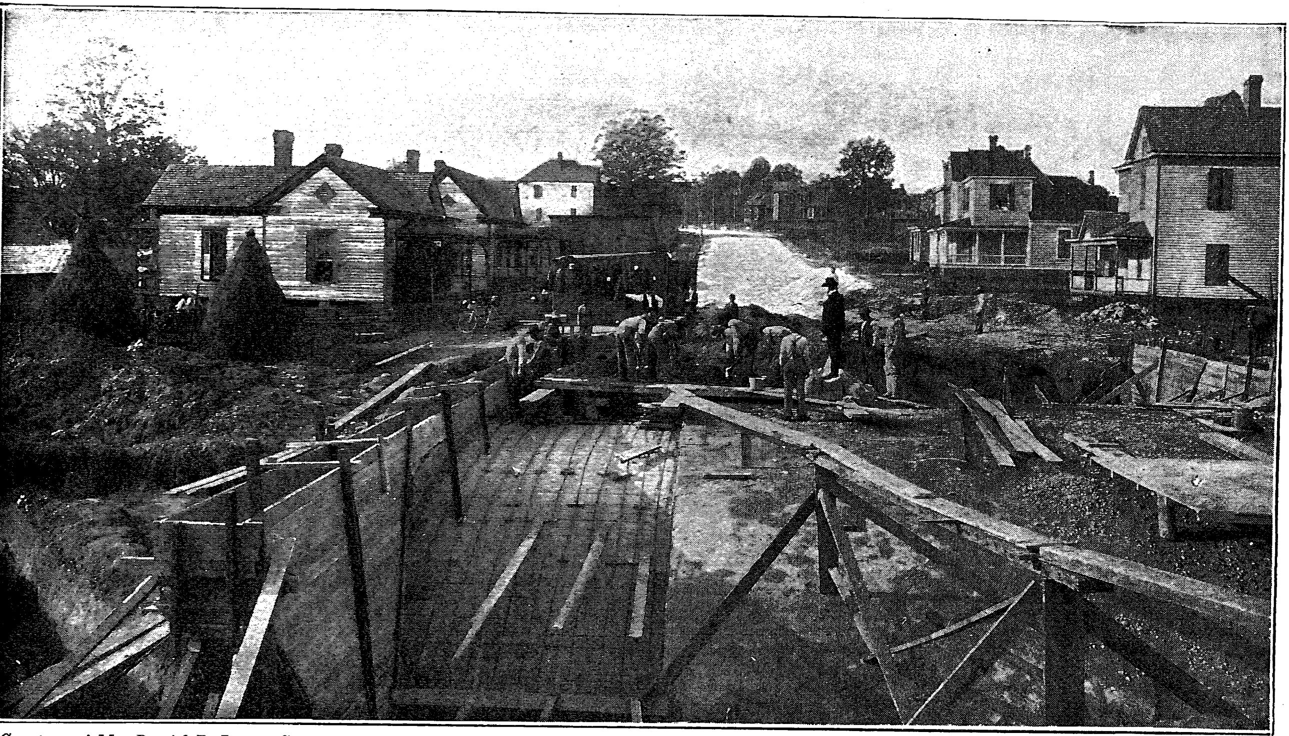
No attempt will be made in this chapter to describe at length the character of the materials required for concrete; the kind of lumber to use for forms; the types of forms for ordinary column, beam, and slab construction; the methods of bending and placing reinforcement; the proportioning, mixing, and placing of concrete; the finishing of concrete surfaces; or the various methods of waterproofing. These subjects are all treated in detail in Volume II. Special consideration, however, must be given in this volume to the methods of arch construction; the centering for arches; and the forms for spandrel walls, piers, and abutments.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

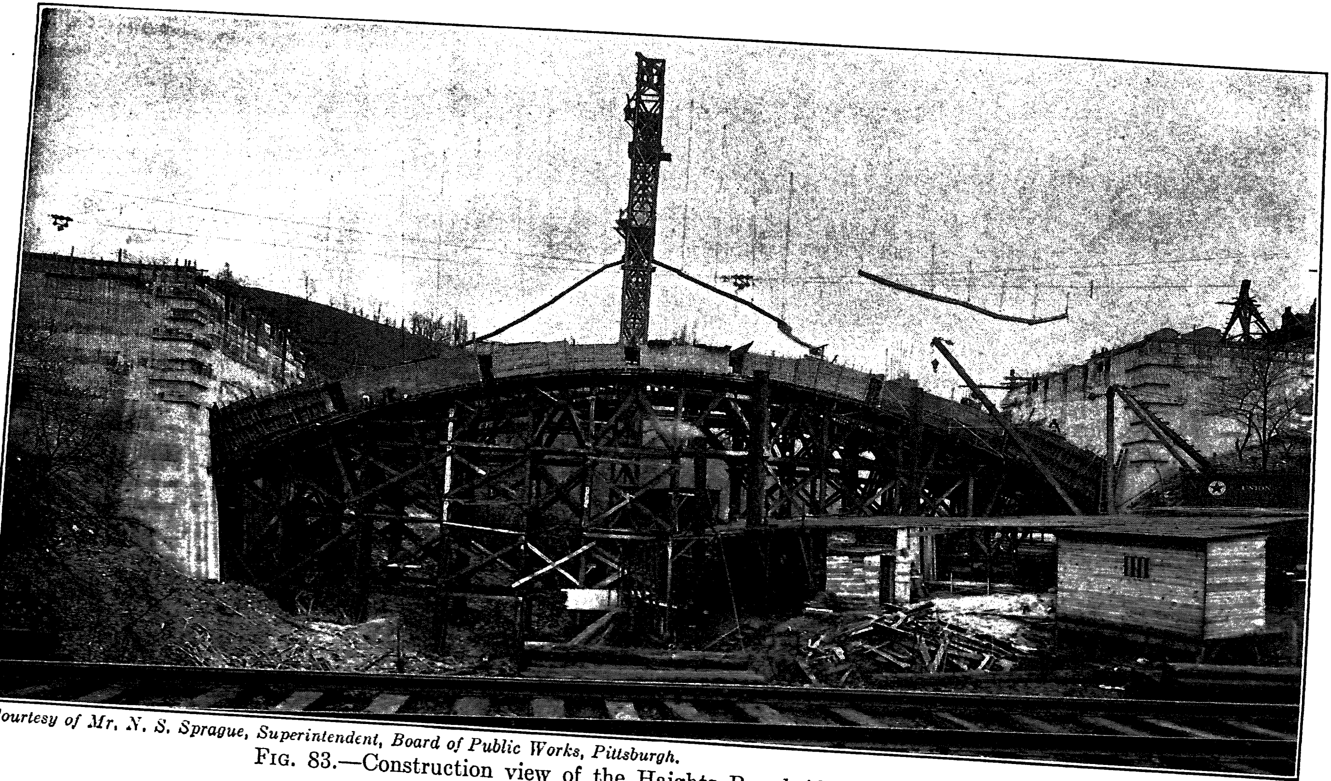
FIG. 81.—Harford Avenue bridge over Hering Run, Baltimore, Md. Showing method of construction of arch ring in longitudinal sections.

53. Arch-ring Construction.—Arch rings with span lengths less than about 90 ft. are usually constructed in longitudinal ribs 3 or 4 ft. wide (Figs. 81 and 82), or in fact of such width that one entire rib can be poured in approximately 1 day's time. In



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

G. 82.—Elizabeth Avenue bridge, Charlotte, North Carolina. View showing arch concreted in longitudinal rings; reinforcement in place for one 12-ft. concentric ring. Forms constructed for spandrel wall.



ourtesy of Mr. N. S. Sprague, Superintendent, Board of Public Works, Pittsburgh.

FIG. 83.—Construction view of the Haights Run bridge, Pittsburgh, Pa.

narrow arches the entire arch ring is sometimes poured at one operation. This method of construction has been successfully used for much greater spans than 90 ft. but, unless special care is taken to make the centering very stiff, the construction of any one rib may deform the arch center to such an extent as practically to strike the center under the completed ribs. Of course, the ribs should be poured continuously from each abutment toward the crown so as to obtain a symmetrical loading on the falsework and thus eliminate distortion of the centering as far as possible.

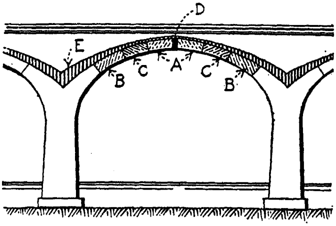


FIG. 84.

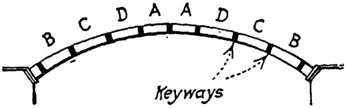
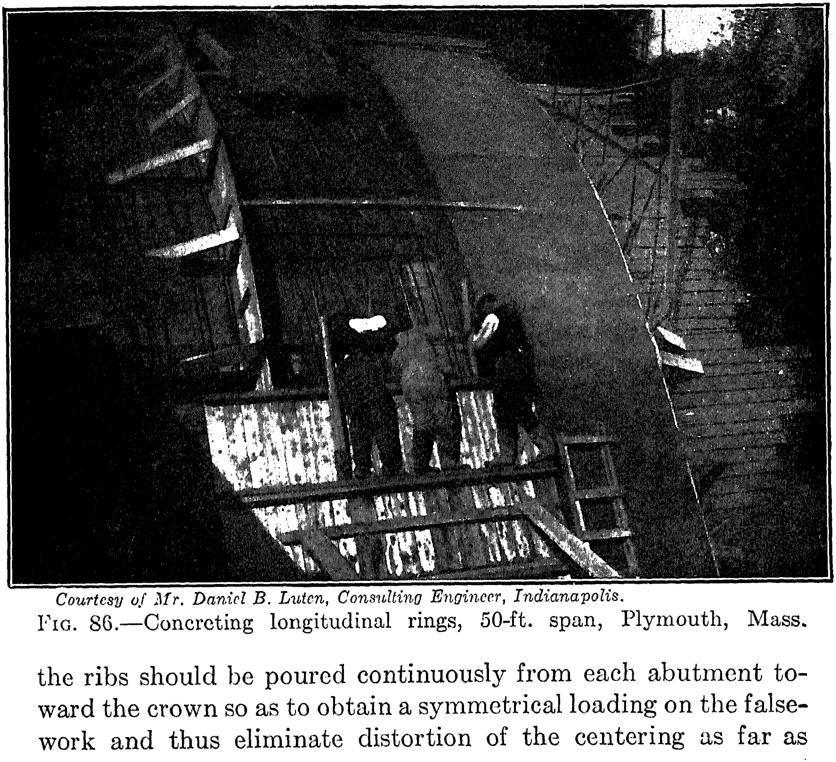
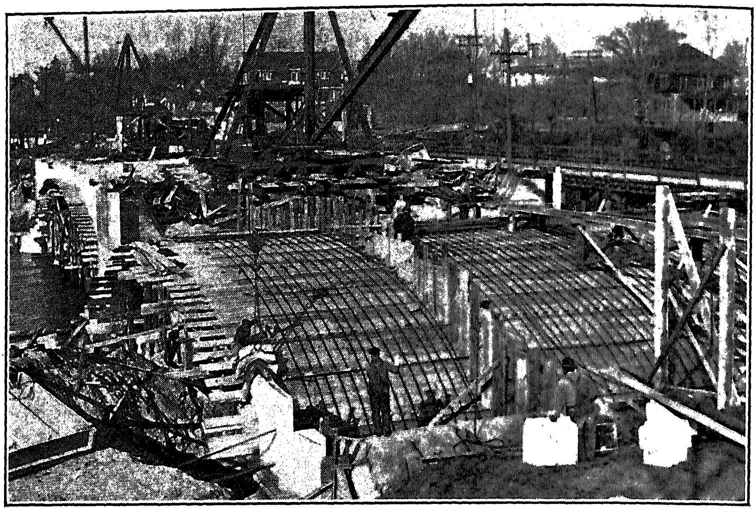


FIG. 85.



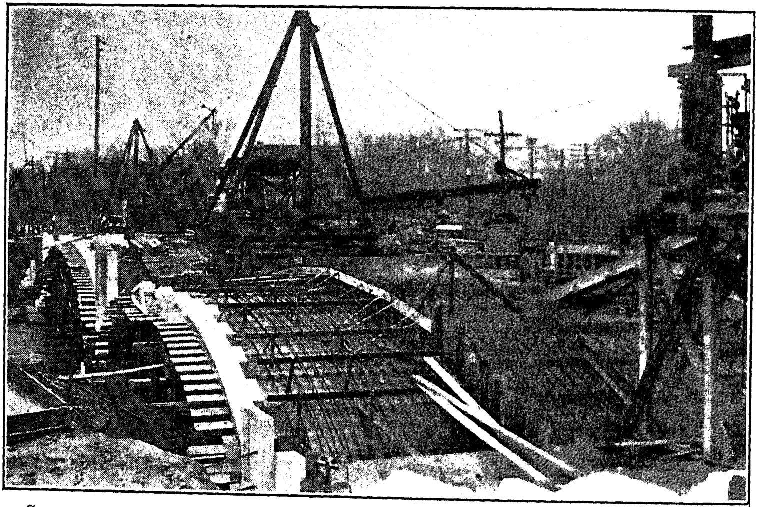
Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 86.—Concreting longitudinal rings, 50-ft. span, Plymouth, Mass.



Courtesy of Mr. Charles W. Cole, City Engineer, Mishawaka, Ind.

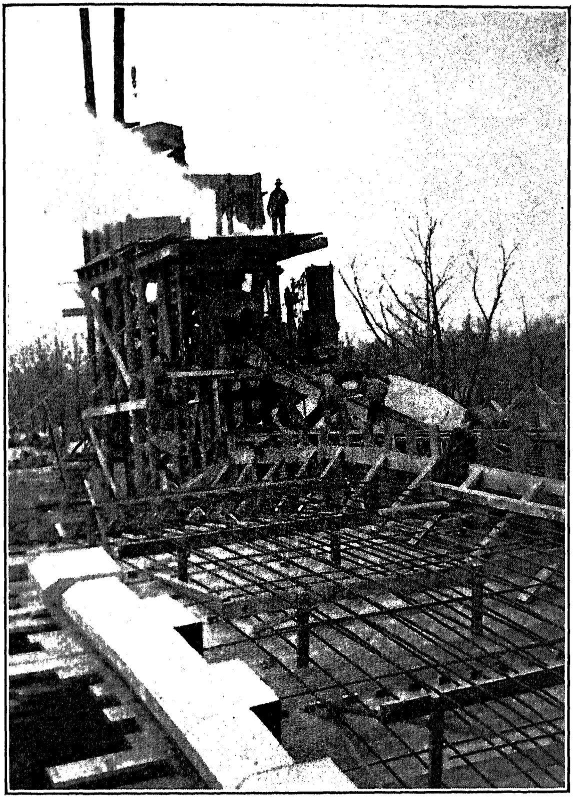
FIG. 87.—Construction view of North Michigan Street bridge, South Bend, Indiana.



Courtesy of Mr. Charles W. Cole, City Engineer, Mishawaka, Ind.

FIG. 88.—Construction view of North Michigan Street bridge, South Bend, Indiana.

For spans of 90 ft. or over it is usually preferable to construct an arch ring or arch rib by what is known as the alternate block or voussoir method. The arch is constructed in transverse blocks (Fig. 83) of such size that each block can be completed at one pouring, or with about a day's work. Obviously this method reduces shrinkage stresses in the arch ring to a minimum.

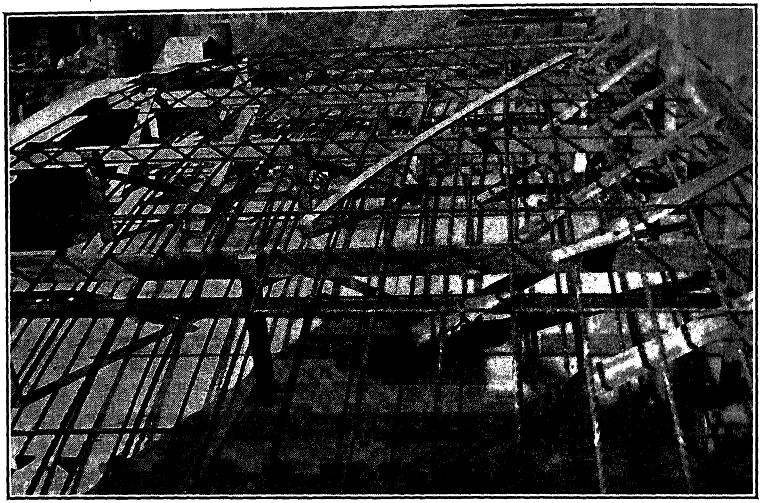


Courtesy of Mr. Charles W. Cole, City Engineer, Mishawaka, Ind.

FIG. 89.—Construction view of North Michigan Street bridge, South Bend, Indiana.

For the best results the blocks should be poured in such order as to give a uniform settlement of the centering, and also prevent the crown of the arch from rising as the lower arch loads are placed. If blocks close to the crown section are not placed before the blocks at the haunch and springing sections, the centering will rise at the crown and the placing of the crown loads will be likely to cause cracks at the middle of the haunch. Even in the construction of an arch by the longitudinal rib method, a temporary loading of the crown is often necessary.

The order followed in the construction of the Philadelphia and Reading R. R. bridge across the Delaware River at Yardley, Pa.—an earth-filled bridge with clear span of 90 ft. 9-in.—is shown in Fig. 84, the sections being concreted in the alphabetical order shown. The section *D* is the keying section, and the section *E* a haunching section (quite unusual construction) which was added after the lower portion of the arch ring was completed. The part of the arch ring close to the springing lines was placed monolithic with the piers and abutments, making what is called an umbrella form for the piers. In large arches this umbrella type of construction is frequently adopted. The pier forms in such cases are more expensive, but this increase in expense for the piers is usually more than offset by the saving in the falsework for the arch ring.



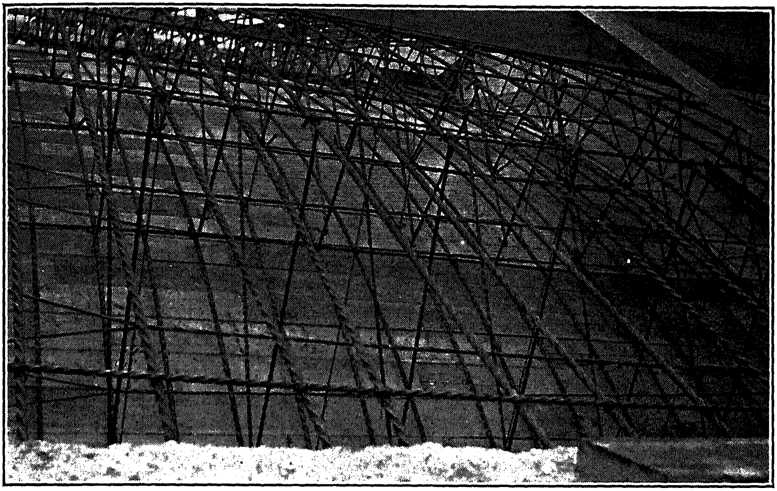
Courtesy of Mr. Charles W. Cole, City Engineer, Mishawaka, Ind.

Fig. 90.—North Michigan Street bridge, South Bend, Indiana. Reinforcement supported in place.

Fig. 85 shows the method of constructing the Larimer Ave. bridge at Pittsburgh—a bridge of the open-spandrel type, with two ribs, having a clear span of approximately 300 ft. The blocks were placed in alphabetical order and later the keys between them were concreted to make the closure.

In constructing an arch rib or arch ring by the alternate block method the individual sections or block spaces are closed off at the ends by timber bulkheads. On the steepest slopes of the lagging these bulkheads adjoining keying sections are held in place by temporary struts between voussoirs. A top form is usually needed for the block sections near the piers and abutments. This top form should be laid up as the concreting progresses. Fig. 86 shows a top form being used in the construction of an arch ring by the longitudinal rib method.

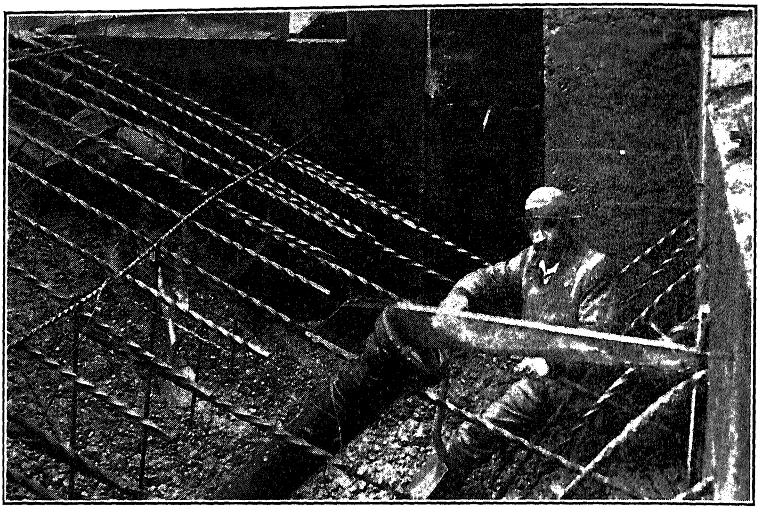
If arch reinforcement for large arches is put in place in long lengths, the settlement and deformation of the centering during the pouring of the concrete will cause buckling of the steel which will prevent the reinforcement from lying in its theoretical position. For this reason steel lengths should not exceed about 30 ft. and the splicing should occur in the keyways. An effort should be made to stagger the splices of adjacent rods and to locate the splices where the tension in the steel is a minimum.



Courtesy of Mr. Charles W. Cole, City Engineer, Mishawaka, Ind.

FIG. 91.—North Michigan Street bridge, South Bend, Indiana. Reinforcement in place held by overhead timbers.

Figs. 87 to 92 inclusive show the reinforcement in place in the North Michigan Street bridge at South Bend, Ind. The lower steel was wired together and blocked up as shown in Fig. 87; props were then erected for the upper steel (Figs. 88, 89, and 90); after which the diagonal rods were placed. Instead of knocking out these blocks and spacing boards as the concrete was brought up (a method sometimes followed), all the wood used in erection



Courtesy of Mr. Charles W. Cole, City Engineer, Michawaka, Ind.

Fig. 92.—North Michigan Street bridge, South Bend, Indiana. Concrete being put in place.

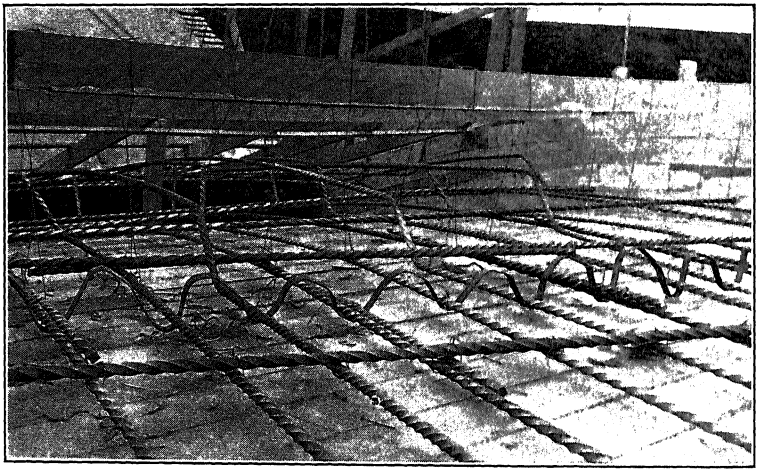
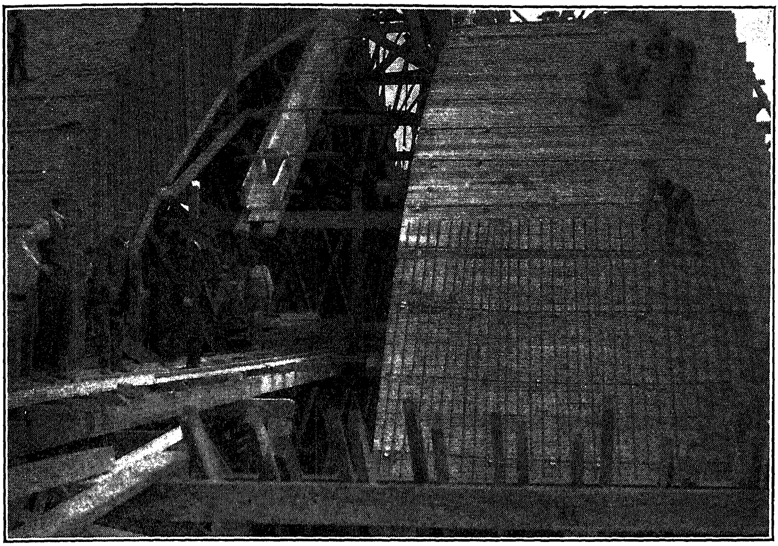


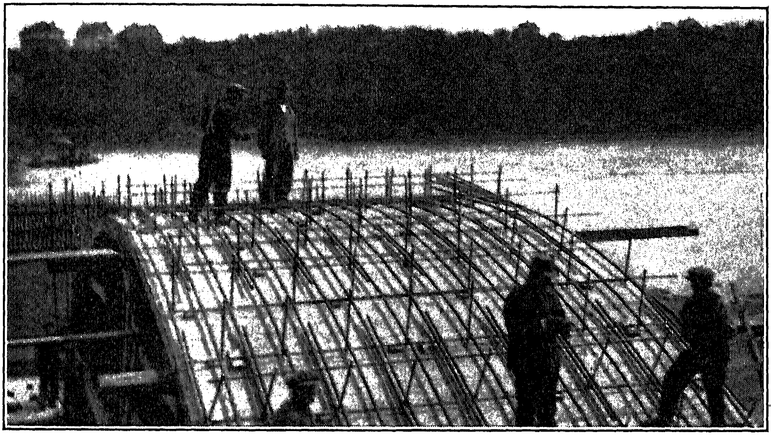
Fig. 93.—Arch reinforcement held in place by being wired to transverse timbers supported above the surface of the finished concrete. Bridge of Luten Design.

was removed before concreting was started and the system of rods was held in place by being wired to transverse timbers supported above the surface of the finished concrete (Fig. 91). By this method the likelihood of disturbing the steel during the pouring



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

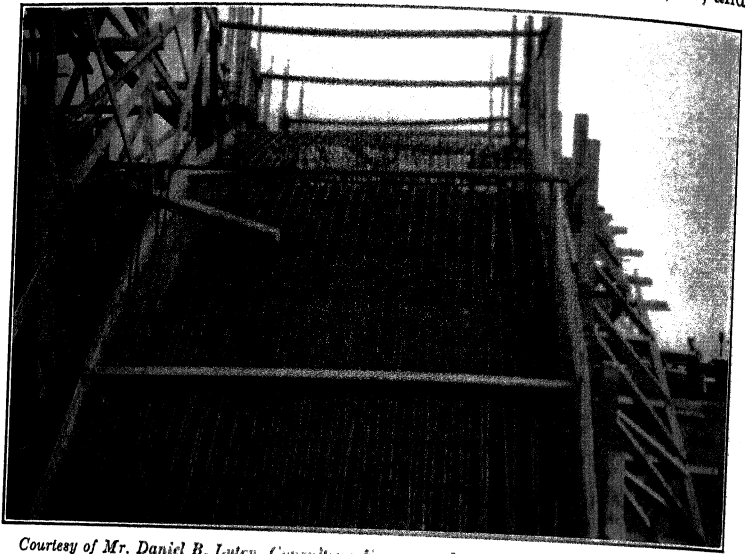
FIG. 94.—Bridge over Saskatchewan River at Saskatoon, Canada. Placing reinforcement in arch ribs.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 95.—Bridge over Saskatchewan River at Saskatoon, Canada. Reinforcement in place on one of the arch ribs.

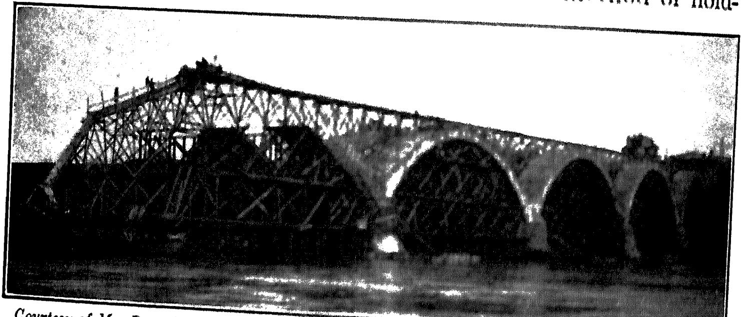
of the concrete was reduced to a minimum and, of course, there was no chance of trouble arising from failure to remove all spacing blocks. The bridge in question has a stone facing. Some of the stone work may be seen in place in Figs. 87, 88, and



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis

FIG. 96.—Bridge over Saskatchewan River at Saskatoon, Canada. Arch rib ready for concreting.

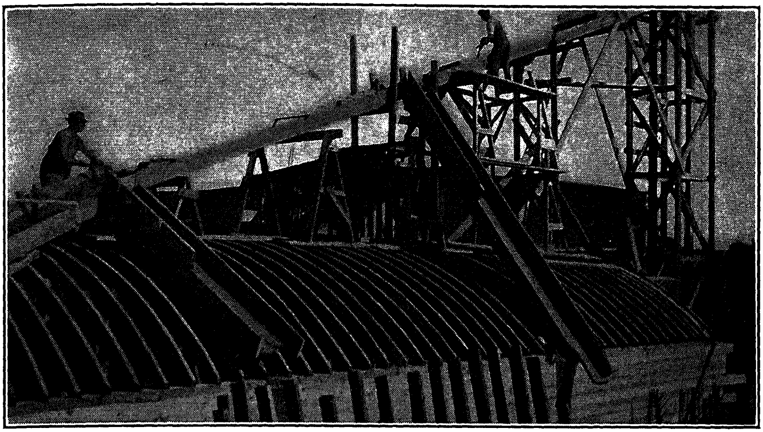
89. Narrow gauge distributing track should be noted, also longitudinal bulkheads used in the longitudinal rib method of arch-ring construction. Fig. 93 shows a similar method of holding steel in position employed in a Luten Design bridge (see Chapter XII) at Collinsville, Ohio.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis

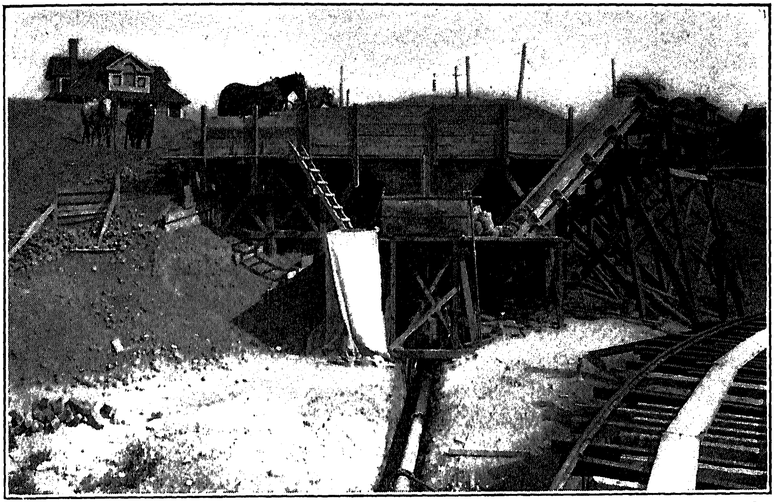
FIG. 97.—Construction view of bridge over Saskatchewen River at Saskatoon, Canada.

Figs. 94, 95, and 96 show the placing of reinforcement in a two-ribbed open-spandrel arch bridge at Saskatoon, Saskatchewan, Canada. Two tracks were used for distributing materials, one



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

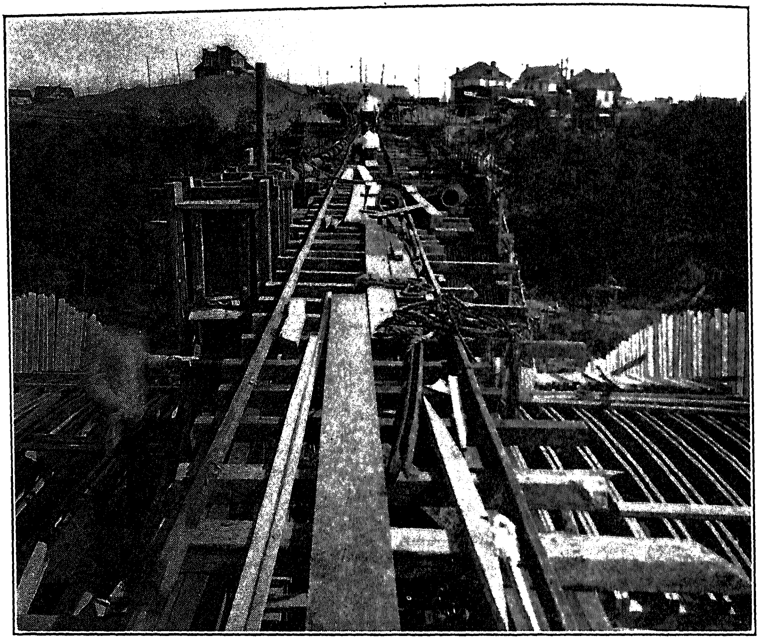
FIG. 98.—Bridge over Saskatchewan River at Saskatoon, Canada. Placing concrete by hoist and chutes.



Courtesy of Mr. A. P. Linton, Asst. Chief Engineer, Board of Highway Commissioners, Sask.

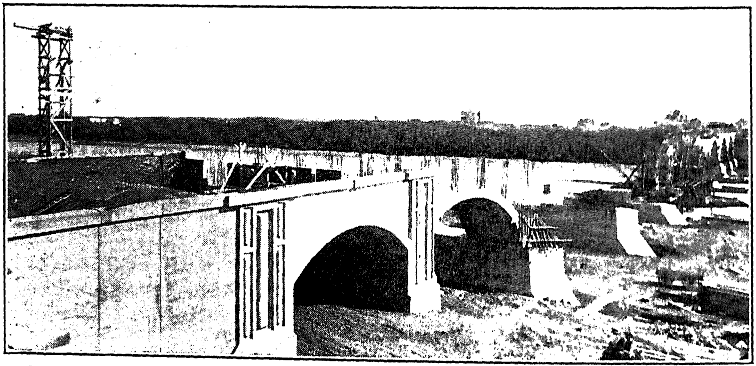
FIG. 99A.—Pneumatic mixing plant at bridge over Saskatchewan River, Saskatoon, Canada. View showing hopper for measuring gravel; tank for measuring water; method used in handling cement; pneumatic mixing machine; and pipes leading to discharge box.

just above the spring line shown in Fig. 94, and another on top of



Courtesy of Mr. A. P. Linton, Ass't. Chief Engineer, Board of Highway Commissioners, Sask.

FIG. 99*B*.—Pipe line from pneumatic mixer at bridge over Saskatchewan River, Saskatoon, Canada. Notice discharge box. (Troughs are not shown in this view.)



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 100.—Bridge over Saskatchewan River at Saskatoon, Canada. General construction view.

bridge was placed by compressed air; the remainder by hoist and chutes (Fig. 98). In Fig. 99*A* is shown the pneumatic mixing plant used in the construction of this bridge with pipes leading to discharge box on the upper material track (Figs. 97 and 99*B*). Troughs leading from the discharge box to the work are shown in Fig. 97 but not in Fig. 99*B*. In Fig. 99*A* should be noted the hopper for measuring gravel, tank for measuring water, and method of handling cement. A general view of the Saskatoon bridge during construction is shown in Fig. 100.

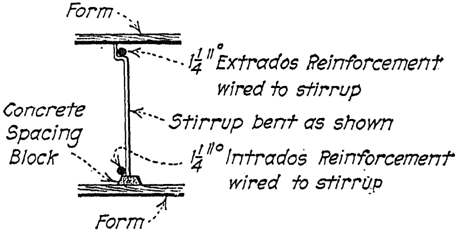
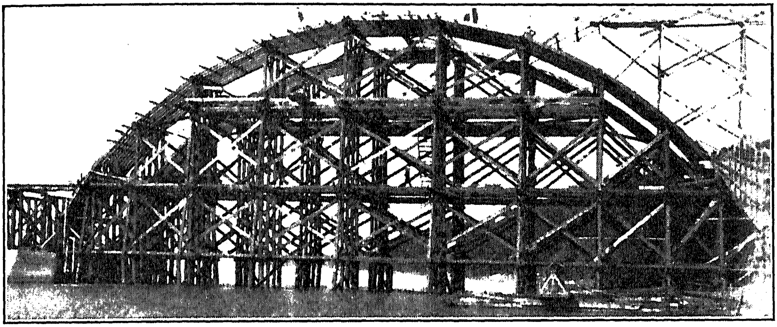

Form
1¼"° Extrados Reinforcement wired to stirrup
Concrete Spacing Block
Stirrup bent as shown
1¼"° Intrados Reinforcement wired to stirrup
Form

FIG. 101.

The short radial rods in the arch ring, shown in Figs. 95, 96, and 101 were not looped around either upper or lower steel, but simply wired thereto. They were intended simply for framing.

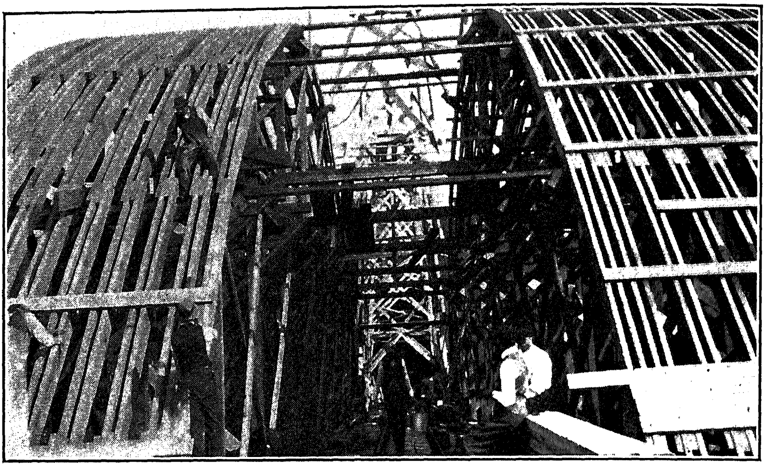


Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 102.—Timber falsework and centering for arches. Bridge over Saskatchewan River at Saskatoon, Canada.

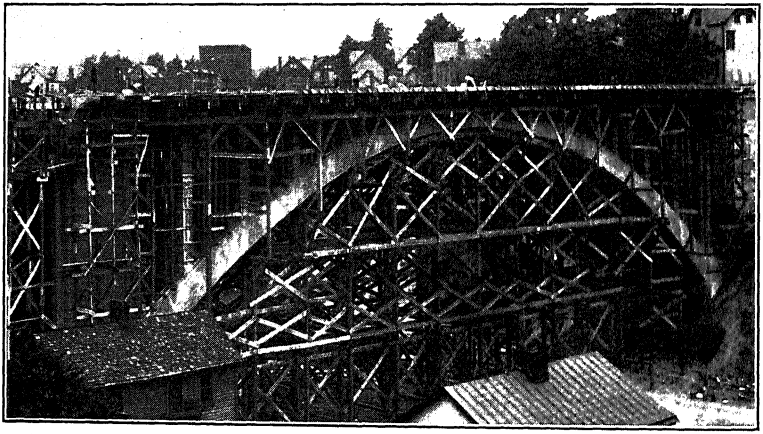
When steel ribs (either rolled sections or built-up lattice girders) are used as arch reinforcement, great care should be taken to fix the ribs in the proper position, and in this position they should be braced until the concrete is placed. The use of such ribs is known

54. Centering.—The bent type of timber falsework (Figs. 102, 103, 104, and 105) is the type of centering generally employed in arch construction except where a deep gorge is to be spanned or where a large clearance under the arch is necessary while the bridge is under erection. Timber arches, Howe trusses, and bowstring trusses are sometimes employed when it is impossible to use the bent type of centering, but these forms are expensive to build.



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapilos.

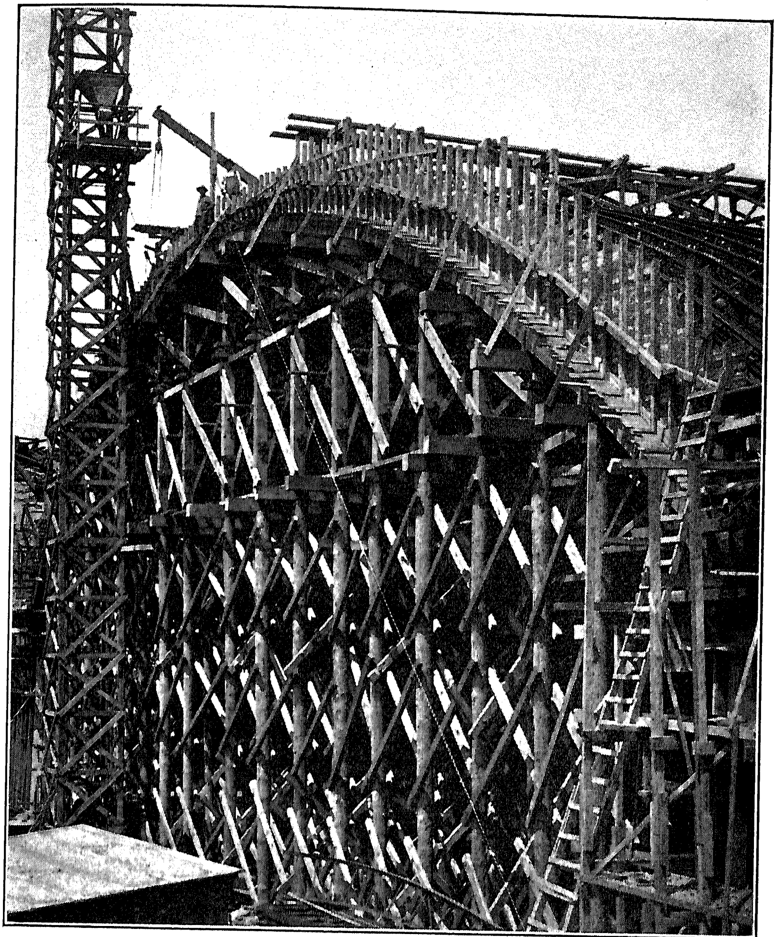
FIG. 103.—View of upper part of centering. Bridge over Saskatchewan River at Saskatoon, Canada.



Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

FIG. 104.—Centering for Meadow Street bridge, Pittsburgh, Pa.

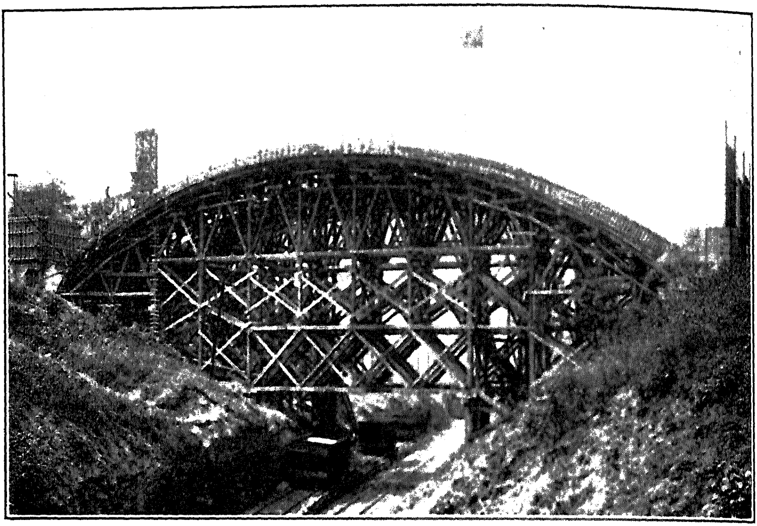
deform badly under loading, and have but small salvage value. (Fig. 106 shows a combination of the bent and Howe-truss types of centering.) Before using any of these types, consideration should be given to the use of steel centers (Fig. 107); or, in some cases, to the use of suspended centering (Figs. 108, 109, and 110), if the writer may be permitted to refer in this manner to patented methods of arch construction. In some special cases



Courtesy of Marquette Cement Mfg. Co.

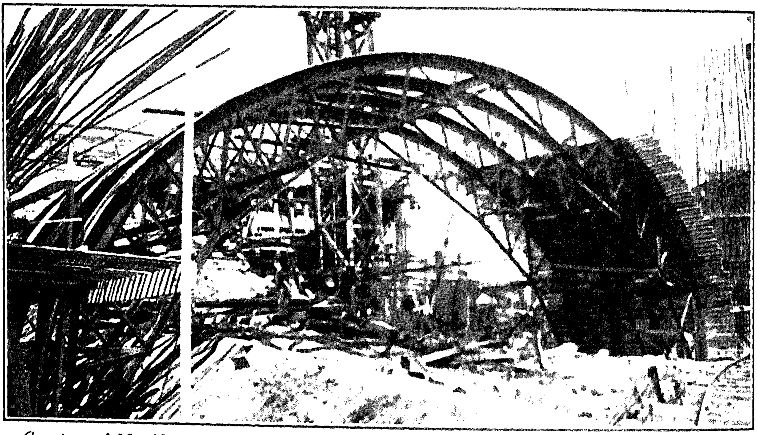
FIG. 105.—Centering for Grand Avenue viaduct, Milwaukee, Wis.

to carry a suspended arch form together with the dead load of the wet concrete (Fig. 111).



Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

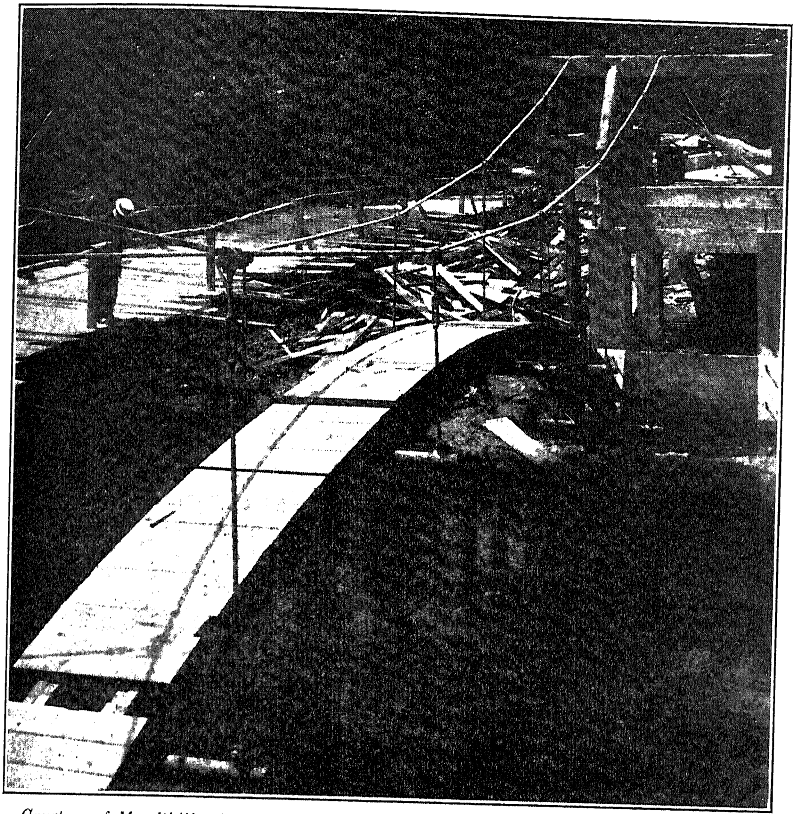
FIG. 106.—Centering for Atherton Avenue bridge over Pittsburgh Junction R. R., Pittsburgh, Pa.



Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

Fig. 107.—Steel centering for North Side Point bridge, Pittsburgh, Pa.

Timber Centers.—A simple and common form of timber centering for arches of low rise is shown in Fig. 112. The lagging had not been placed at the time this photograph was taken, but some of the joists for supporting the lagging were already in place. The joists, of course, extended from abutment to abutment and were supported by transverse bents of round timber resting on sills the full length of each bent. The falsework rested on the concrete floor of a spillway channel so that mud sills, piles, or specially-constructed concrete footings were not necessary. Wedges were placed at the bottom of the posts so that the center might be lowered conveniently after the arch ring was completed and ready to bear its load. Fig. 113 shows a similar type of centering partly removed. In the constructing of this bridge, however, the wedges were placed at the top of the posts, as can be plainly seen in the



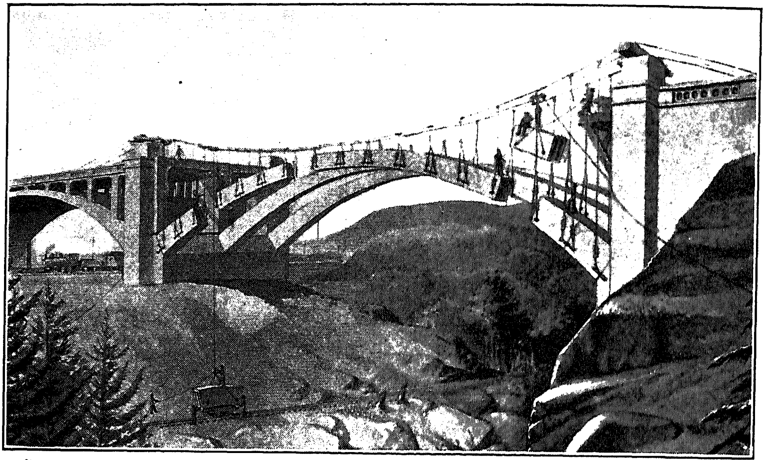
Courtesy of Mr. Philip Aylett, Civil Engineer, St. Louis.

FIG. 108.—Construction view of Chickahominy River bridge near Richmond, Va. Suspended centering in place.

Timber centering varies so greatly in design to meet different conditions, that plans and photographs of a number of centers which have actually been built, with a few statements (quoted or otherwise) concerning each, will do more to give the reader a clear idea of this type of construction than any general descriptions which might be given.

The centering used in the Third Avenue bridge at Cedar Rapids, Iowa is shown in Fig. 114. In a paper presented before the Western Society of Engineers, April 13, 1914, Mr. Barton J. Sweatt described the construction of this centering as follows:

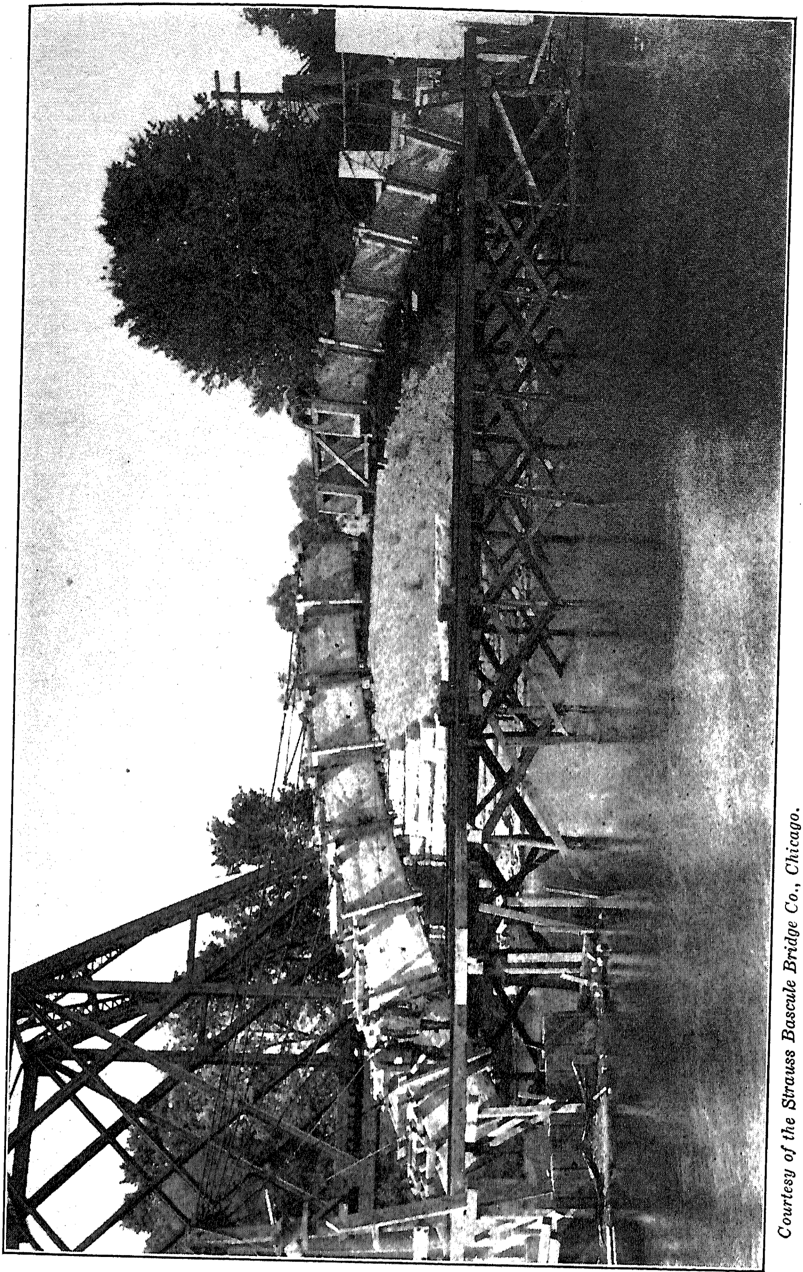
"The falsework for supporting the arches consisted of pile bents, the first bent being 6 ft. 6 in. from the face of piers and abutments, the second bent 12 ft. 6 in. from the first, and the intermediate bents were 14 ft. 6 in. centers. Oak piles were used and as a rule were driven to bed rock, the spacing was 6 ft. 0 in. for the three outside piles and 8 ft. 0 in. for the intermediate. The caps used were 12 in. by 12-in. yellow pine, false caps 6 in. by 10 in., joints 4 in. by 14 in., spaced 24 in. on centers and the lagging was 2 in. by 8 in. The proper curve for the intrados was obtained by the use of 2-in. strips cut to the proper curve and tacked to the regular joists. Oak wedges were used between the main and false caps. These wedges were placed in pairs and spaced about 4 ft. apart. Small wedges were used under the ends of the joists to bring them to the proper height.



Courtesy of Mr. Philip Aylett, Civil Engineer, St. Louis.

FIG. 109.—Pre-casted voussoir method of arch construction.

"In constructing the centering ...

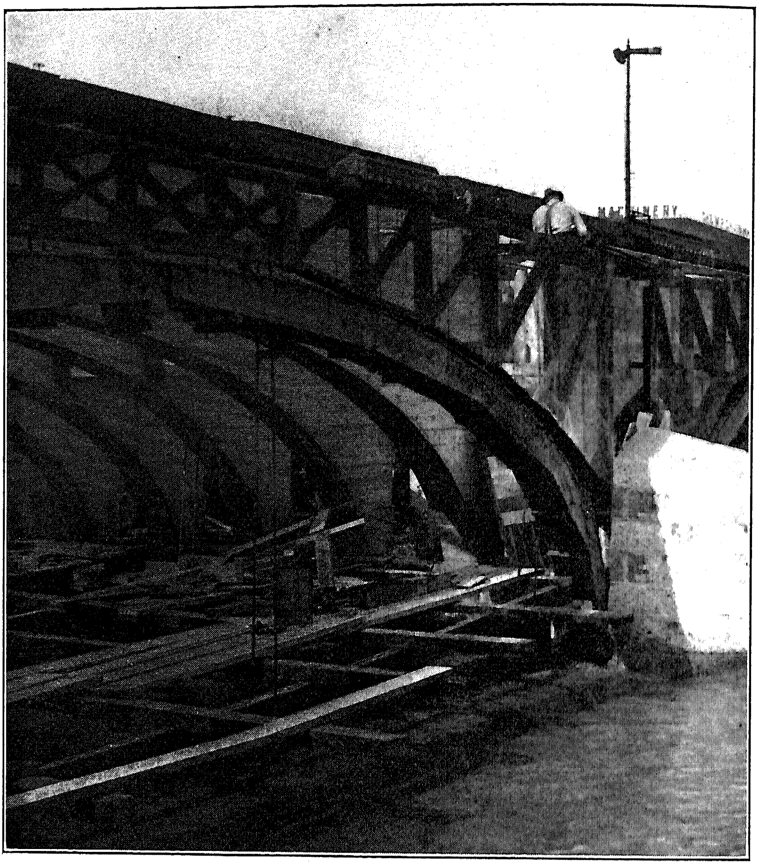


Courtesy of the Strauss Bascule Bridge Co., Chicago.

FIG. 110.—Strauss system of bridge construction.

camber and ½ in. for settlement after the centering was removed. The actual settlement of the crown after removing the centering was ⅜ in."

An article in Cement Age, March, 1912, describes the construction of the centering shown in Fig. 115 in the following manner:



Courtesy of Mr. John F. Skinner, Prin. Ass't. Engineer, Dep't. of Engineering, City of Rochester.

FIG. 111.—Bridge at Central Avenue, Rochester, N. Y. Structural reinforcement and arch-rib forms in place.

"The centering for the arch consisted of four pile bents of four piles each, and two center pile bents of five piles each. These bents were capped, top of caps being elevation of spring line of arch, and four lines of 6 in. by 8-in. stringers were placed continuous from abutment to abutment, the ends, at elevation of the spring line bearing 4 in. on the concrete holes in the abutments filled with concrete. On these four lines of stringers were placed a set of pine wedges over each pile, there being four and five sets of wedges to each bent; 3 in. by 12-in. timber was laid on the wedges over each bent through the width of the arch. On these 3 in. by 12-in. timbers, the arch bents were erected, each having four and five 6 in. by 8-in. vertical posts V-braced, with 6 in. by 8-in. caps set edgeways, top of caps 12 in. below intrados of arch. On these caps were placed 2 in. by 12-in. ribs, dapped to take square bearing on caps. These ribs were placed 18 in. centers across the arch and were cut from timber of sufficient dimensions to make them lap over alternate caps. The ribs were covered with 2-in. planking to form the intrados of the arch.

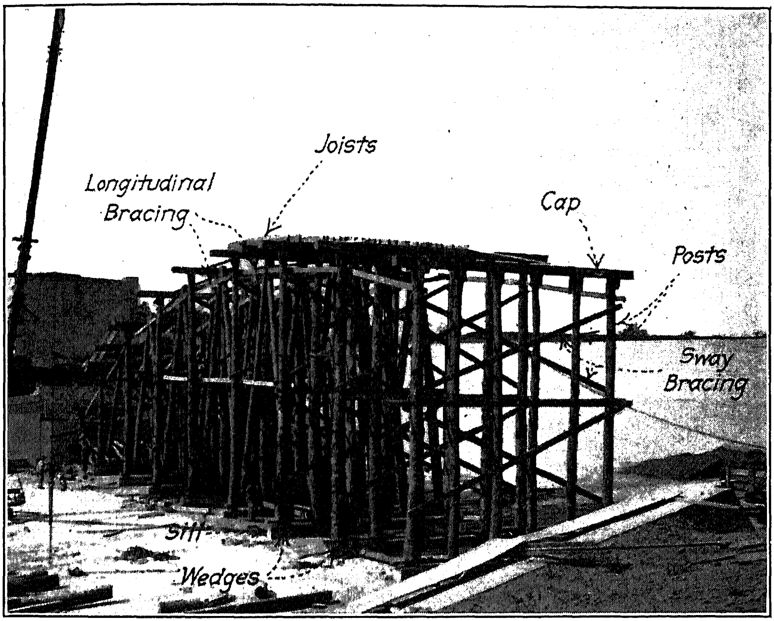
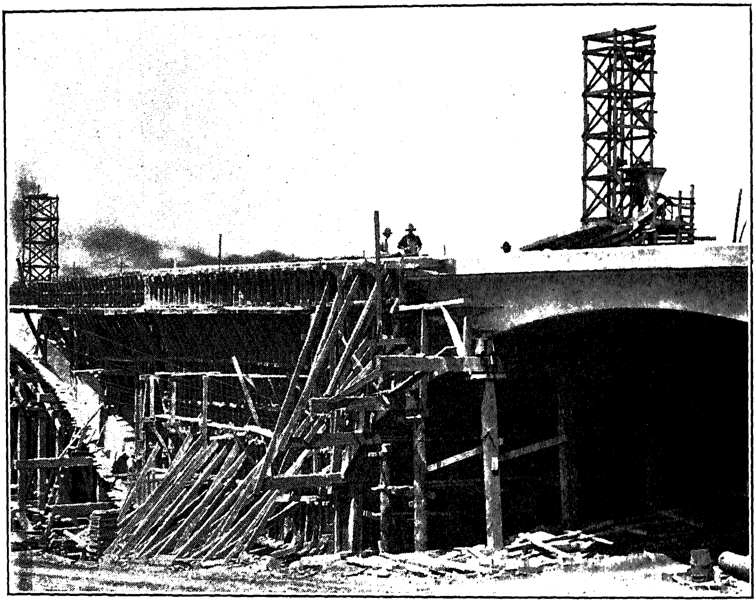


FIG. 112.—Common form of timber centering for arches of low rise.

"A very simple and accurate method of laying out the ribs was used which consisted of laying out the full-size arch intrados radii on a level place near the bridge site. The timber for the ribs was, therefore, marked by a full-size drawing. Probably the most interesting feature of this arch centering was the simple straight work giving maximum strength and maximum safety in every respect at the lowest cost. The five steel floor beams of the old bridge were utilized to make an 18-ft. clear opening of maximum height in the centering. This was econompurpose was to allow drift to pass through in case of high water during construction."

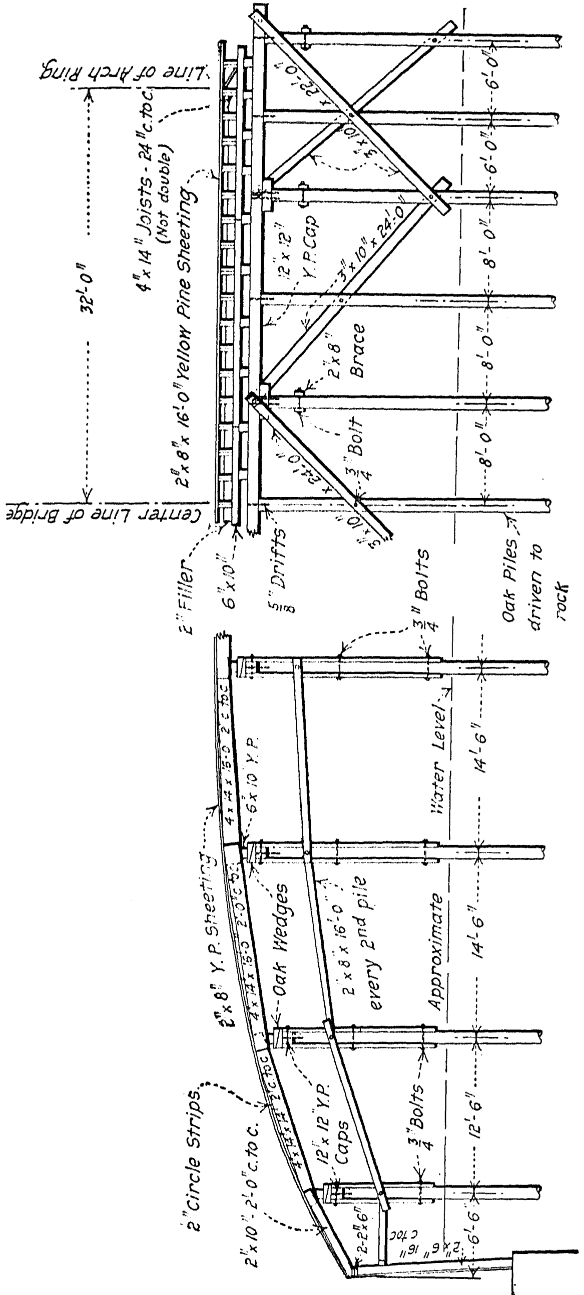
Figs. 116, 117, 118, and 119 show timber centers similar to the one just described.

A patented type of centering is shown in Figs. 120*A*, 120*B*, 120*C*, and 120*D*, known as the Luten arch centering (see Chapter XII). The idea in this center is to dispense with the usual wedges employed in lowering the falsework. The top part of the uprights consist of two thin members with major dimensions transverse to each other. These are arranged in the form of a T-column, and wired together at frequent intervals. Each member separately is made too light to carry its loading so that clipping the wires permits each member to buckle, which lowers the center. Fig. 120*B* shows the first stage in construction. The joists are nailed to the uprights which consist originally of only one member each. The transverse members of the uprights are then added to form the T-columns. The V-bracing is put in position after all the joists and uprights are in place.



Courtesy of Marquette Cement Mfg. Co.

FIG. 113.—Centering for arches of low rise, Grand Avenue viaduct, Milwaukee, Wis.



Section showing Half of Center Bent

Half Side View

FIG. 114.—Centering for Third Avenue bridge, Cedar Rapids, Iowa.

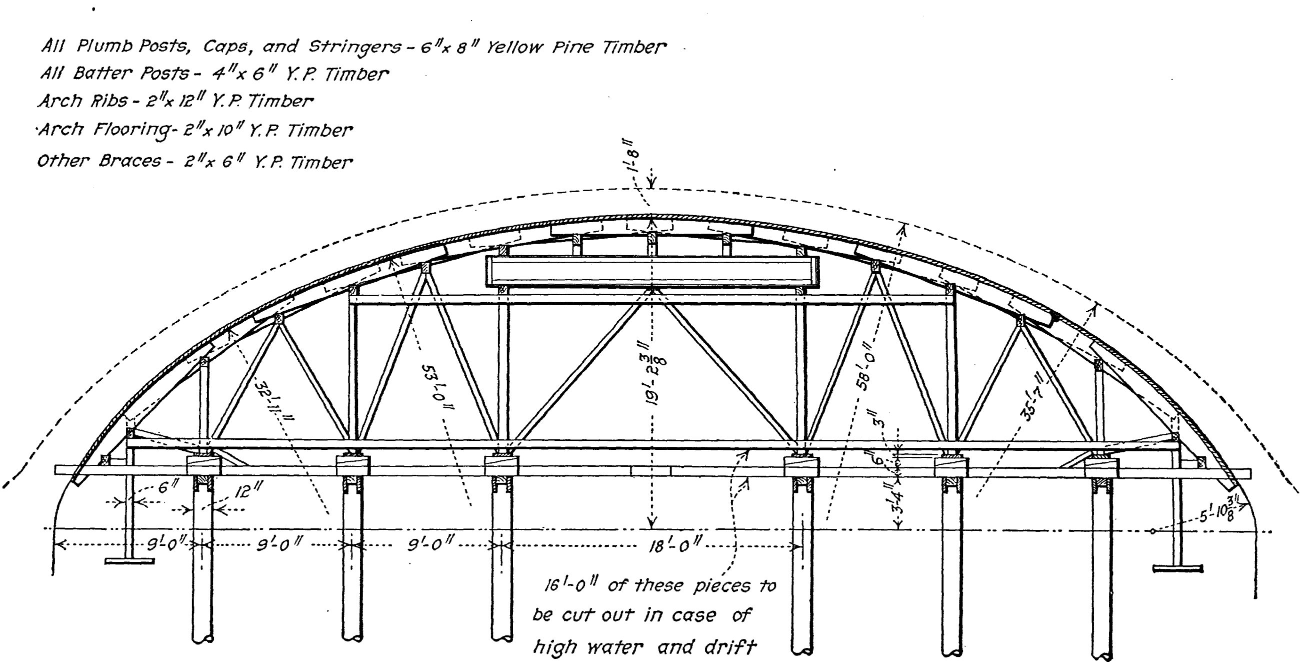
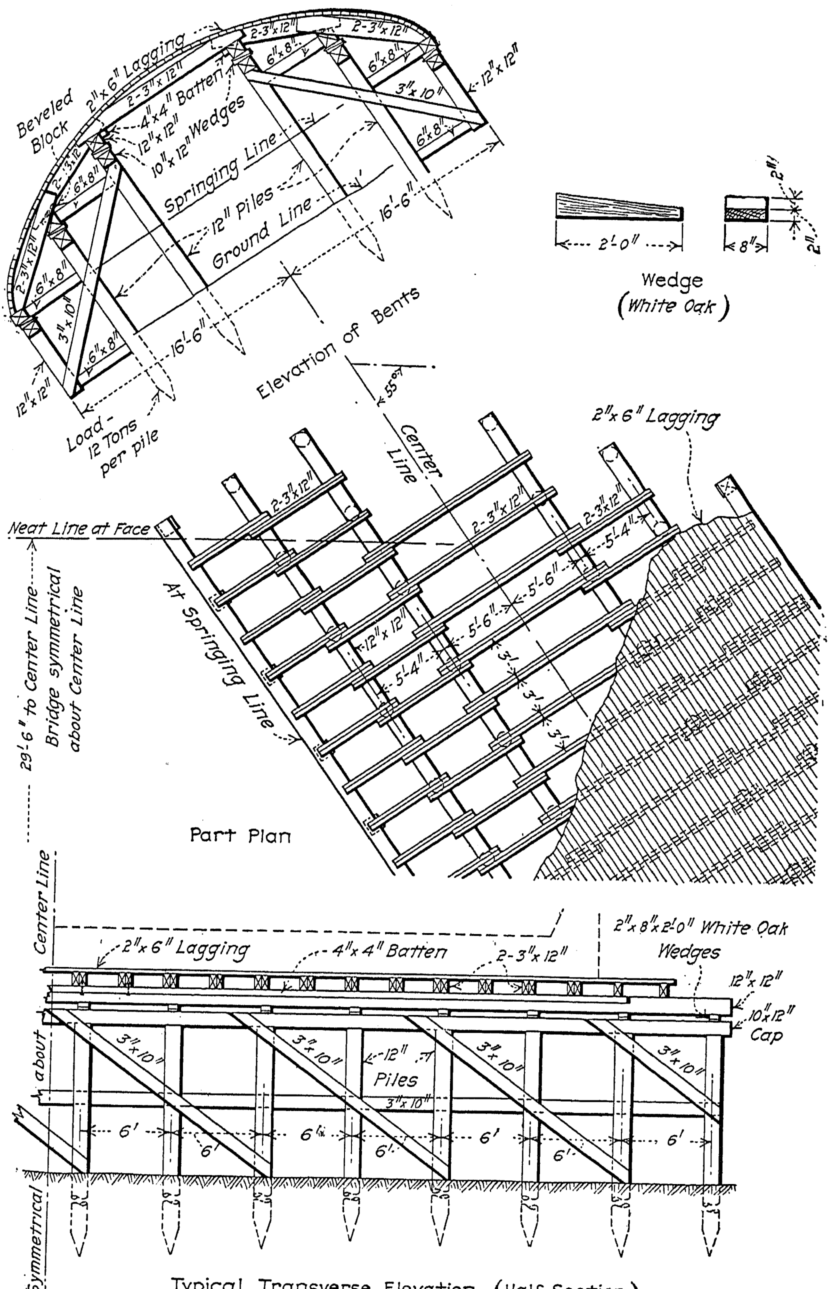

All Plumb Posts, Caps, and Stringers - 6"x 8" Yellow Pine Timber
All Batter Posts - 4"x 6" Y.P. Timber
Arch Ribs - 2"x 12" Y.P. Timber
Arch Flooring - 2"x 10" Y.P. Timber
Other Braces - 2"x 6" Y.P. Timber
1'-8"
32'-11"
53'-0"
19'-2$\frac{3}{8}$"
58'-0"
35'-7"
3"
6"
3'-4"
6"
12"
5'-10$\frac{3}{8}$"
9'-0"
9'-0"
9'-0"
18'-0"
16'-0" of these pieces to be cut out in case of high water and drift

FIG. 115.—Falsework and centering for Cleveland Avenue bridge, Kansas City, Mo. Note structural steel girder carrying the center of the span to allow for flood.


Beveled Block
2"x6" Lagging
2-3"x12"
4"x4" Batten
12"x12"
10"x12" Wedges
6"x8"
3"x10"
12"x12"
Springing Line
12" Piles
Ground Line
16'-6"
16'-6"
Load - 12 Tons per pile
Elevation of Bents
55°
Wedge (White Oak)
2'-0"
8"
2"
2"
Center Line
2"x6" Lagging
2-3"x12"
Neat Line at Face
12"x12"
5'-4"
5'-6"
5'-6"
5'-4"
3'
3'
3'
At Springing Line
29'-6" to Center Line
Bridge symmetrical about Center Line
Part Plan
Center Line
2"x6" Lagging
4"x4" Batten
2-3"x12"
2"x8"x2'-0" White Oak Wedges
12"x12"
10"x12" Cap
3"x10"
12" Piles
3"x10"
about
6'
6'
Symmetrical
Typical Transverse Elevation (Half Section)

Mr. W. W. Washburn in an article in Concrete-cement Age, August, 1914, writes as follows in regard to the centering shown in Fig. 121:

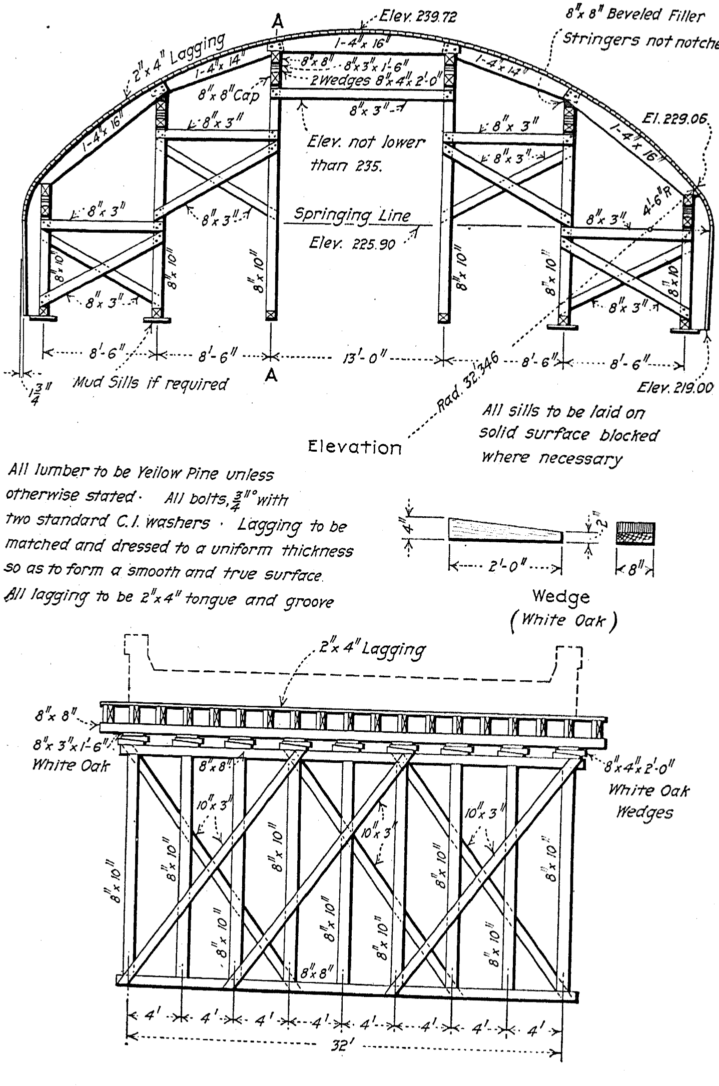
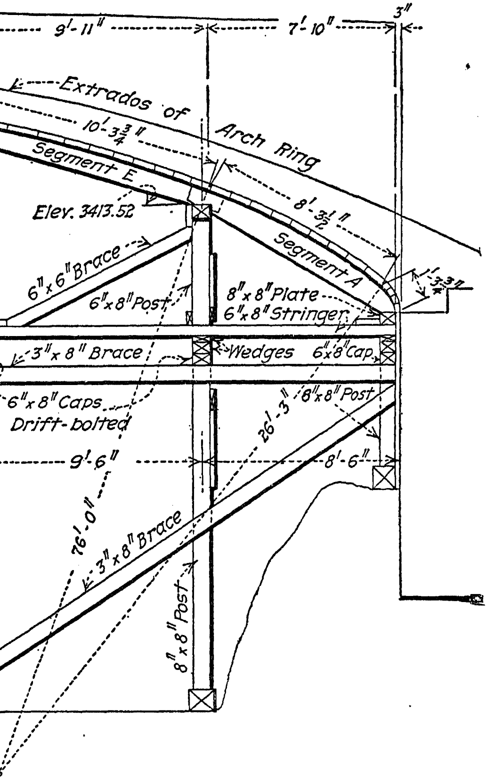


FIG. 117.—Details of arch centers for Center Street bridge, Phillipsburg, N. J.

“Since Buffalo Bayou is a navigable stream, it

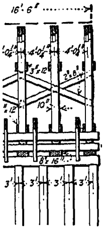


Material.

	No.	Size	Ordered Length	Location
	18	3"x8"	24'-0"	Bracing
	9	3"x8"	22'-0"	"
	9	3"x8"	18'-0"	"
	55	3"x8"	16'-0"	"
	9	3"x8"	14'-0"	"
	116	3"x8"	18'-0"	Lagging D·&M.
	116	3"x8"	16'-0"	"
s	116	3"x8"	12'-0"	"
gers				
	16	3"x6"	16'-0"	Bracing
	40	3"x6"	14'-0"	Bracing
	11	3"x4"	18'-0"	Lagging S-2-S & 2-E
	11	3"x4"	16'-0"	" (See Sketch)
	11	3"x4"	12'-0"	" "
& A				
	140	6"x8"	2'-0"	Wedges
& C	140	¾"⌀x1'-4"	Drift Bolts without heads or	

e average
passage-
ng in the
ning nine
ed in the

much as
wn as the
l, special
care of all
at right



Half
ection A-A

Vedges are to be
rk, surfaced on
d thoroughly soaped

. R. R.

ed a load

li River,
Record,

ovide for
ne shown

3 ft. at a
between
he top of
ted with-
centering
e lagging

M
Aug
in I

A
01
n
n
s
ℓ

during construction. This opening was 24 ft. vertically above average water level and 38 ft. wide. Protection piles on each side of the passageway were necessary, consequently the total span of the opening in the arch centering was 49 ft. To carry the load over this opening nine 30-in., 200-lb. I-beams were used. These I-beams will be used in the construction of other bridges.

"Longitudinal bracing was arranged so as to counter, as much as possible, the 'bucking up' tendency of the centering at the crown as the arch was concreted. On account of the arch being skewed, special study was given during the placing of braces, etc., so as to take care of all side thrusts. Struts were placed diagonally between bents at right angles. Spacing of centering piles was such that no pile received a load of over 12 tons."

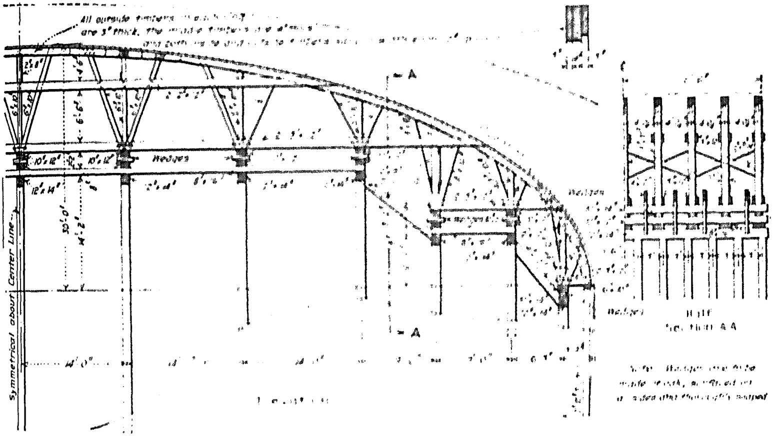
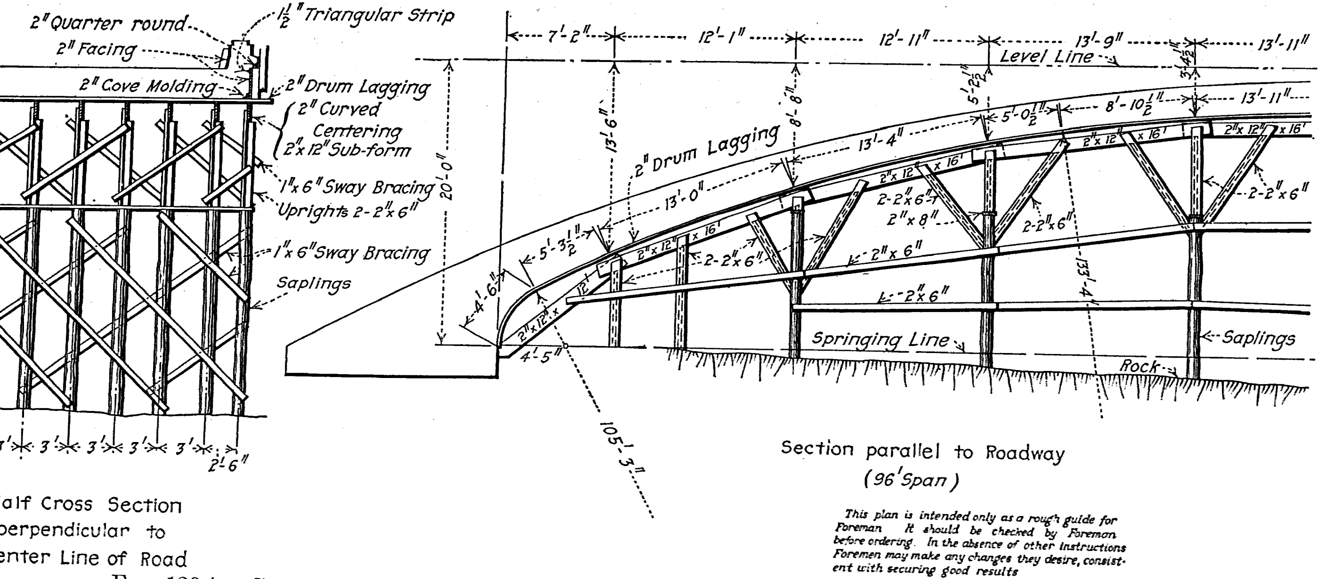


FIG. 119. Falsework for bridge over Big Muddy River, I. C. R. R.

Timber centering for a bridge over railroad tracks at Fall River, Mass., is shown in Fig. 122 and described in Engineering Record, issue of April 26, 1913, as follows:

"In building the arch centering it was necessary to provide for certain requirements that necessitated a design similar to the one shown in the accompanying drawing.

"These requirements called for an overhead clearance of 18 ft. at a point 11 ft. 3 in. from the property line, a clear span of 40 ft. between inside vertical posts, and a minimum clearance of 15 ft. from the top of rail to the under side of the truss. The work had to be conducted without interruption of train service. The uprights supporting the centering



Half Cross Section perpendicular to Center Line of Road

Section parallel to Roadway (96' Span)

This plan is intended only as a rough guide for Foreman. It should be checked by Foreman before ordering. In the absence of other instructions Foremen may make any changes they desire, consistent with securing good results

FIG. 120A.—Centers of Luten Design for Cicott Street bridge over Wabash River, Logansport, Ind.

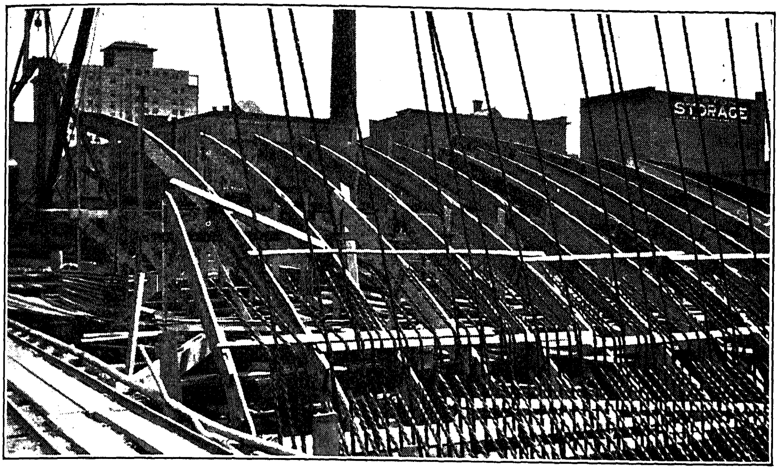


FIG. 120*B*.—Luten arch centering.

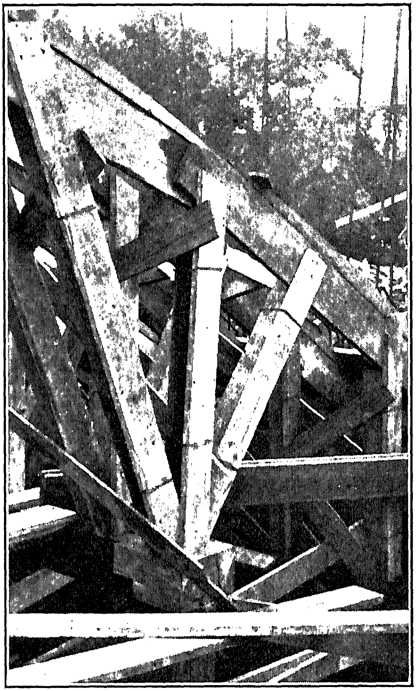


FIG. 120*C*.—Luten arch centering.

was used over the entire arch ring. The centering was designed for a deflection equal to one eight-hundredth of the span."

In the construction of the Haights Run bridge shown in Fig. 83 the two arch ribs were constructed in sections and each section cast at one operation. The false work consisted of braced timber posts supporting I-beams which carried the lagging for the arch ribs. All adjustments for the arch rib were made by wedges immediately under the I-beams. Two bents of posts of the false work were located under each section of the arch rib, and located so that the center of gravity of the section fell between the posts which were braced together like a single tower; thus each section of the rib was independently supported and adjusted. In designing the falsework no soft lumber was subjected to cross-grain compression, hard wood being used for sills and caps. Posts and bracing were of yellow pine.

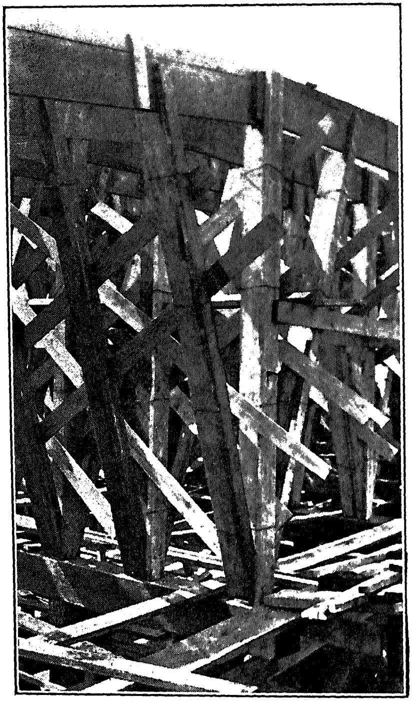
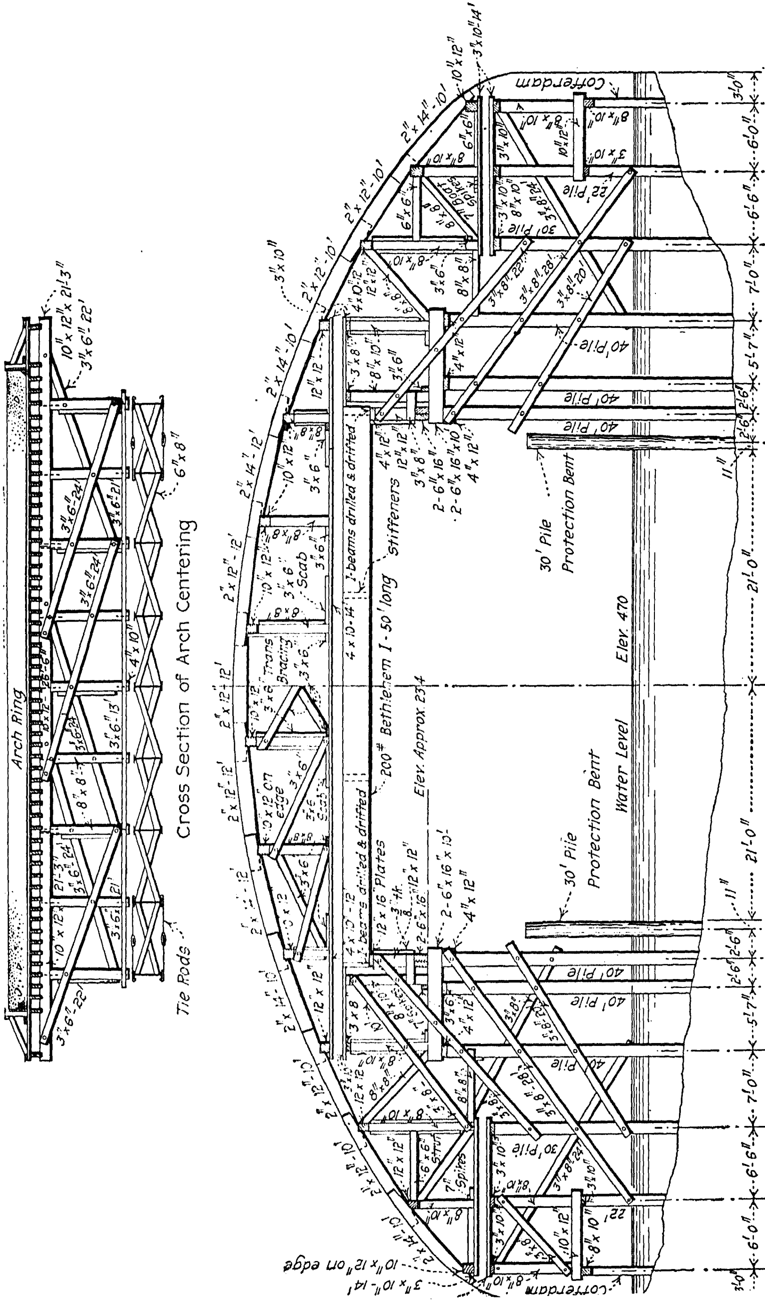


Fig. 120*D*.—Luten arch centering.

Figs. 123*A* and 123*B* show the type of arch centering used in



G. 121.—Details of arch centering and supports for 110-ft. span of San Jacinto bridge, Houston, Texas. Note size of opening for navigation.

constructing a three-ribbed arch on the Mississippi River Boulevard, St. Paul, Minn. Boxing for the ribs is also shown.

In Figs. 124*A* and 124*B* a bent type of centering is shown, but it should be noted that the portion of the center above the post caps (which were placed at about the level of the springing line) was framed and erected in a similar manner to a trussed center. The additional carpenter work required on this type of centering over the ordinary bent type is likely to make this form uneconomical, except perhaps in special cases.

Posts in timber centering have sometimes been placed approximately normal to the arch soffit, but such instances are quite rare. Since specially-constructed footings are necessary for inclined members, this form of center may be used economically only when rock or other suitable foundation lies near the ground surface.

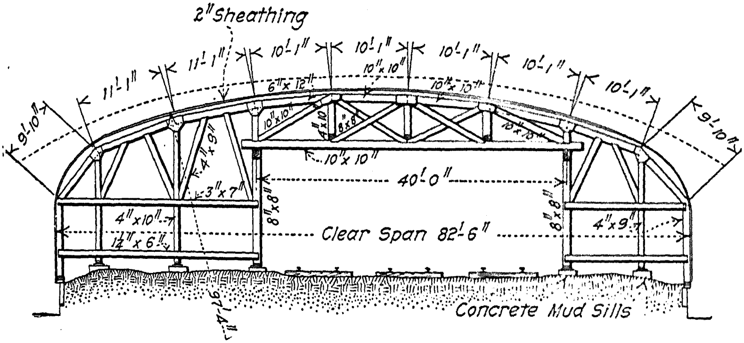
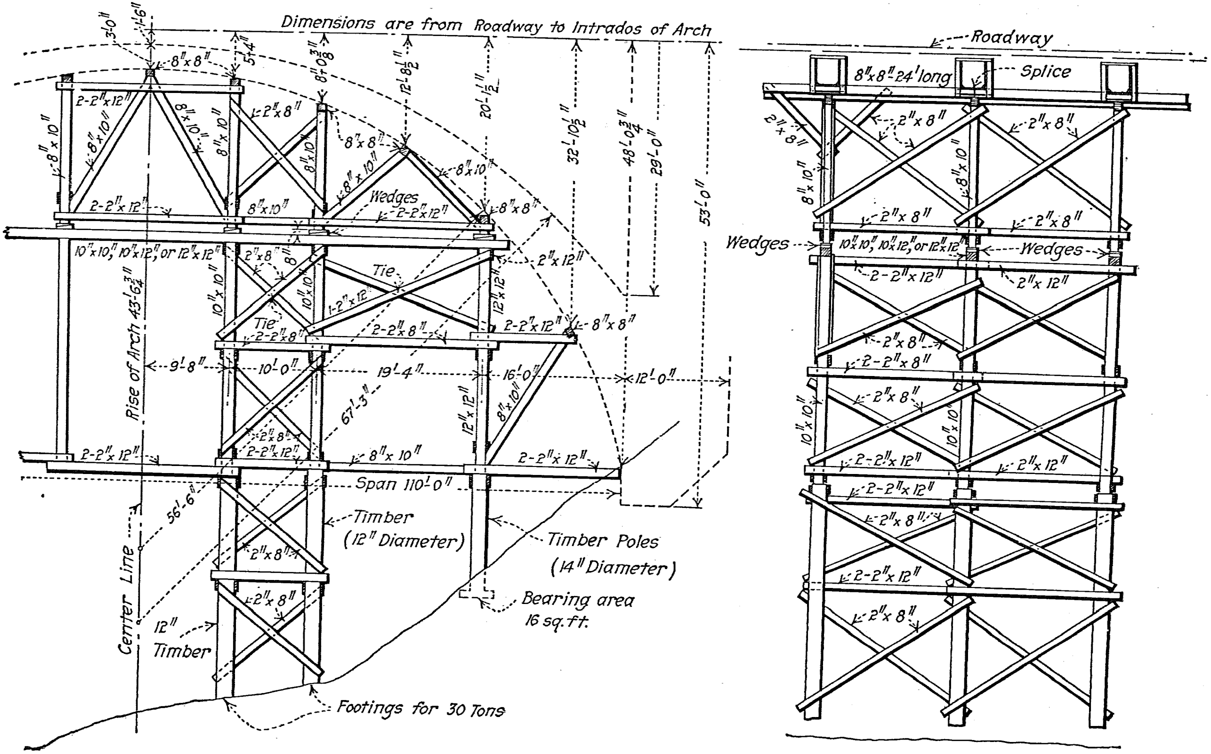


FIG. 122.—Highway bridge at South Park, Fall River, Mass. over tracks and right-of-way of the N. Y., N. H. & H. R. R.

Sand boxes have been used to a very limited extent in this country in place of wedges for the striking or lowering of arch centers. These boxes have given satisfaction in most instances, but great care must be taken to keep the sand dry while the arch ring is being constructed. This type of lowering device is expensive, but the extra first cost may be offset in large arches by the high cost of striking wooden wedges.

The chief disadvantage of using sand boxes lies in the fact that the sand will compress as the weight on the centering increases. The amount of this compressibility is considerable, greatly increasing deflection unless the sand is put under an initial compression, which is seldom feasible.



Elevation

Section on Center Line

FIG. 123A.—Centering for bridge over ravine on Mississippi River Boulevard, St. Paul, Minn.

A sand box used in the main arch of the Edmondson Avenue bridge, Baltimore, is shown in Fig. 125. The center was lowered by allowing the sand to run out through a 1-in. circular hole in the oak bottom of the steel-plate cylinder. This hole was closed by a wooden plug while the centering supported its load. A disadvantage in the use of sand boxes lies in the fact that the centering cannot be raised before the arch ring is poured in order to adjust the top members to the curve of the arch intrados.

In the design of large arch centers an uncertainty exists regarding the pressures from voussoirs placed on the steepest portions of the lagging. Either of two assumptions are usually made as to the forces acting on the centering due to the weight of such voussoirs. In the common method of design, the assumption is made that the centering sustains only the radial components of the voussoir weights, the tangential components being resisted by temporary struts between voussoirs so as to be transferred to the abutments. The more accurate method is to assume that tangential pressures (in addition to the radial pressures) act on the centering which, from any voussoir, may be as great as the product of the radial component and the coefficient of friction between the voussoirs and lagging. The original tangential component is then reduced by this amount.

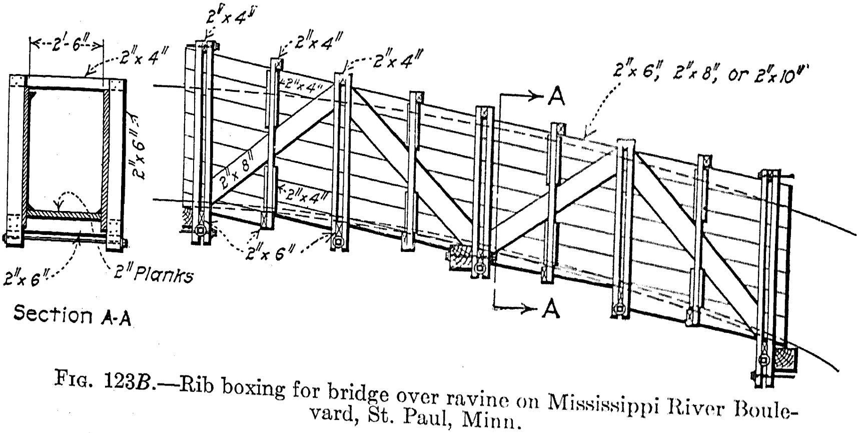


FIG. 123*B*.—Rib boxing for bridge over ravine on Mississippi River Boulevard, St. Paul, Minn.

Since a timber center is only a temporary structure and has a high salvage value, great accuracy in the design of the separate members is not necessary. The method of design need only be such that the size of each member is well on the safe side. Then too, rigidity is quite as important as strength, so that all things considered, close figuring is out of the question. Obviously the weight of centering may be omitted except for high arches. For the method of designing lagging, joists, and posts see Art. 88, Volume II.

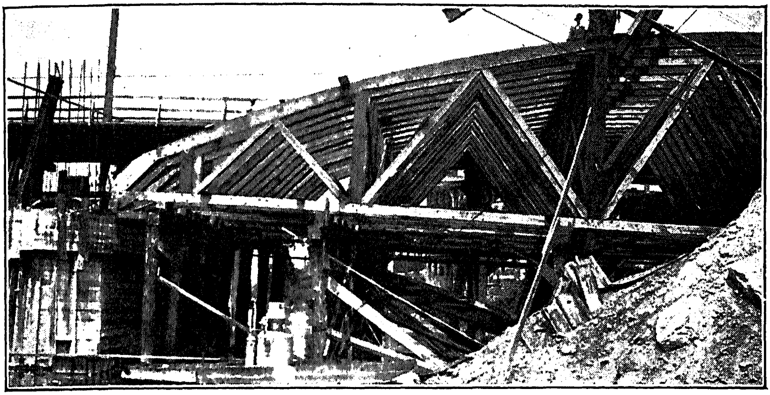


FIG. 124*A*.—Centering for High Street bridge, Hamilton, Ohio.

As a rule, only hard wood should be used for caps and sills, although long-leaf pine may be sufficiently hard in many cases. Wedges, however, should be made of hard wood without exception.

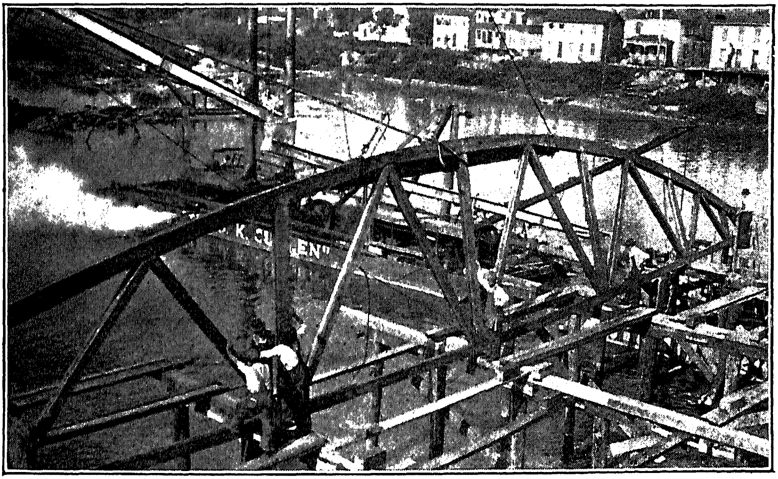


FIG. 124*B*.—Centering for High Street bridge, Hamilton, Ohio.

compression to a minimum on account of the low bearing value of timber across the grain. Steel distributing plates are of advantage in this connection.

Care should be taken to prevent lateral displacement of vertical posts due to radial pressure from the arch ring. This may be avoided either by proper longitudinal bracing or by notching out the joists and shimming them tight against the caps.

Many practical notes on the design and erection of falsework may be found in Section 7 of the American Civil Engineers' Pocket Book.

In striking arch centers, wedges should be lowered gradually beginning at the crown and working toward the springing lines. The lowering should be done symmetrically with respect to the center of the arch ring. In a series of arches, centers between abutments or abutment piers should be struck simultaneously. As a rule, centers should not be struck from arches in less than 28 days under favorable weather conditions, and it is desirable that a longer period should elapse if possible.

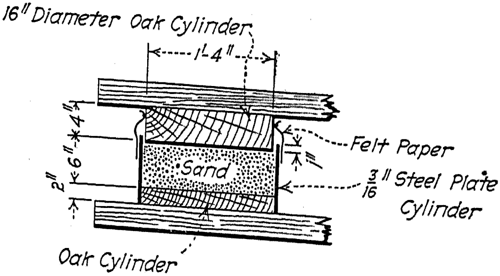
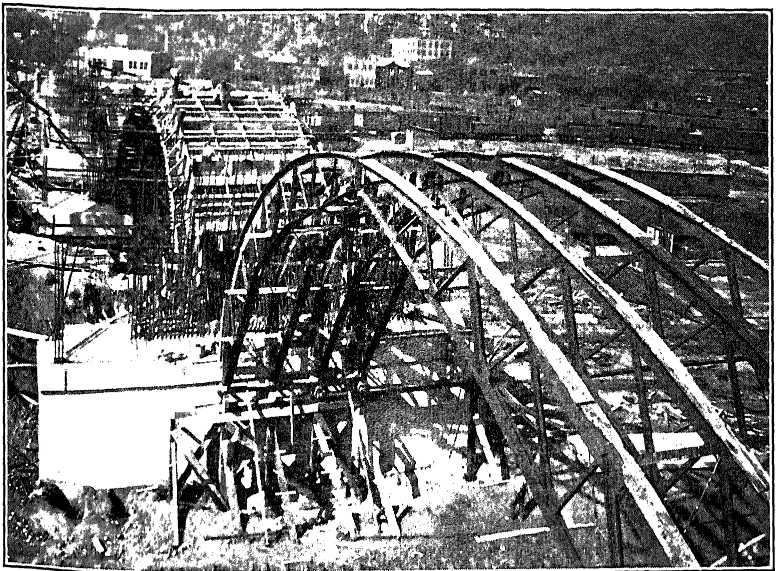


FIG. 125.—Sand box.

Steel Centers.—Steel centers of the arch-trussed type (Fig. 126) should receive consideration when arches are to be built in series or where the character of the stream or crossing renders timber and pile falsework impossible or expensive. Undoubtedly the cost of a steel center is usually high, but if it can be used a number of times, as in a large series of arches, it may not prove any more costly than timber. Mention should be made of the fact that in the Cummings-Watson system, to be described later on, the steel arch centering consists of triangular units which can be adapted to any shape and can be used for any span or curve by connecting and adjusting the units to fit. It is quite likely that with these units as part of a regular contractor's equipment, the steel center may be used with economy even in ordinary construction.

It is generally recognized that there are some well-defined advantages in using three-hinged arch centers. In the first place, the crown deflection using steel centers is usually much less than that obtained by employing timber falsework. Furthermore, it is possible to compute the deflection of each point of a steel center with some degree of accuracy while, in the case of a wooden center, the probable settlement at each bent is pretty much a matter of guesswork. Steel centers also have the additional advantages of allowing an obstructed opening for railroad or other traffic and of eliminating danger from flood and ice in the construction of arches over streams. The advantage of allowing the deflection to be quite accurately computed makes it possible to give the centers a preliminary camber so that when the concrete is in place and the centering withdrawn, the arch ring will assume its true position.



Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

Fig. 126.—Steel centers for North Side Point bridge, Pittsburgh, Pa.

One disadvantage of using steel in arch centering lies in the fact that it is materially affected by temperature changes. For this reason, in constructing large arches, only the alternate block

The steel centering used in constructing the three-span earth-filled arch structure which carries Atherton Avenue in the city of Pittsburgh across the four tracks of the Pennsylvania Railroad is shown in Fig. 127. This centering, fabricated by the Blaw Steel Construction Company, consisted of steel arch trusses spaced 5 ft. 5¼ in. on centers. The trusses carried timbers and lagging, and were supported on framed trestle bents placed close to the pier faces. Sufficient trusses were at first erected to concrete one-half the width of each arch ring, then the centers were shifted transversely to themselves and the concrete placed for the second half of the structure. The method used in construction and the details of the arch centering are described in Engineering and Contracting, issue of February 19, 1913, as follows:

"A six-post bent was erected on the footing shelf of the pier, the idea being to have a bent-post under each arch rib. On the bent caps over each post was placed a block and double wedge and on these supports a 12 in. by 12-in. plate on which rested the ribs. Between each pair of ribs a dolly was fastened to the bent-caps.

"The shifting of the center to construct the second half of the arch was accomplished as follows: Jacks were set up on the bent-caps alongside the dollies, and a strain taken on them until the wedges were loosened sufficiently to be easily removed. The jacks were then lowered until the weight of the centers rested on the dollies. To prevent the lagging and cross timbers from being lifted off the ribs by sticking to the soffit of the arch ring, one end of the center was lowered ahead of the other so as to give a stripping action in freeing the lagging. When lowered into the dollies the whole center was shifted sidewise rubbing on the dollies, until it rested on the six-post bents under the second half of the arch. The jacks were then placed on the caps of the second bents and the center raised and the blocks and wedges inserted. A steamboat ratchet was used to pull the center on the dollies. Four men working 8 hours shifted a center. Incidentally the tie rods connecting the opposite ends of the ribs were found, when planked across, to provide a most convenient bridge for the workmen engaged in shifting and adjusting the centers.

"The lateral thrust on the centers due to their skewed position was taken care of by suitable lateral bracing of the ribs. In anticipation of the center rising at the crown in concreting from the haunches upward, the ribs were anchored back to the pier masonry as shown in Fig. 41. The joining carried by the ribs consisted of cross-timbers over which were notched stringers with curved top edges. The stringers were spaced 11½ in. apart and were lagged with ⅞-in. boards. The bearings of the stringer ends against the piers were formed by wedges."

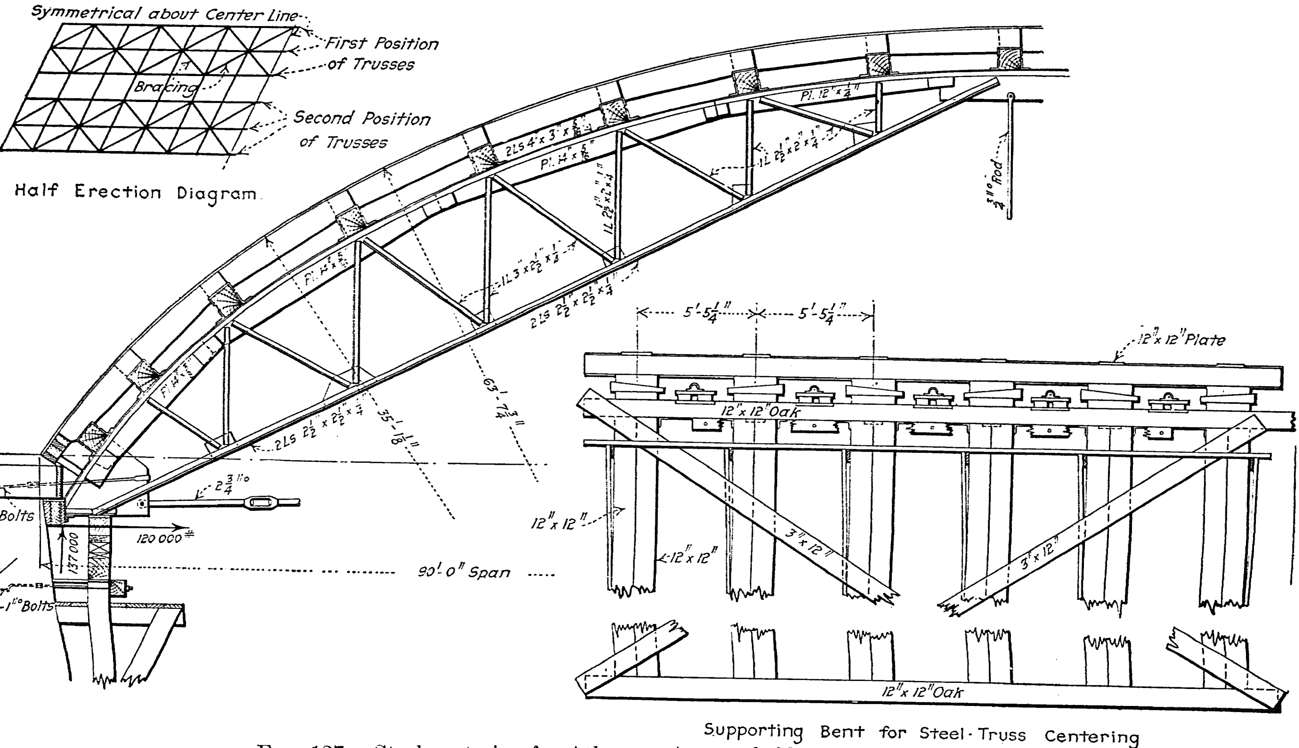


FIG. 127.—Steel centering for Atherton Avenue bridge, Pittsburgh, Pa.

Steel centering employed in the construction of the South Eighth St. Viaduct, Allentown, Pa. (a two-ribbed arch structure of nine 120-ft. spans) is shown in Fig. 128. The Engineering News, issue of April 17, 1913, describes this centering as follows:

"For the nine 120-ft. arches three full sets of steel arch centers were used, using each set for three of the arches. Each set of centers consisted of two independent-trussed arches of the outlines shown in Fig. 128, each arch supporting one of the twin concrete arch ribs and being itself made up of two steel arch ribs interbraced with steel struts. Across the upper chords of these steel ribs, which were curved to the curve of the concrete arch, was bolted the wooden lagging on which the concrete was deposited. The twin centering arches were held together by a timber cross-beam and diagonal steel rods.

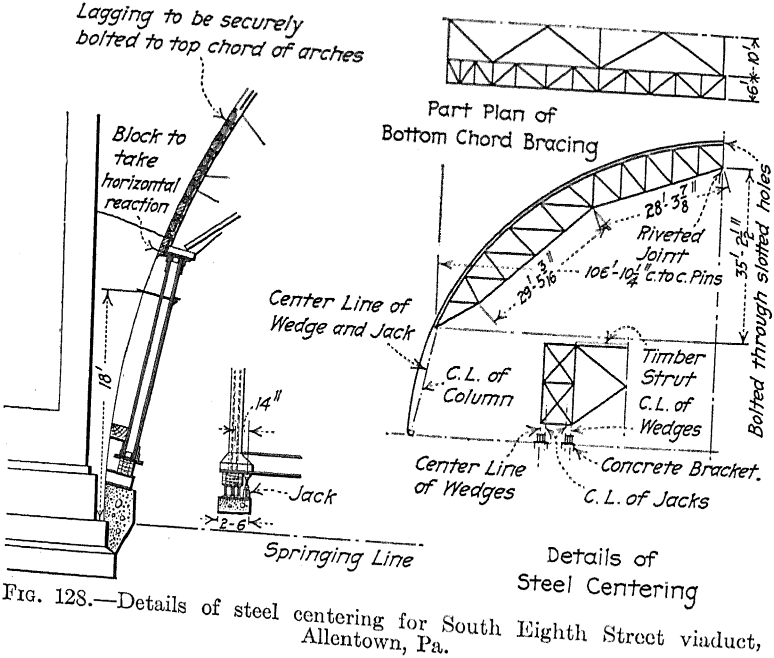
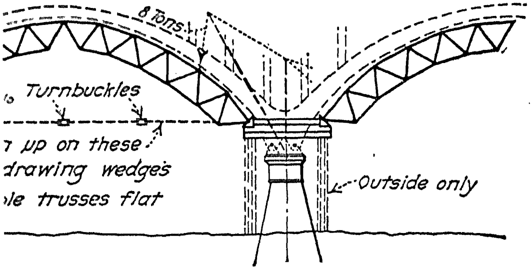
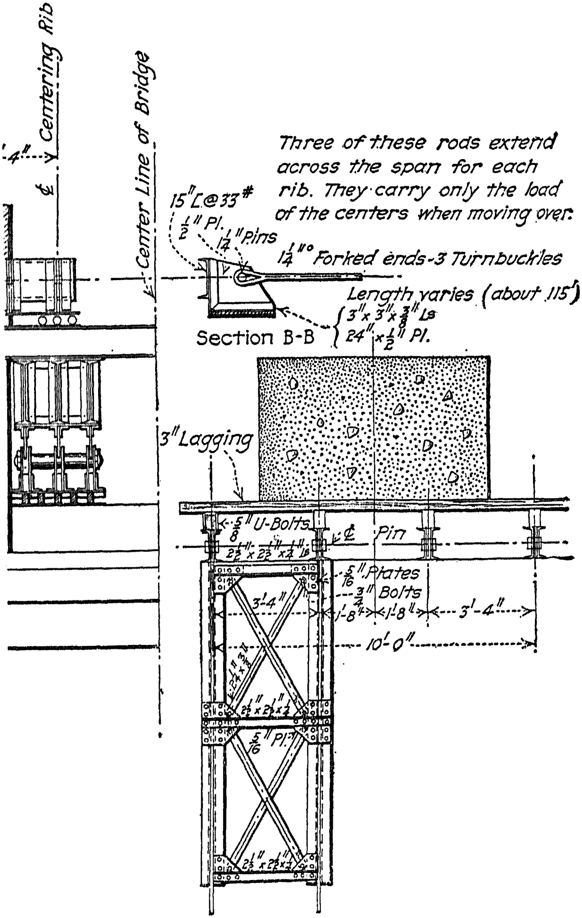

Lagging to be securely bolted to top chord of arches
Block to take horizontal reaction
18'
14"
Jack
2-6
Springing Line
Center Line of Wedge and Jack
Part Plan of Bottom Chord Bracing
10'
6'
28'-3 7/8"
Riveted Joint
29'-5 3/16"
106'-10 1/4" c.to c. Pins
35'-2 1/2"
Bolted through slotted holes
C.L. of Column
Timber Strut
C.L. of Wedges
Center Line of Wedges
Concrete Bracket
C.L. of Jacks
Details of Steel Centering

FIG. 128.—Details of steel centering for South Eighth Street viaduct, Allentown, Pa.

"The arch trusses were fabricated in six sections and riveted on the ground into semi-arches, which were lifted by derricks into place, to be bolted at the base to the supporting columns. At the crown it was riveted solidly in the bottom chord, but bolted through slotted holes at the upper chord, to insure the stress passing through the lower chord.

"The centers were supported on inclined steel columns which footed on concrete steps purposely projected from the main section of the pie





of the
illages,
en the
sed to
wed to
which
n. bolt
lges to
or the

tion.
n off

ting
ela-
able
rip-
pter

own
pin-
pers
to

Ei
of
N

us
sis
ea
m

—

=

—

=

—

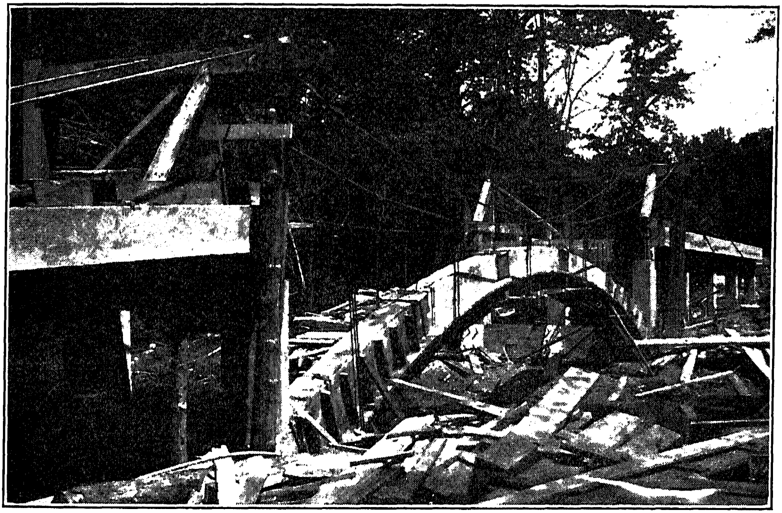
F

th
co
de
cr

gr
bc
ri
at

or

and cut off after the centers were struck. The base plates of the columns rest on cast-iron wedges which in turn rest on I-beam grillages, footing on the afore-mentioned concrete projection. Between the column base and the projection, 10-ton screw jacks are interposed to aid in the alignment and leveling of the centers; they are allowed to remain in place, though the load passes directly to the wedges which are used for striking centers. A U-shaped clamp, made of a 1-in. bolt (not shown in the drawing), is passed around each pair of wedges to prevent any possible lateral motion. A similar bolt is used for the same purpose higher up on the main column."



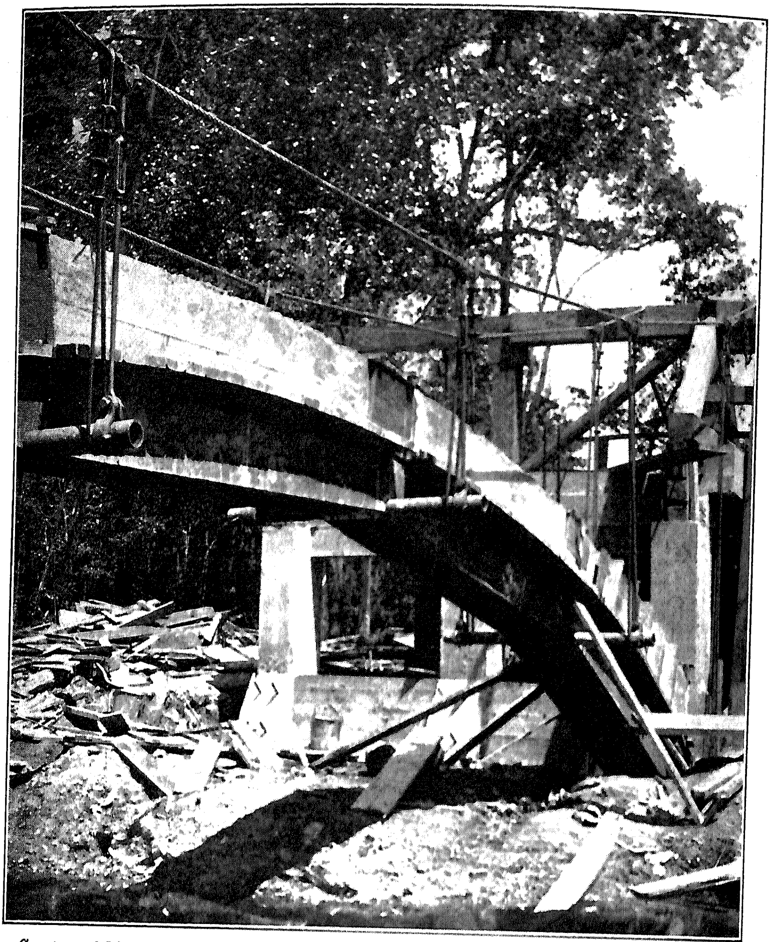
Courtesy of Mr. Philip Aylett, Civil Engineer, St. Louis.

FIG. 130.—View of Chickahominy River bridge during construction. Key spaces have just been filled with concrete. Forms are being taken off rib in order to hasten setting of concrete.

The unique feature in the steel centering used in constructing the Tunkhannock Creek viaduct on the relocation of the Delaware, Lackawanna, and Western Railroad was an adjustable panel at the crown of the steel arch trusses. A detailed description of these trusses and their erection may be found in Chapter XXXII.

The Cummings-Watson system of steel arch centering is shown in Fig. 129. The centering consists of triangular units, pin-connected at their apices, and with extra adjustable members which serve to connect the units and at the same time serve to form the lower chords of the arch trusses.

Suspended Systems.—Suspended systems of arch construction refer only to arches of the ribbed type.



Courtesy of Mr. Philip Aylett, Civil Engineer, St. Louis.

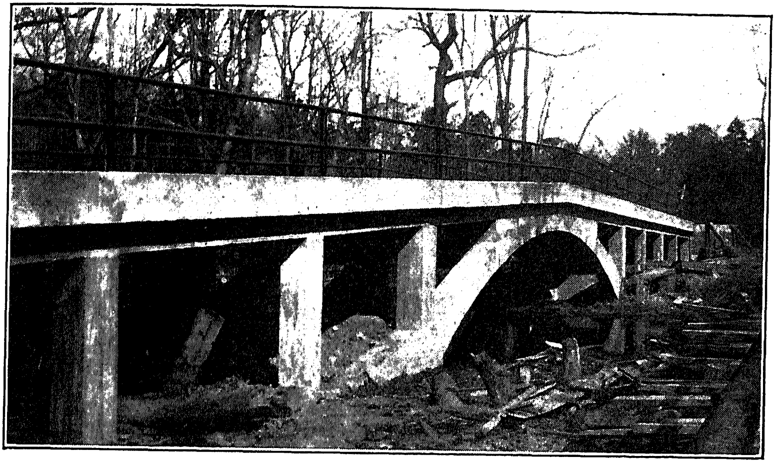
FIG. 131.—View of Chickahominy River bridge during construction. One rib completed and suspended centering ready to be removed.

The Aylett System (patented)[1] of suspended centering, previously illustrated in Fig. 108, is shown in detail in Figs. 130 and 131, which are construction views of the Chickahominy River bridge, Richmond, Va. (Fig. 132 is a view of the completed arch structure.) Another Aylett method of arch construction is shown in Fig. 133, known as the pre-casted voussoir method.

---

[1] See Engineering and Contracting, June 21, 1911, or Cement Age for June, July, and September, 1911, for detailed account of this method of construction.

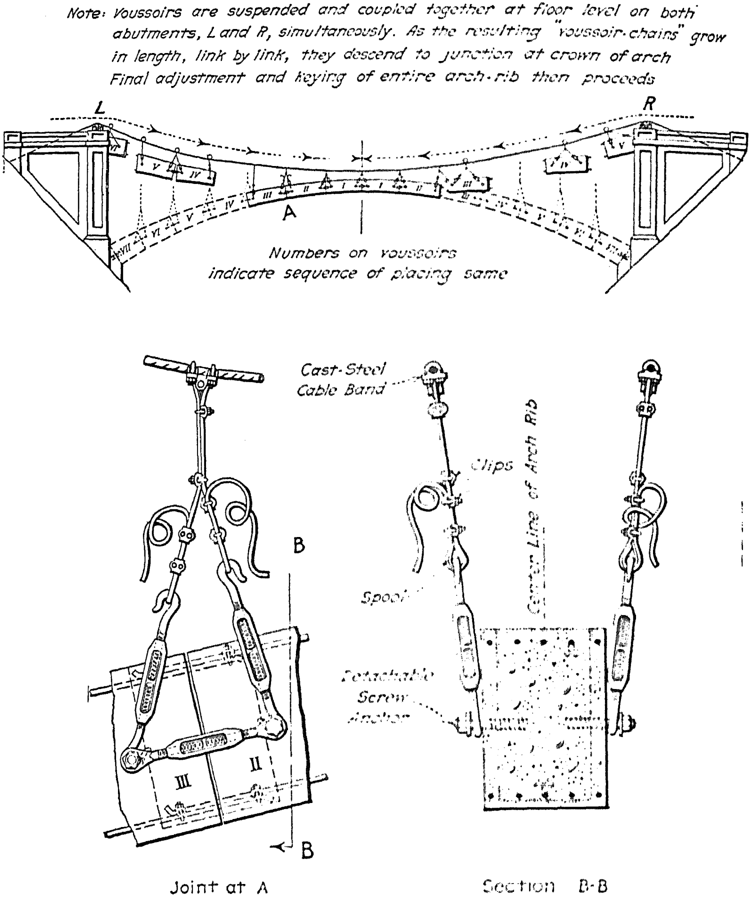
A patented system of arch erection without falsework, originated by Mr. J. B. Strauss and tried out in the construction of a bridge over the Kiswaukee River near Belvidere, Ill., in 1906, is illustrated in Fig. 110. Voussoir forms are made of reinforced concrete in sheet-metal molds (Fig. 134). After these forms are raised to position they are held in place by suspension rods which pass over A-frames supported on top of the piers and anchored by means of steel rods. The concrete forms when self-supporting form an arch trough in which the reinforcing steel and concrete are placed. The concrete forms are designed to safely carry their own weight and the weight of the concrete deposited therein. (Fig. 135 shows the completed Belvidere bridge.) Fig. 110, previously referred to, shows a traveller on falsework used in connection with the construction of the Belvidere bridge. This is not the way, however, that the Strauss System is intended to be used, since the particular object of the construction is to avoid any falsework whatever in the stream. In the erection of the Belvidere structure it was impossible to procure a cableway in time to start the work, and the traveller method was used on that account.



Courtesy of Mr. Philip Aylett, Civil Engineer, St. Louis.

FIG. 132.—View of completed Chickahominy River bridge near Richmond, Va.

arch rib forms are shown in Figs. 111 and 136. These trusses were designed to carry the weight of the arch forms together with the dead load of the wet concrete, both as to tension and compression, the result being that the steel reinforcement of the arches was stressed by the weight of the forms and wet concrete


Note: Voussoirs are suspended and coupled together at floor level on both abutments, L and R, simultaneously. As the resulting "voussoir-chains" grow in length, link by link, they descend to junction at crown of arch
Final adjustment and keying of entire arch-rib then proceeds
L
R
VII
VI
V
IV
III
II
I
A
Numbers on voussoirs indicate sequence of placing same
Cast-Steel Cable Band
Clips
Spool
Detachable Screw Anchor
Center Line of Arch Rib
B
B
III
II

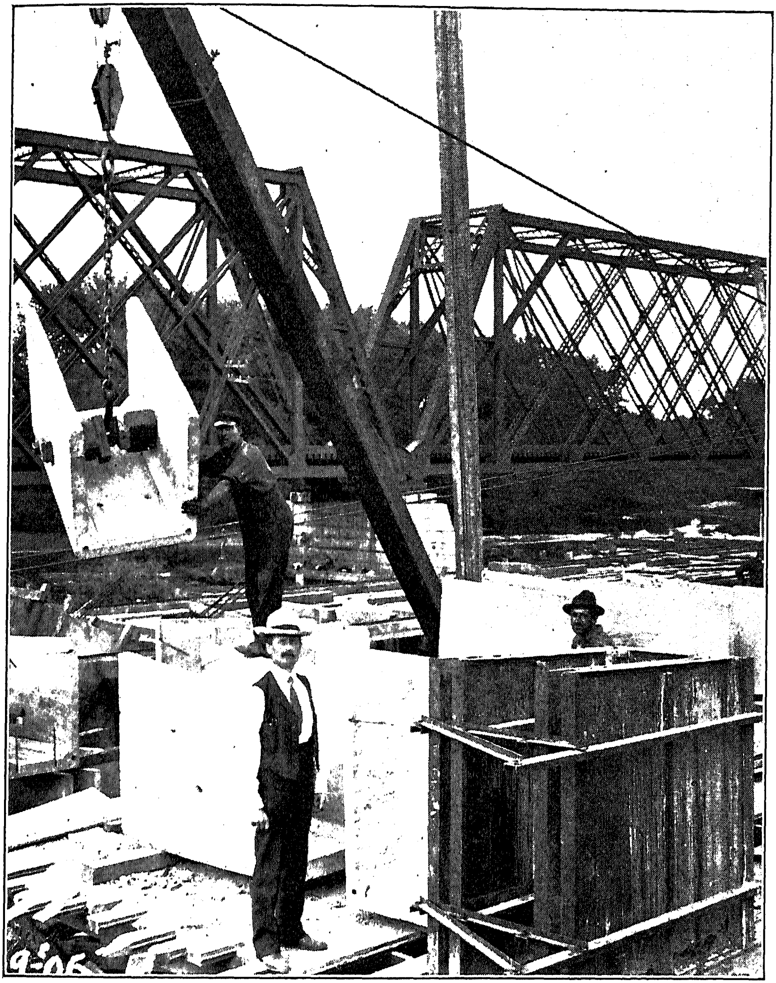
Joint at A

Section B-B

FIG. 133.—Pre-casted voussoir method of arch erection.

before the concrete obtained its set. In the finished structure the reinforcement was also considered to do its share in carrying the live load.

In Fig. 111 a wire mesh is shown on the outside of the outer line of arch ribs. This was stiffened by means of small vertical rods and supported from the steel arch trusses at a distance of 2 in. from the face form. The white Portland cement and crushed granite aggregate were carried up next the form and

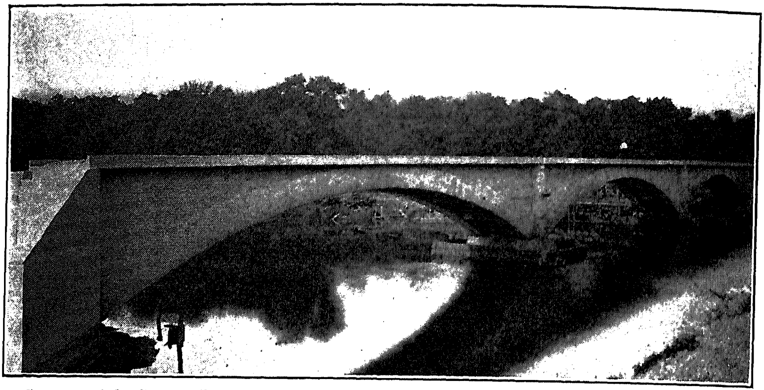


Courtesy of the Strauss Bascule Bridge Co., Chicago.
FIG. 134.—Voussoir forms used in the Strauss system of bridge construction.

outside of this mesh, a little ahead of the gray Portland cement concrete in the body of the arch. The mesh of the wire screen was large enough to admit stones from the granite aggregate passing through, but too small to allow the gray limestone of the main aggregate from passing outward toward the face. The advantage of this construction was, of course, the carrying up of the two masses simultaneously, and did not require the pulling of an intermediate form with the consequent dislodgment of material and breaking of the bond between them.

Figs. 136 and 137 show that the upper chords at the ends of the arches were each connected by two tension rods. Boxes of clay were arranged back of the nuts of these rods, so that in case of any lengthening of the arches the rods would slide through their bearings, but would come into action in tension under heavy loads.

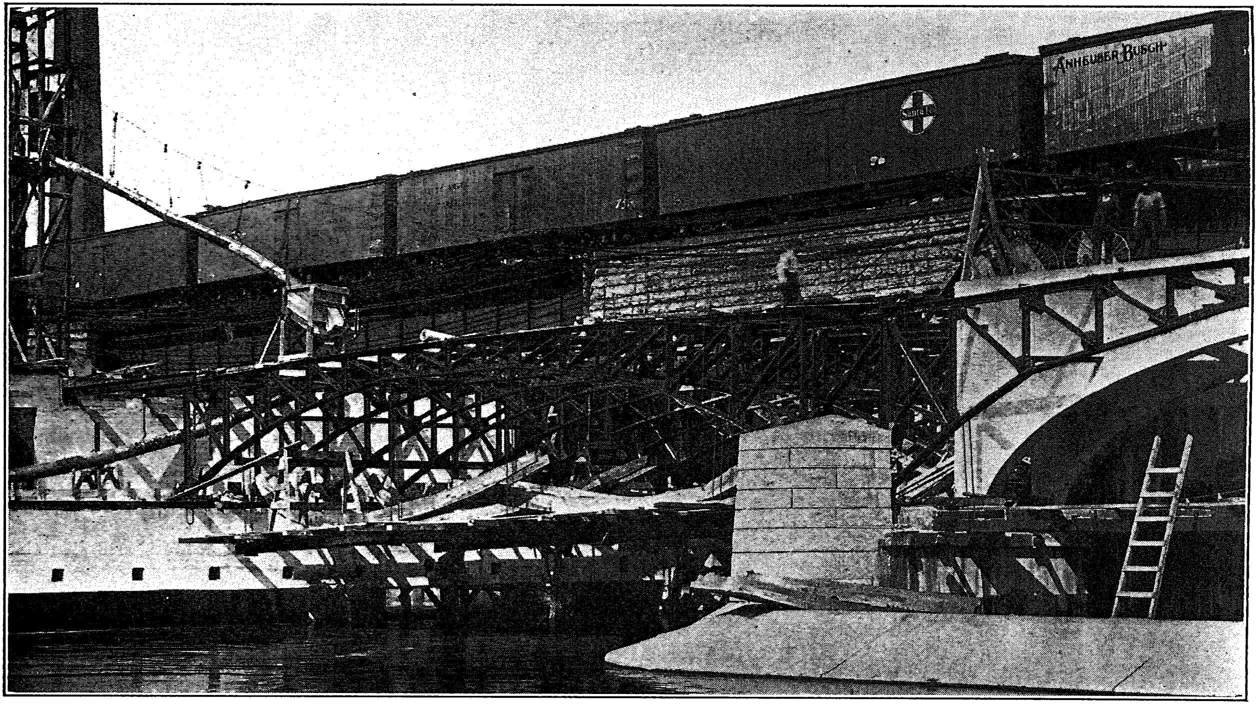
Figs. 111 and 136 show also working platforms suspended from the steel arch ribs. The reason for suspending these platforms rather than building up falsework may be seen in Fig. 111, where high water is passing under the bridge and a low power dam crosses the river 10 ft. upstream. A 98-ft. fall also occurs a few hundred feet downstream. Fig. 138 is a view of the completed structure. Incidentally it might be stated that the floor has a 2-ft. camber in its entire length, the ends of the structure being at the same elevation.



Courtesy of the Strauss Bascule Bridge Co., Chicago.

FIG. 135.—Bridge over the Kiswaukee River near Belvidere, Ill. Built by the Strauss system.

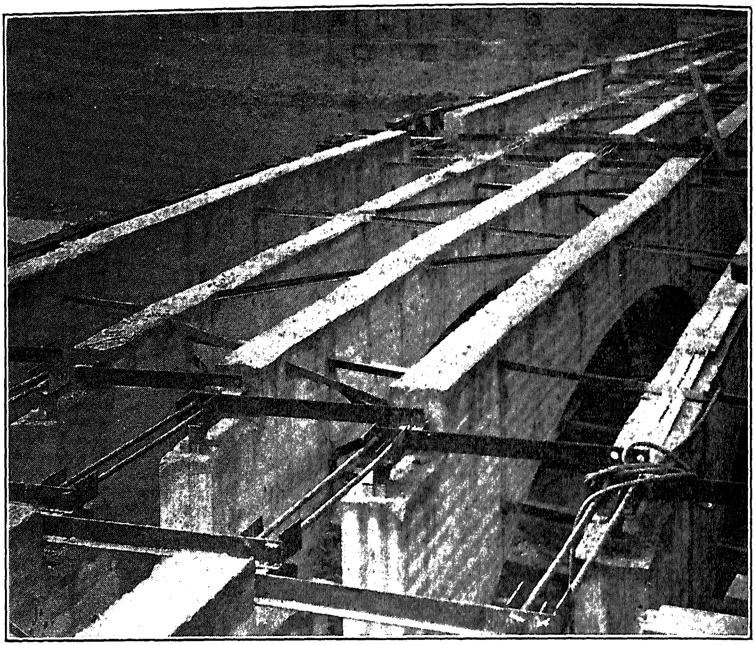
55. Forms.—Forms for piers and walls are usually constructed of either 1-in. or 2-in. plank nailed to studs and held by horizontal waling pieces, with tie bolts extending across the pier or wall between opposite wales. The wales, which consist generally of two planks fastened together but separated by



Courtesy of Mr. John F. Skinner, Prin. Ass't. Engineer, Dep't. of Engineering, City of Rochester.

FIG. 136.—Bridge at Central Avenue, Rochester, N. Y. under construction.

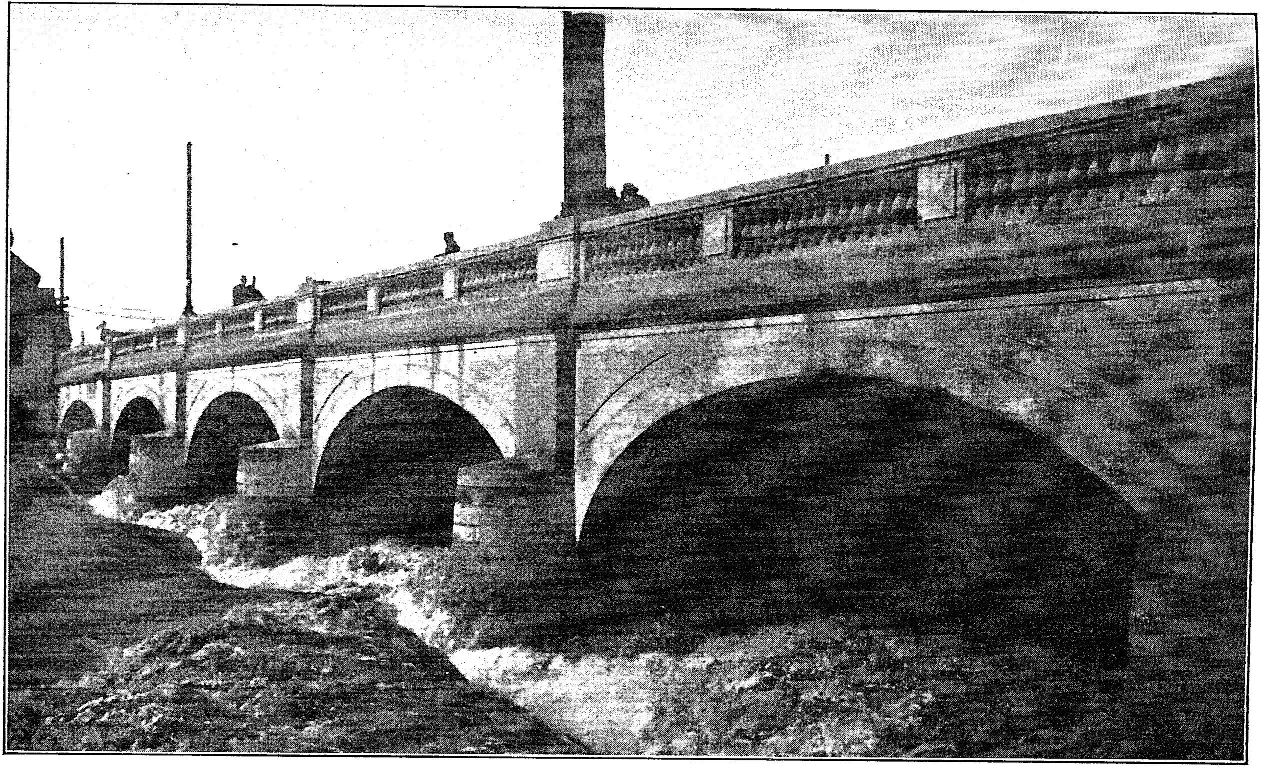
spacing blocks, are set edgewise against the form studs and the tie bolts are carried through the openings which occur in the waling pieces. Wire is sometimes used for bracing and is tightened either by wedges or by twisting. The wire pulls against spreaders which are inserted between forms and which are removed as the concrete level rises. As in most form work, bridge forms are either erected in sections of a size for easy handling or built in place (Fig. 139). Forms of small height may be braced by only battered posts outside.



Courtesy of Mr. John F. Skinner, Prin. Ass't. Engineer, Dep't. of Engineering, City of Rochester.

FIG. 137.—Bridge at Central Avenue, Rochester, N. Y. under construction.

Where bolts are employed in pier and wall construction, a number of different methods are used for withdrawing the bolts. One method is to cover each bolt with old pipe cut somewhat shorter than the inside dimensions of the forms, and to place a wood washer at each end of the pipe. When the forms are taken down, the bolts are easily drawn out of the pipes, the wood washers are then cut out of the face of the concrete, and the holes pointed up. Another method is to make the bolts



Courtesy of Mr. John F. Skinner, Prin. Ass't. Engineer, Dep't. of Engineering, City of Rochester.

FIG. 138.—View of completed bridge at Central Avenue, Rochester, N. Y.

in three pieces, with the middle piece occupying the same position between the forms as the pipe above described. This middle section is connected with the end pieces by means of ordinary unions. When the concrete has set sufficiently, one turn releases the end sections and the holes left in the work are plugged with mortar.

The following specification for the construction of forms is taken from the "Specifications of the Piney Creek concrete bridge, Washington, D. C."

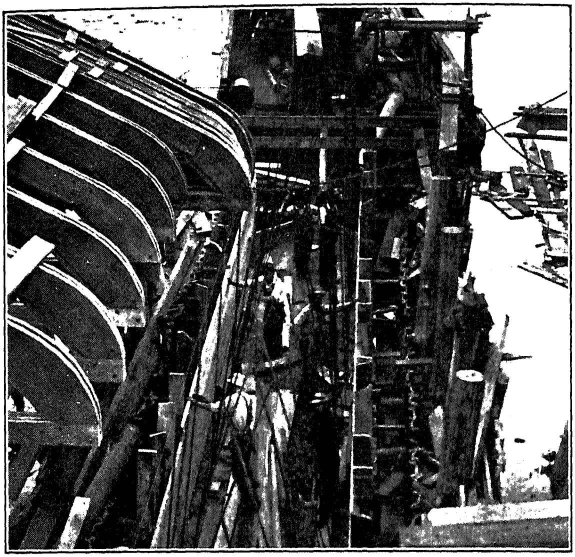
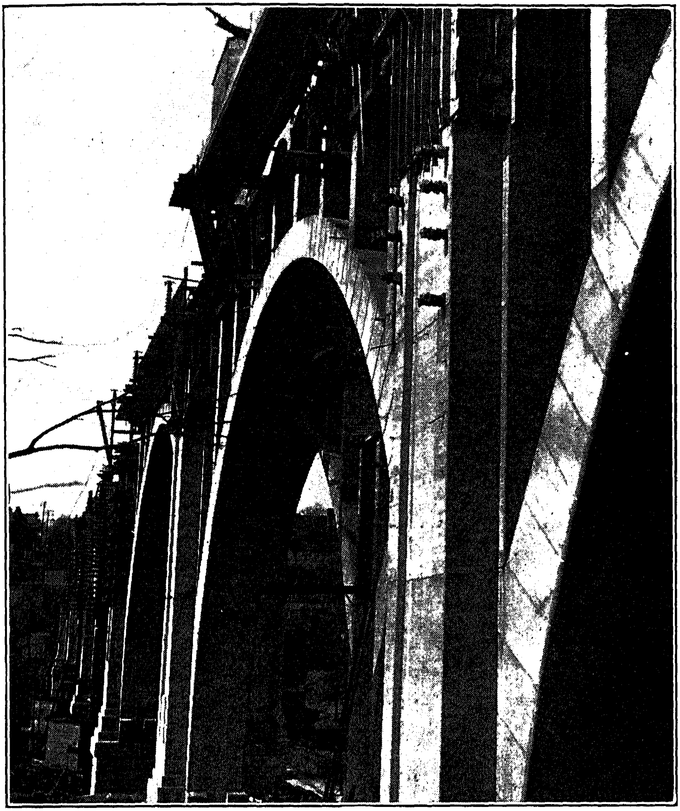


FIG. 139.—Constructing pier of Main and High Street bridge, Hamilton, Ohio.

"All forms for all classes of concrete shall be closely laid and strongly braced. The contractor shall, before proceeding with the work, submit drawings of the forms to the engineer for his approval. All lagging shall be tongue-and-grooved, and the studding for all the work shall be dressed or sawed to an even thickness. All forms, except by the consent of the engineer, shall be held in place by means of bolts, so made that the outer 3 in. of the bolts can be removed after the forms are taken down and the remaining holes shall be filled with mortar. If $\frac{7}{8}$-in. lagging is used, the studs shall not exceed 18 in. on centers; and, if 2 in. by 8-in. studs are used, the wales shall not be less than 8 in. by 8 in. These wales on a basis of 2 in. by 8-in. studs shall not be farther apart than

8 ft., nor shall the bolts which hold them and which have diameters of $\frac{3}{4}$ in. be farther apart than 8 ft.

"If lagging, studs, wales, or bolts are proposed by the contractor other than those described herein before, they shall be such as to make a form of equal strength and stiffness to that described.



Courtesy of Lehigh Portland Cement Co.

FIG. 140.—Construction view of Eighth Street viaduct, Allentown, Pa.

"Washers shall be used under all bolt heads and nuts, and, before proceeding with the concrete work, forms shall be brought true to line and grade, and all bolts shall be taut."

Where especially good work is desired, forms are lined with galvanized iron. For high piers or walls, the forms are constructed in large panels. After the concrete has been constructed to a proper height and the last course has set several days, the panels are disconnected and hoisted to a higher position and then reassembled for concreting, and so on. The same practice is followed in pilaster construction (Fig. 140).

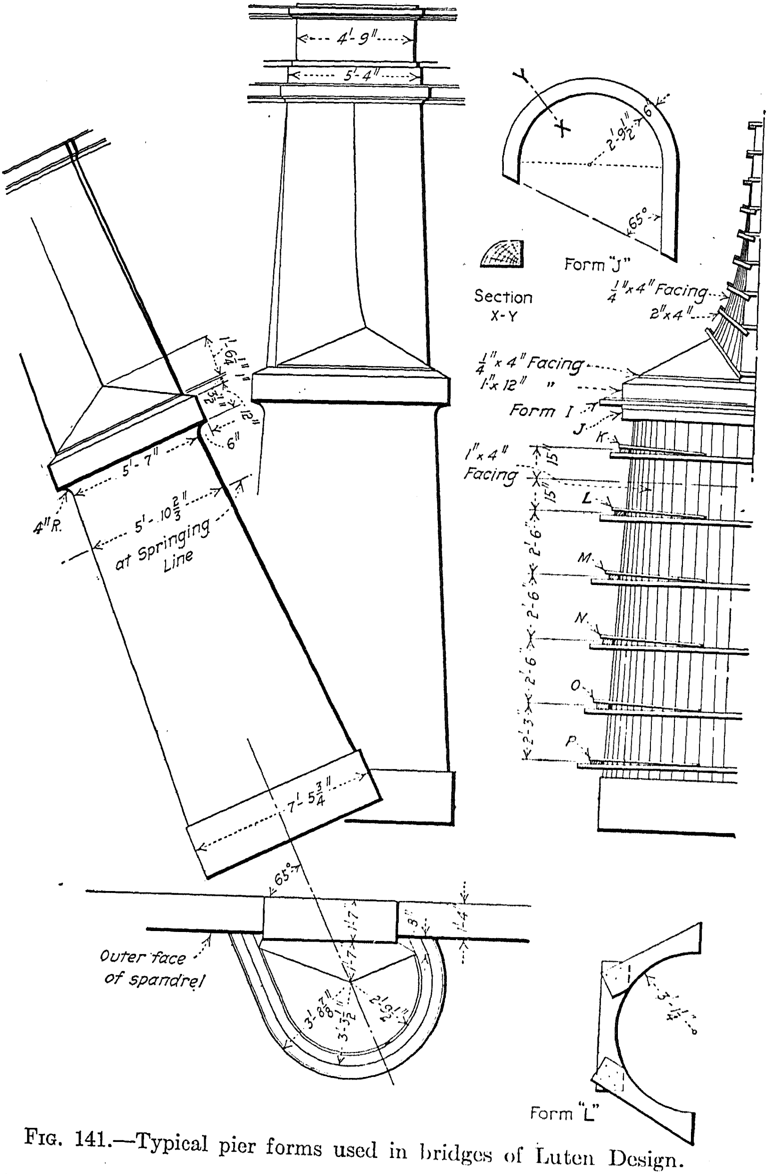
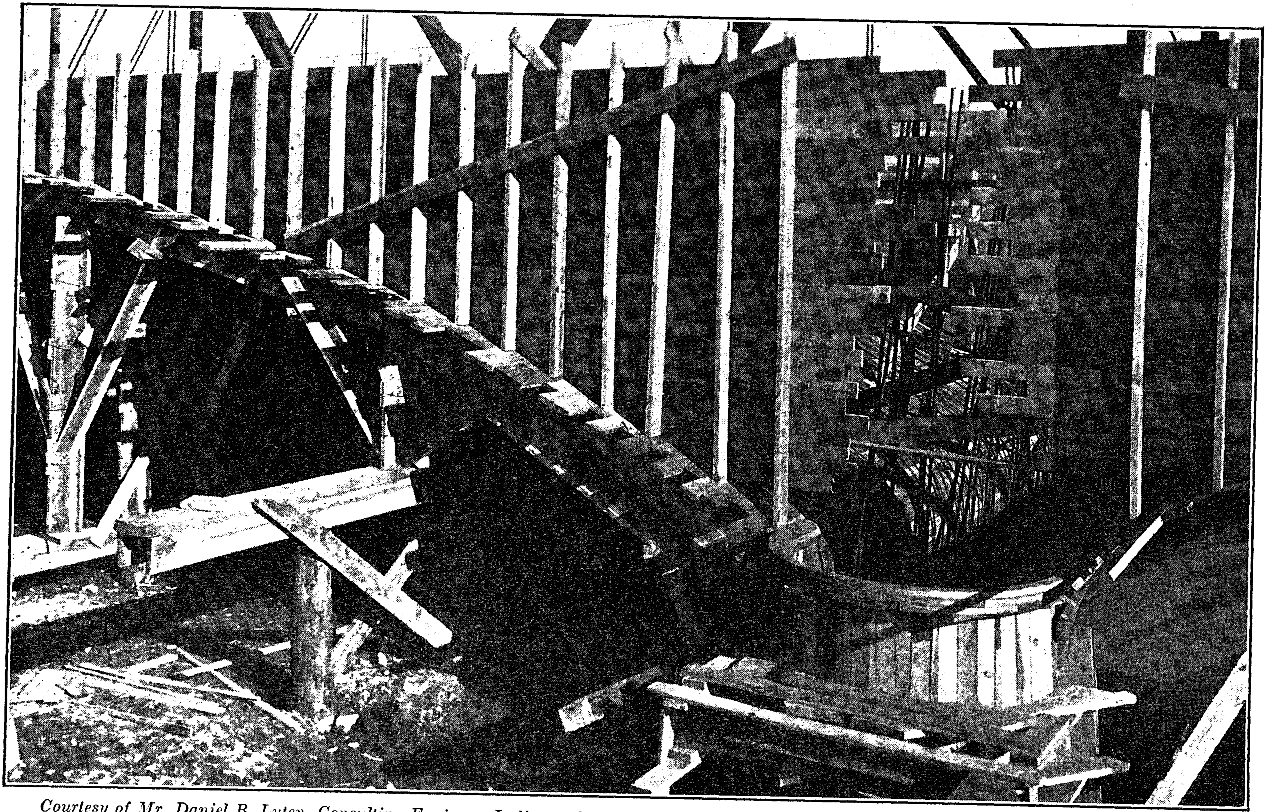


FIG. 141.—Typical pier forms used in bridges of Luten Design.

When the ends of piers are rounding, special forms are necessary. In the construction



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 142.—Centering and curved pier forms used in bridges of Luten Design.

the Pennsylvania R. R. tracks in Pittsburgh, the forms for the curved ends of the piers were built of 1 in. by 2-in. strips nailed to horizontal segmental wales. These wales were nailed to the wales of the side forms. The rounding forms were kept in place by wiring to dowels set in the foundation concrete. Before starting the erection of the form work, a flexible panel was made by nailing galvanized-iron sheets to the 1 in. by 2-in. strips. This panel was bent against the wales, which acted like hoops.

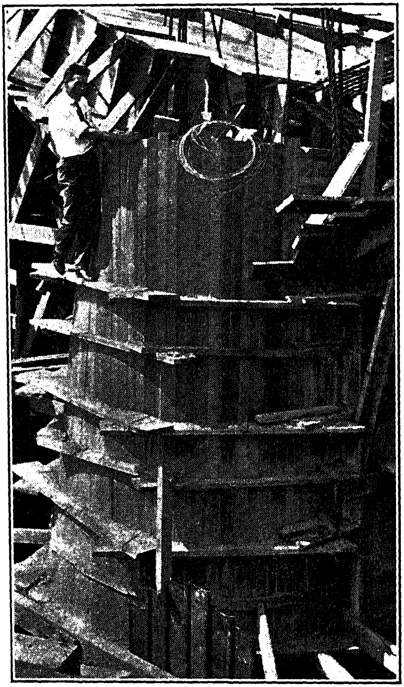
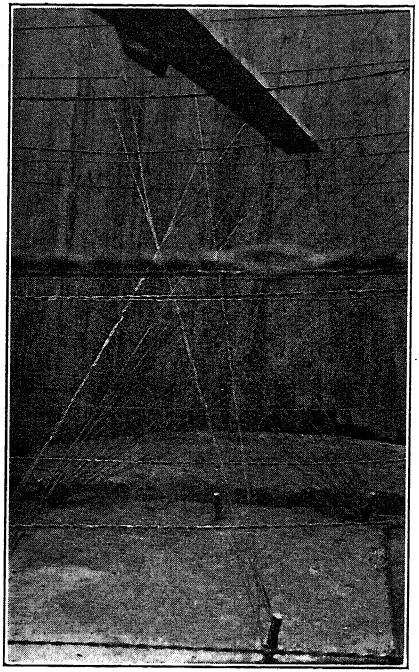


Fig. 143.—Detail view of curved pier forms.

Curved pier forms used in bridges of Luten Design are shown in Figs. 141, 142, and 143. Fig. 144 shows the wiring of the forms to prevent spreading. Fig. 145 shows the pier form removed, and Fig. 146 shows it set in place for use in constructing another pier end.

In the bridge shown in Fig. 147 the pier starlings were constructed of granite from an old bridge. Pilaster forms are seen resting on the upper course of stone.

The common form of longitudinal bulkhead used in arch ring



Courtesy of Mr. W. C. Giffels.

FIG. 144.—Wiring of curved pier forms to foundation concrete.

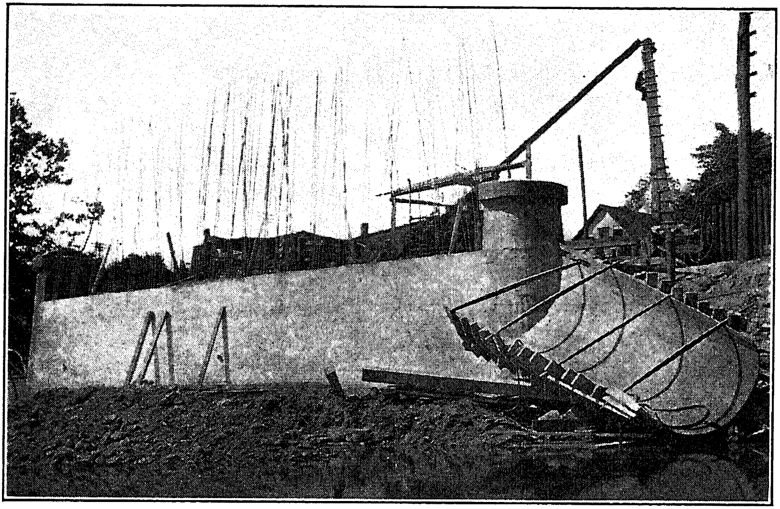
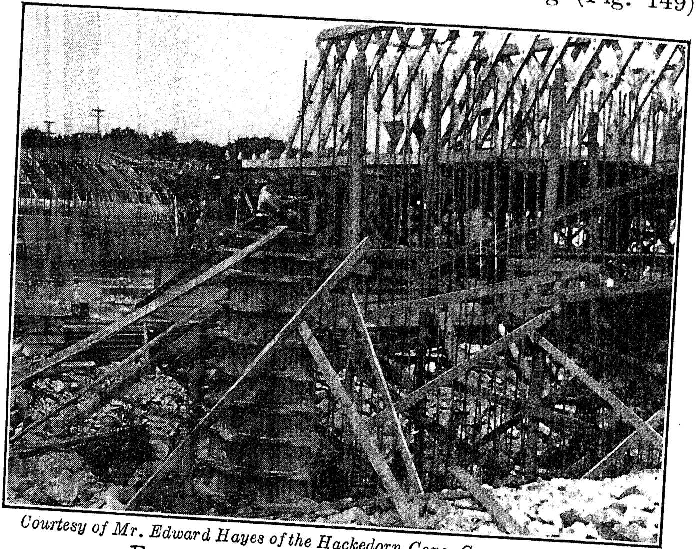


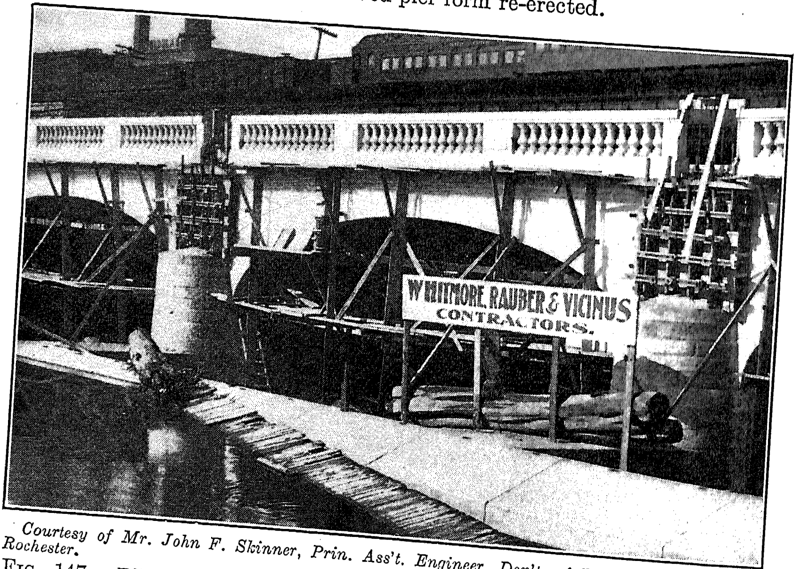
FIG. 145.—Curved pier form removed.

construction is shown in Fig. 148. Spandrel walls for earth-filled arches are either built on top of the arch ring (Fig. 149), or



Courtesy of Mr. Edward Hayes of the Hackedorn Cons. Co.

FIG. 146.—Curved pier form re-erected.



Courtesy of Mr. John F. Skinner, Prin. Ass't. Engineer, Dep't. of Engineering, City of Rochester.

FIG. 147.—Pilaster forms. Bridge at Central Avenue, Rochester, N. Y.

include a portion of the arch, the bottom inner edge of the spandrel retaining wall lapping a short distance over the pleted arch ring (Fig. 150). Forms for spandrel walls of the latter class are shown in Figs. 151 and 152.

Prepared sheet metal forms are often employed for special work such as cantilever beams, capitals on spandrel columns, and other small projections. These sheet metal forms are fastened to the wood forms at successive positions.

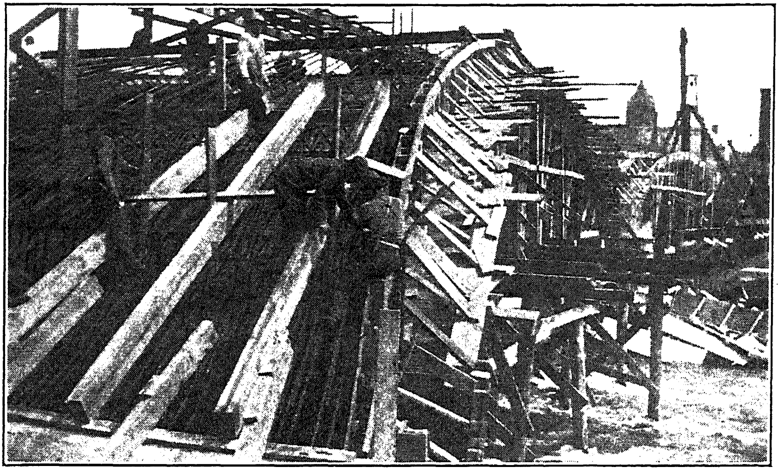


FIG. 148.—Longitudinal bulkhead used in arch-ring construction.

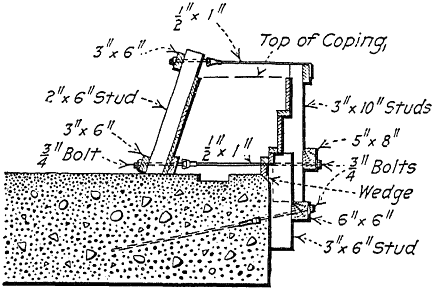
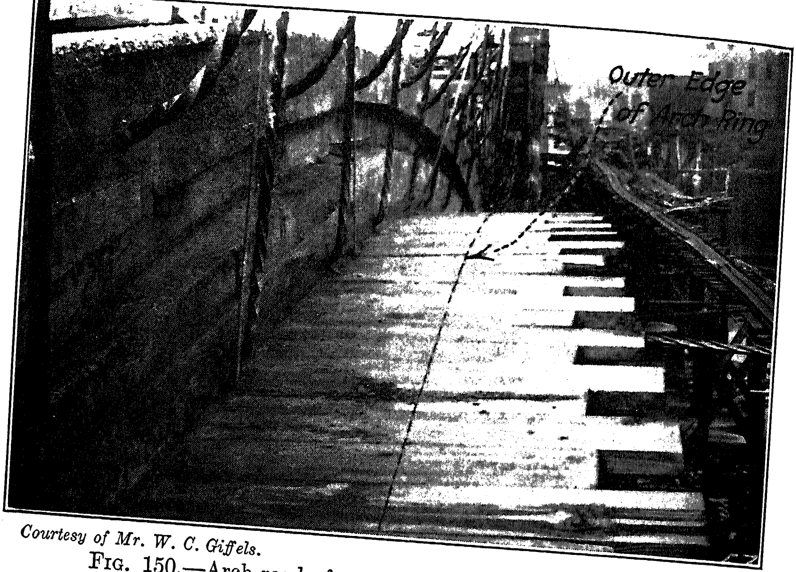


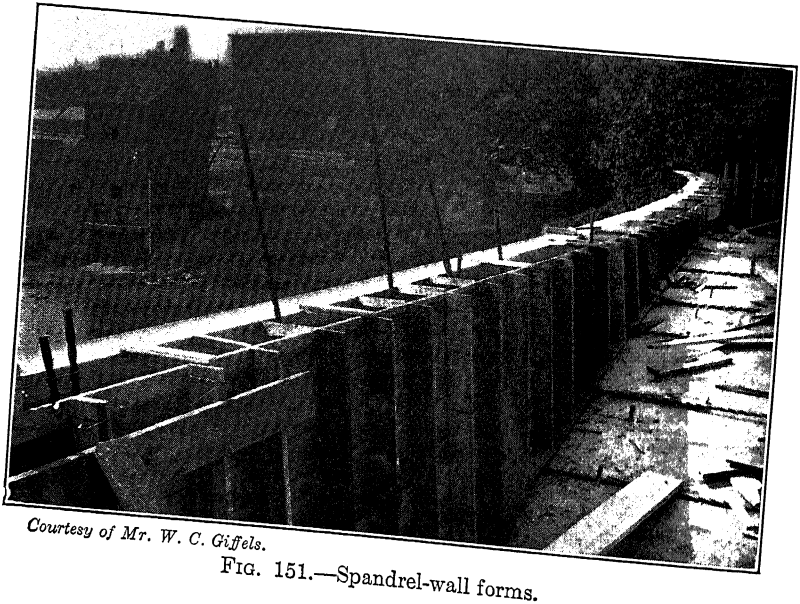
FIG. 149.—Spandrel wall forms used in constructing the Yardley bridge, Philadelphia & Reading Ry. Co.

The railing or balustrade of Greek cross panels shown in Fig. 153 was cast in units on a horizontal plane. Cement Age, issue of March, 1912 describes the construction of the panels of this balustrade as follows:



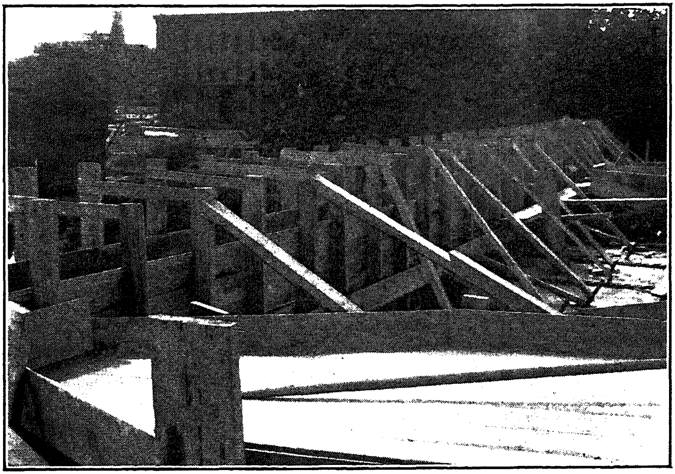
Courtesy of Mr. W. C. Giffels.

FIG. 150.—Arch ready for the building of spandrel walls.



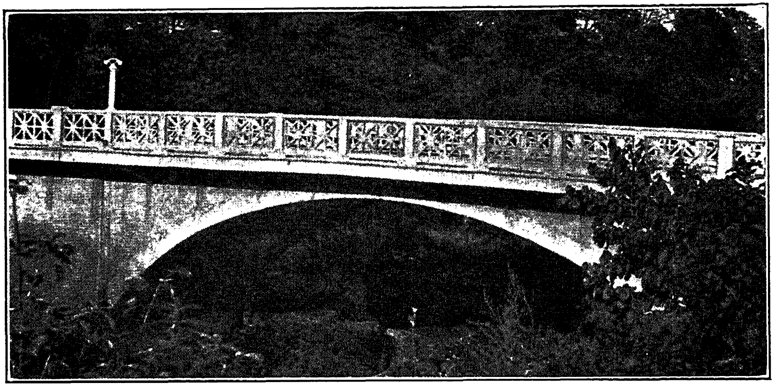
Courtesy of Mr. W. C. Giffels.

FIG. 151.—Spandrel-wall forms.



Courtesy of Mr. W. C. Giffels.

FIG. 152.—Spandrel-wall forms.



Courtesy of Waddell & Harrington, Consulting Engineers, Kansas City.

FIG. 153.—Cleveland Avenue bridge, Kansas City, Mo.

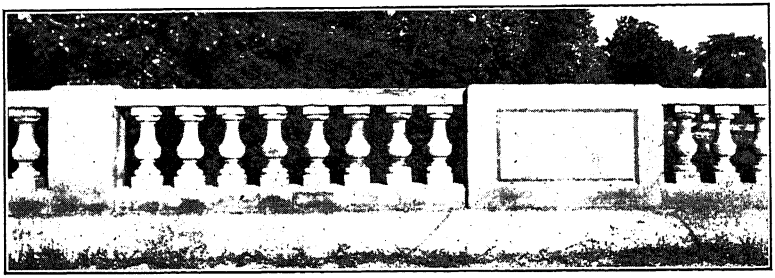


FIG. 154.

"The forms consisted of a wooden base 6 in. wider and longer than the concrete panel. A wooden rail the thickness of the panel was bolted around the outer edges of this base, the measurements between the rails being the size of the panel. On this base were also bolted triangular

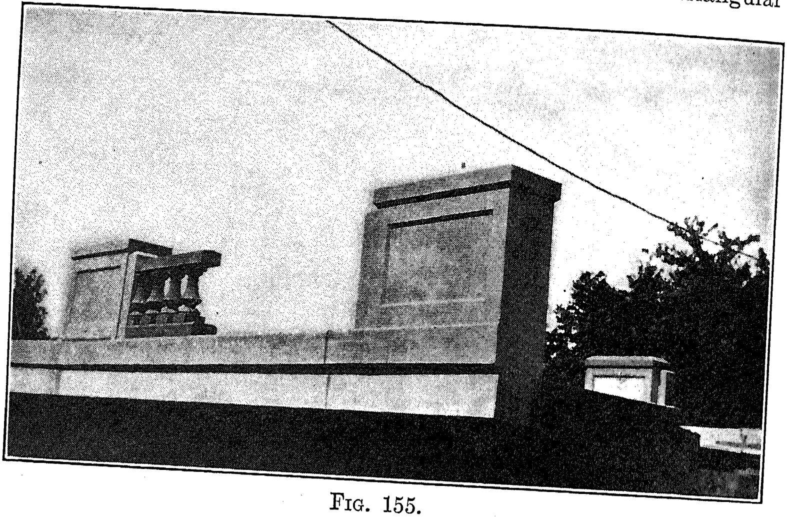


FIG. 155.

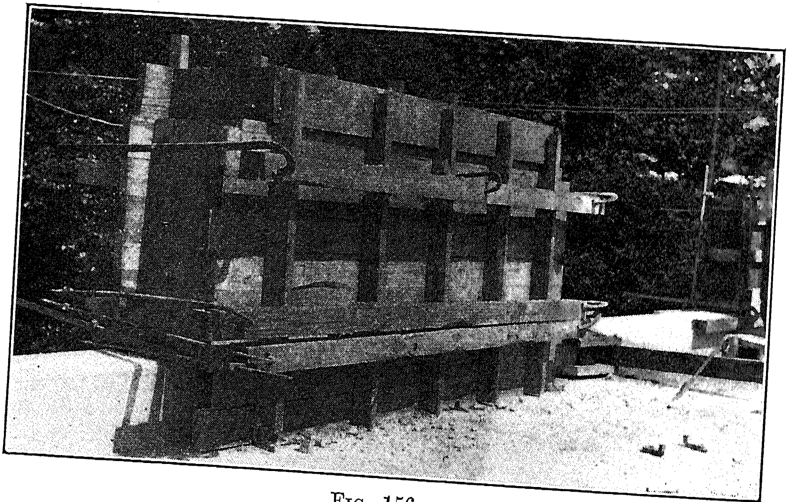


FIG. 156.

blocks, split in half horizontally, with a slight bevel or draw to each half. These blocks were bolted together, and, bolted to the base in place, form the triangular openings of the Greek cross design of the panel. After the forms were bolted together the base was laid level horizontally on two horses or bents, about 3 ft. above the work floor level. The form was then filled with water and allowed to soak over 1 night. The following morning the water was let out; the form cleaned and relevelled; $\frac{1}{2}$ in. of concrete was put in bottom and well tamped with small hand tampers; one set of reinforcing wires were placed; 3 in. more concrete was put in, well tamped; a second set of reinforcing wires was then placed; concrete finally finished to the top and struck off with a straight edge run over top of outer railing and triangular blocks, all of which were made the exact thickness of the panel. After the concrete was sufficiently set the top was finished with smoothing trowel. Concrete in the panel was allowed to stay in the form undisturbed 3 days; the outer railing of the form was then removed and the base to which the triangular blocks were bolted and on which the concrete was resting, was set up on edge, the panel resting on a piece of timber on the ground. The blocks were then unbolted from the base and the base

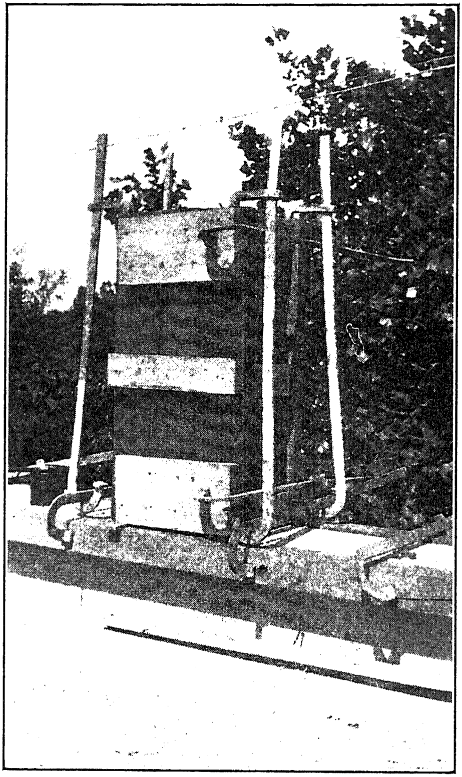


FIG. 157.

removed. Forty-eight hours from the time the base was removed, the triangular blocks were withdrawn from the concrete panel by taking one-half of the block out from one side and the other half from the opposite side, the slight bevel and shrinkage of the blocks allowed them to be removed without injury to the edges of the concrete."

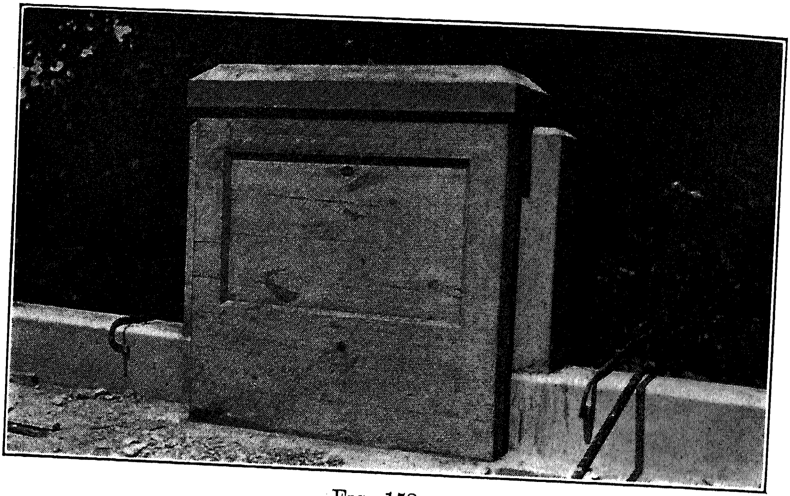


Fig. 158.

Railing of the type shown in Fig. 154 may be either constructed entirely in separate units and afterward erected, or the spindles or balusters may be poured separately in wooden or cast-iron molds and the top and bottom rails and posts cast in place. Figs. 155 to 158 inclusive show the construction of a railing of this balustrade type.

CHAPTER XI

THREE-HINGED ARCHES

56. General Discussion.—An arch with three hinges is statically determinate and consequently can be analyzed much more readily for a given loading than is possible in the case of a fixed-ended or solid arch. Furthermore, three-hinged arches do not need to be analyzed for temperature changes, the hinges allowing contraction and expansion of the ribs without causing any stress throughout the arch. Obviously this statement does not take into account the effect which results from friction on the hinges, but such effect is usually considered to be negligible. Whether or not hinge friction is likely to cause appreciable error in the analysis of three-hinged arches is still a matter, however, in regard to which there seems to be a decided difference of opinion.

Three-hinged arches are especially adapted to sites where abutments and piers must be founded on compressible soil or on piles. The hinges permit of considerable settlement without failure of the arch or without causing the huge cracks which are sure to develop in a fixed-ended structure under like conditions. Of course, a solid arch may be designed on the assumption that the abutments are yielding, but this is rarely done and such computations in any event could not take into account such settlement as might come from an unexpected source.

Hinges in arch-bridge construction are likely to be an expensive detail, especially in short-span structures. The claim is made, however, that in arches of large span, the saving in concrete as compared with the fixed-ended type much more than pays for the hinges.

It is generally admitted that the three-hinged arch is usually awkward in appearance and has not the graceful form that is characteristic of the majority of the solid arch structures. This lack of artistic proportions is caused by the increased thickness at the haunches, and cannot be avoided in economical design.

57. Methods of Analysis.—Three-hinged arches should be analyzed for at least the same conditions of loading that are in Art. 16 for arches with fixed ends. It should be stated, however, that, for large arches, an analysis by influence lines for the exact maximum loadings is much to be preferred to the common method referred to in the above article, by reason of the fact that the common method assumes the live loads as fixed at only a few definite locations on the span.

Common Method.—Consider first the general case of an unsymmetrical three-hinged arch subjected to a number of vertical concentrated loads. By referring to Fig. 159, it is seen that there are four unknown quantities—namely: the horizontal and vertical components of each reaction—and four independent equations are necessary to solve for these unknowns. We have the following three equations from statics:

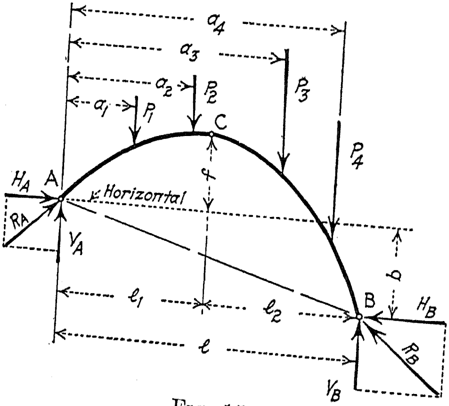


FIG. 159.

ΣV = algebraic sum of the vertical components = 0.
ΣH = algebraic sum of the horizontal components = 0.
ΣM = algebraic sum of moments of all the forces about any point = 0.

The additional equation may be obtained from the fact that the bending moment is zero at the crown hinge. Thus we have the following four equations with respect to the arch of Fig. 159:

$$V_A + V_B - \Sigma P = 0$$
$$H_A - H_B = 0$$

Taking moments about the left hinge

$$H_B b - V_B l + \Sigma Pa = 0$$

Since the moment at the crown hinge is zero

$$V_A l_1 - H_A f - \Sigma_0^{l_1} P(l_1 - a) = 0$$

These four equations may be solved simultaneously to obtain the horizontal and vertical components of the two reactions.

The calculations may be simplified by resolving each reaction into a vertical force and a force in the direction of the closing chord (Fig. 160). The four equations in this case are as follows (since $H_A = H_B$ from Fig. 159):

$$\begin{aligned} V_1 + V_2 - \Sigma P &= 0 \\ H_1 - H_2 &= 0 \\ -V_2 l + \Sigma Pa &= 0 \\ V_1 l_1 - \Sigma_0^{l_1} P(l_1 - a) - H_1 r &= 0 \end{aligned}$$

or

$$V_1 l_1 - \Sigma_0^{l_1} P(l_1 - a) - H_A c = 0$$

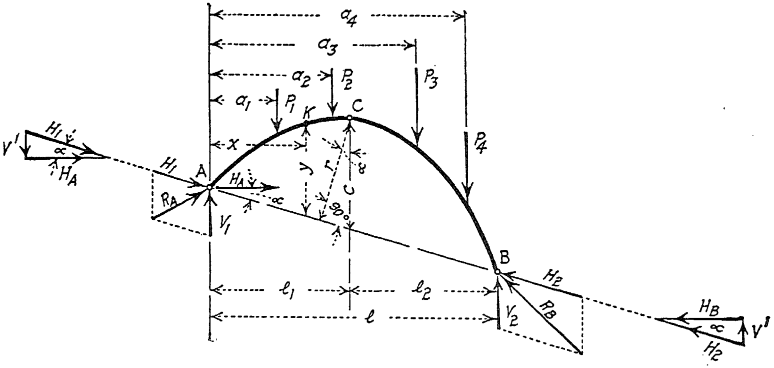


FIG. 160.

(The values of V' have not been considered in the first equation as they are equal and opposite in direction.) With the components of either reaction determined by these equations, the line of thrust may be drawn throughout the arch as described in Art. 12 for the arch with fixed ends.

It should be noted that the values of V_1 and V_2 may be obtained from the above equations (or by using $\Sigma M = 0$ at both points A and B) in the following form:

$$V_1 = \frac{1}{l} \Sigma P(l - a) \tag{1}$$

$$V_2 = \frac{1}{l} \Sigma Pa \tag{2}$$

These forces are thus identical with the reactions of a simple beam of the same span and similarly loaded.

The bending moment at any point K (Fig. 160) may be expressed as follows:

$$M = V_1 x - \Sigma_0^x P(x - a) - H_A y$$
$$= M_K - H_A y \qquad (3)$$

where M_K is the bending moment at the point K of a similarly loaded beam. At the crown hinge, letting M_C denote the moment of the vertical forces about the point C, we have

$$M = M_C - H_A c = 0$$

or

$$H_A = \frac{M_C}{c} \qquad (4)$$

Equations (1) to (4) inclusive are the formulas commonly employed in the analysis of three-hinged arches—supplemented, of course, with the force and equilibrium polygons as in the case of arches with fixed ends.

For symmetrical arches, H_1 and H_2 are horizontal and the line of thrust need be drawn for only one-half the arch when the loading is symmetrical about the crown hinge. In such a case of loading, the thrust at the crown hinge is horizontal and the line of thrust may be determined by trial in the manner described in Art. 26. This trial method gives exact results when applied to a three-hinged symmetrical arch on account of there being two known points (hinge points) on the line of thrust for each half of arch.

The computations for uniform live loading are extremely simple and should be made separately from those for dead load or concentrated live loads. For full loading, with the crown-hinge at mid-span, Formula (4) gives:

$$H_A = \frac{1}{8} \cdot \frac{wl^2}{c} \qquad (5)$$

where w is the uniform load per foot. The following equation, determined by substituting in Formula (3) gives the bending moment at any point (coördinates x and y):

$$M = \frac{1}{2} wx(l - x) - \frac{1}{8} \cdot \frac{wl^2}{c} \cdot y \qquad (6)$$

(For an arch of parabolic form, $M = 0$, and only axial stress occurs throughout the arch for full uniform loading.) With only one-half of the span loaded

$$H_A = \frac{1}{16} \cdot \frac{wl^2}{c} \tag{7}$$

or one-half that due to full loading. The bending moment at any point in the loaded half equals

$$M = \frac{1}{8} wx(3l - 4x) - \frac{1}{16} \cdot \frac{wl^2}{c} \cdot y \tag{8}$$

and in the unloaded half

$$M = \frac{1}{8} wlx - \frac{1}{16} \cdot \frac{wl^2}{c} \cdot y \tag{9}$$

(In Equations (8) and (9), the value of x is measured from that end of the arch which is nearer to the point in question.)

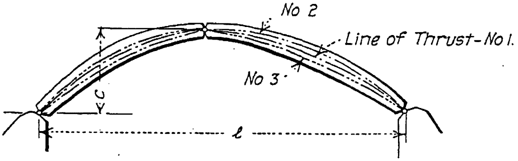


FIG. 161.

A three-hinged arch is commonly analyzed for (1) dead and uniform live load over the entire span, (2) for dead and uniform live load over the right half of span, and (3) for dead and uniform live load over the left half of span. Full loading gives maximum stresses for the sections near the hinges, while the half-span loadings give the greatest stresses near the quarter points of the span. The usual method of design is to locate the hinges at the proper points and to draw the force lines representing the load concentrations. These loads can be determined quite accurately by making a complete design of the spandrels prior to the arch design and by approximating the weight of the arch ring—the arch ring, however, need not be drawn. The lines of thrust for the three conditions of loading stated above are then drawn as shown in Fig. 161. With the lines of thrust known, it then becomes possible to determine the correct thickness of the arch at any point and decide upon a suitable arch ring which, of course, should not differ appreciably in weight or position from arch ring previously assumed or else a second analysis should be made.

Influence-line Method.—The method of analyzing arches by the use of influence lines is explained at length in Chapter V for fixed arches. (See also Chapter VIII.) The method is the same for three-hinged arches but is simplified by the fact that for a single concentration—as, for example, a load of unity at the point L_2 in Fig. 162—the direction of one of the reaction lines is given by the line connecting the two hinges to one side of the load. The direction of the other reaction is then known. Thus, designing sheets similar to Designing Sheets Nos. 11 and 11*A* may be readily prepared and the remainder of the analysis followed in exactly the same manner as for the fixed arch.

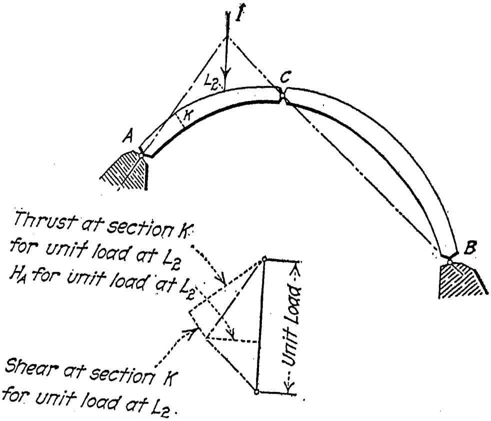
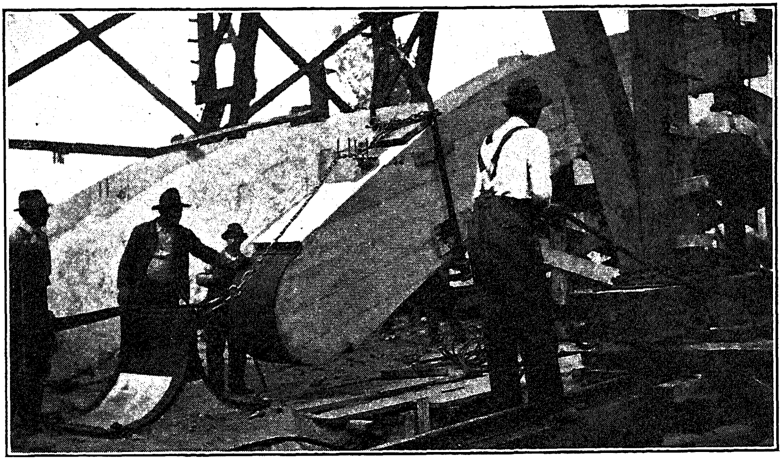

1
L_2
C
K
A
B
Thrust at section K for unit load at L_2
H_A for unit load at L_2
Unit Load
Shear at section K for unit load at L_2.

Fig. 162.

58. Types of Hinges.—The most common form of arch hinge consists of a structural or cast-steel pin bearing on two steel castings. Hinges of this type are shown in Figs. 174 to 177 inclusive. A form similar to the one above mentioned consists of two steel castings with ball-and-socket joints. Details of this type of hinge are given on Plate XL, Chapter XIII.

A patented method of hinge construction is shown in Fig. 198, Chapter XII. The reinforcement of each section of arch rib is connected at the crown end to cylindrical plates of steel having a ball joint mated into a cup in the opposite rib. The lower hinge (see also Fig. 163) consists of a semicircular plate attached to the rib and a cast-iron shoe bolted to the pier or abutment.

A flat lead plate has been used to a limited extent in three-hinged arch construction. A plate of this kind cannot be truly



Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.

FIG. 163.—Thomas Method of arch construction.

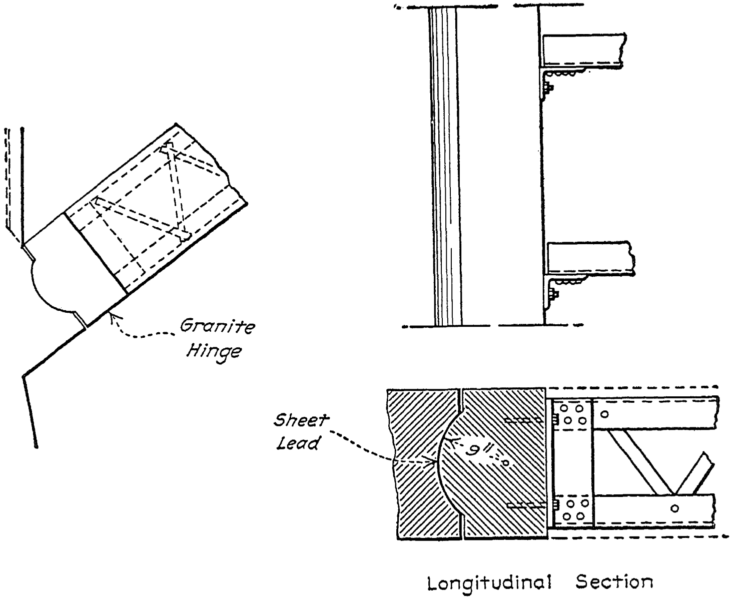


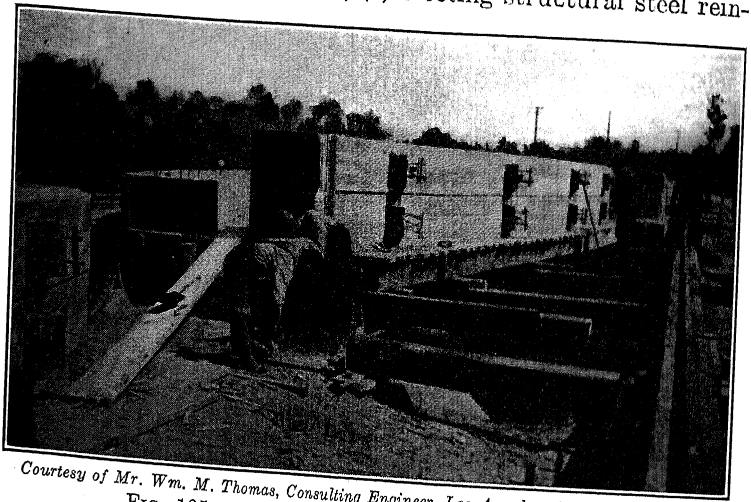
FIG. 164.—Details of stone hinge.

called a hinge, but does limit the line of thrust to a definite area. It should be used only for arches which are to be erected as three-hinged (due to a possible excessive settlement of the centering), but which are afterward to be converted into solid arches by encasing the hinges in mortar or concrete.

A type of stone hinge which has been used in a few bridges is shown in Fig. 164. The details of an unusual hinge are given in Fig. 178.

It should be evident that all types of hinges are not equally effective in definitely locating the line of thrust.

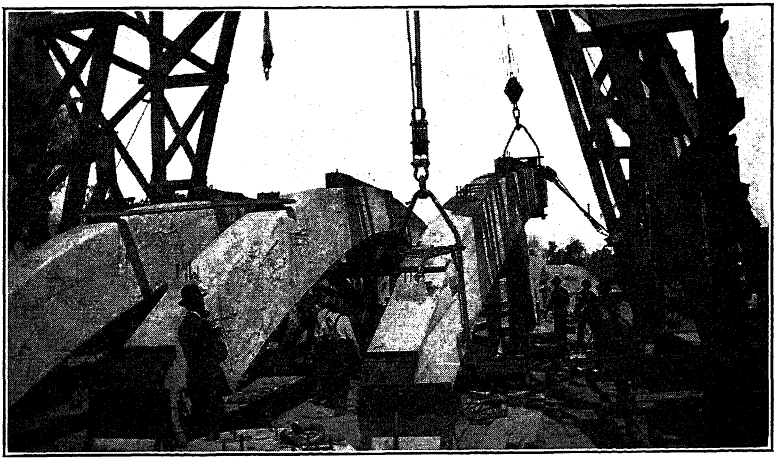
59. Methods of Construction.—Three distinct methods of construction have been employed in the erection of three-hinged arches: (1) casting the concrete ribs in forms on the ground and then hoisting them into place; (2) erecting structural steel reinforcement to be employed in the arch ribs and using this reinforcement to support the weight of the forms and plastic concrete during construction; and (3) employing the usual type of centering and casting the ribs in place. The first method is the one usually followed. Method No. 2 is of advantage when a stream to be spanned is subject to sudden freshets and a minimum of falsework is required. (See Fig. 175 and Plate XL of Chapter XIII.) Method No. 3 is necessary only under unusual conditions.



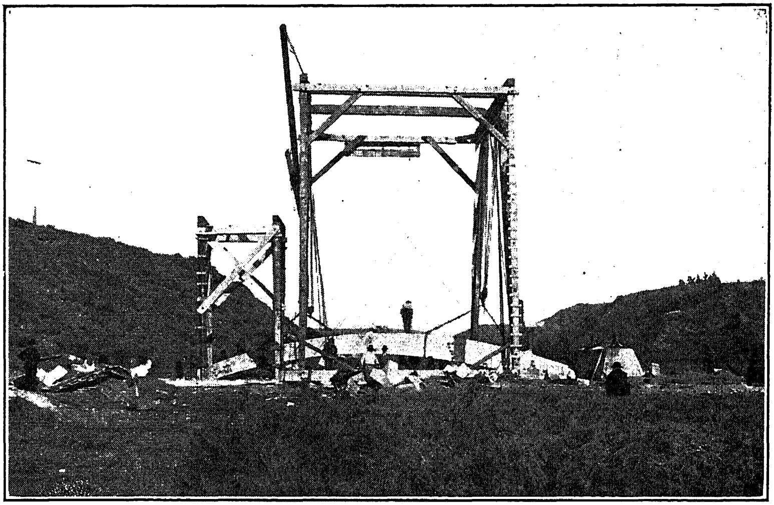
Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.

FIG. 165.—Thomas Method of arch construction.

The cheapest type of the three-hinged arch and also the type that is lightest and best adapted to the use of hinges is one of detached ribs supporting spandrel columns. Such a type of arch lends itself readily to the unit method of construction should this form of erection be desired, and also eliminates the necessity for waterproofing which is a serious problem in the case of a solid filled arch.



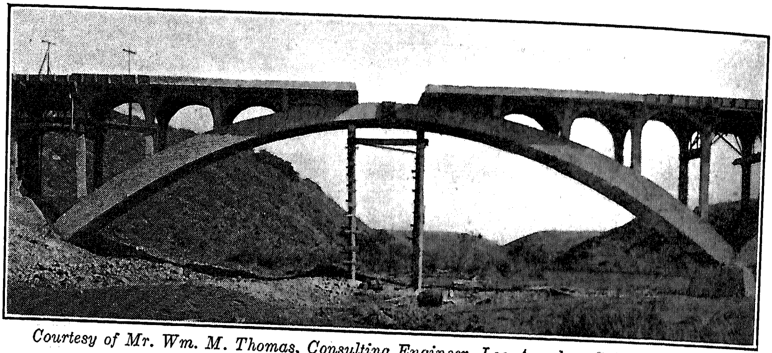
Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.
FIG. 166.—Thomas Method of arch construction.



Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.
FIG. 167.—Thomas Method of arch construction.

The Thomas Method of unit construction is shown in Figs. 165 to 172 inclusive and the designing details in Fig. 178. The illustrations are self-explanatory.

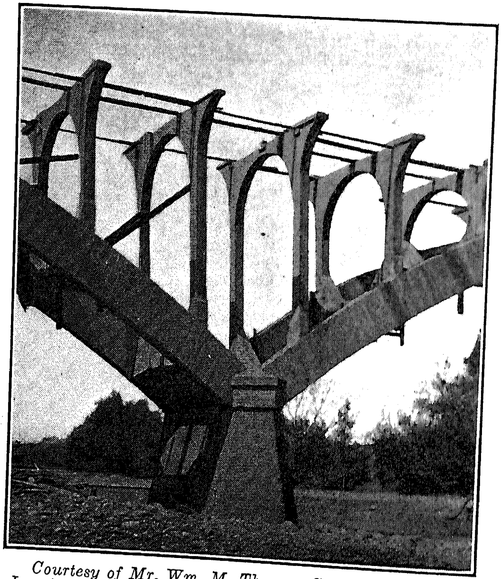
60. Details of Design.—Figs 174 to 178 inclusive give typical details of three-hinged arches.



Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.

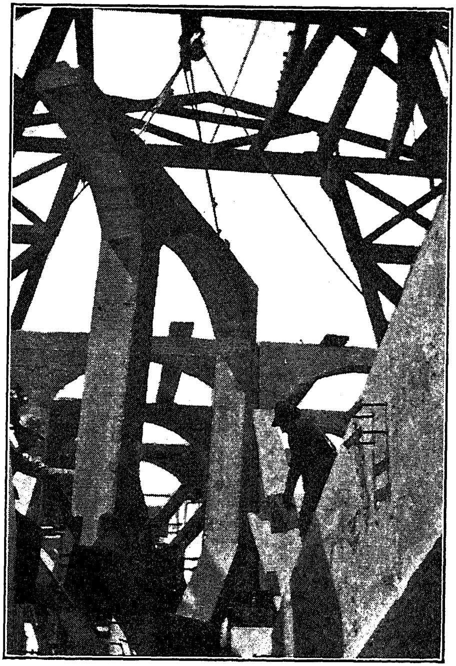
FIG. 168.—Thomas Method of arch construction.

The arch shown in Figs. 173 and 174 is founded on Ohio River mud, Raymond concrete piles being used for the foundations. The reason for the use of the cast hinges in this case is thus apparent, as settlement of foundations was anticipated. No appreciable settlement, however, has ever taken place.



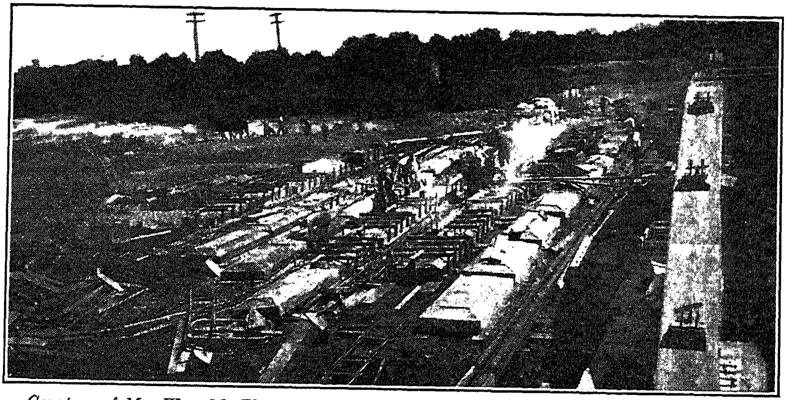
Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.

FIG. 169.—Thomas Method of arch construction.



Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.

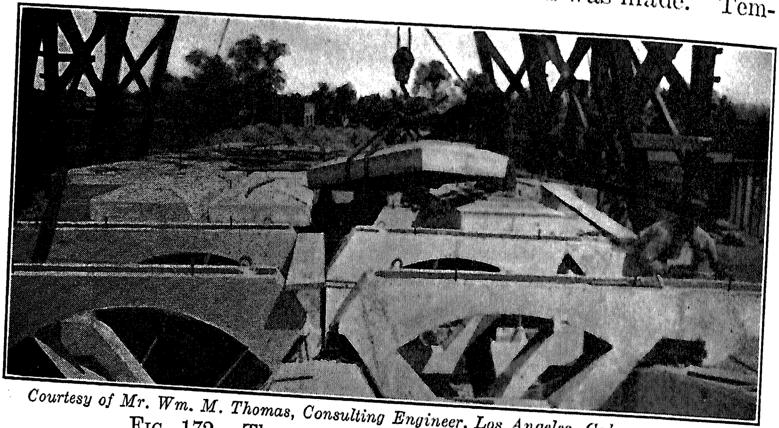
FIG. 170.—Thomas Method of arch construction.



Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.

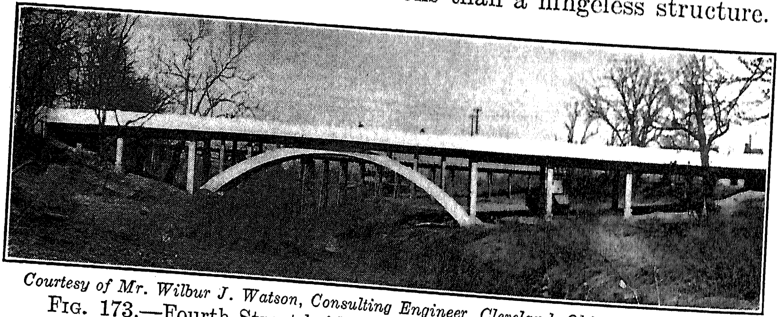
FIG. 171.—Thomas Method of arch construction.

The two halves of each rib of the bridge shown in Fig. 175 were designed to be erected simultaneously, without falsework, by derricks on opposite sides of the stream, and to be self-supporting as soon as the crown hinge connection was made. Temporary sway bracing was provided to insure lateral stability while the forms were being built and filled with concrete.



Courtesy of Mr. Wm. M. Thomas, Consulting Engineer, Los Angeles, Cal.
FIG. 172.—Thomas Method of arch construction.

The three-hinged arch construction with cantilever ends, shown in Fig. 176, is unusual, but was found to be more economical for an arch of this type and dimensions than a hingeless structure. The cantilever ends were rendered necessary on account of the fact that there were no stable foundations for abutments at the top of the fill at the ends of the bridge. The cantilever ends decreased the dead load thrust on the center hinges about one-quarter and decreased considerably the angle which the resultant thrust on the lower hinges made with the vertical, thus decreasing



Courtesy of Mr. Wilbur J. Watson, Consulting Engineer, Cleveland, Ohio.
FIG. 173.—Fourth Street bridge over Island Creek, Paducah, Ky.

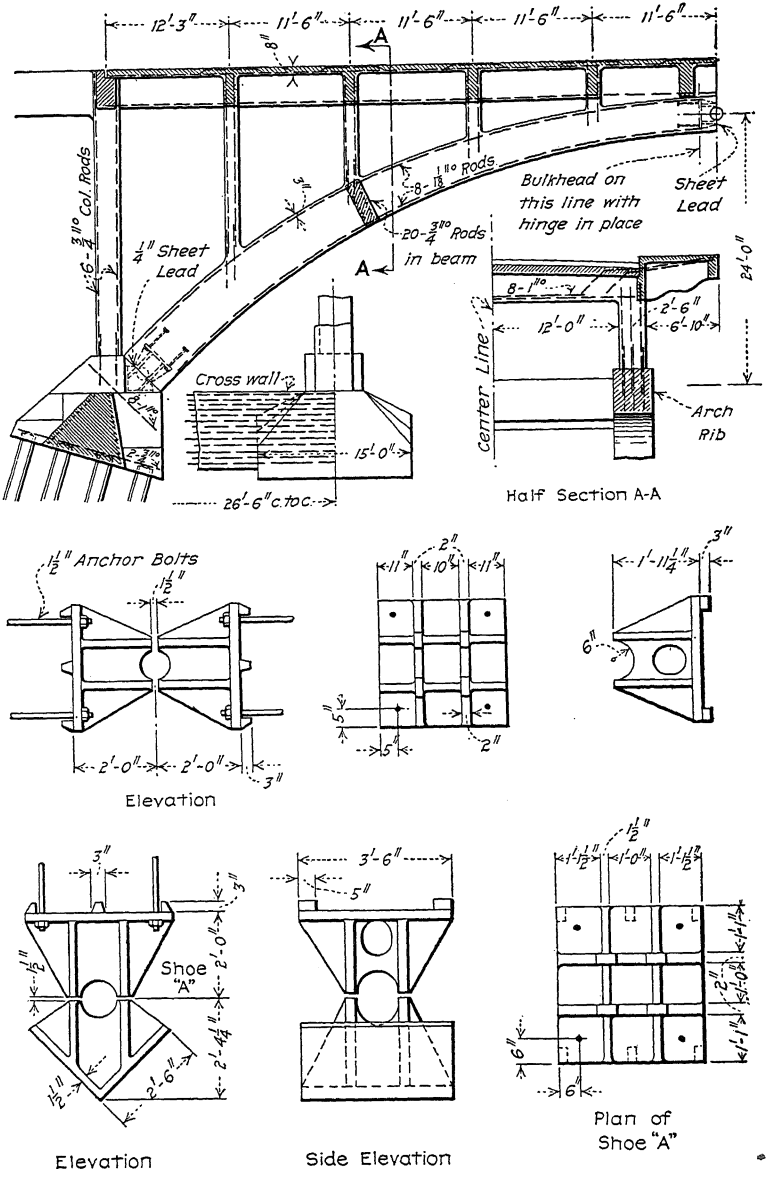


Fig. 174.—Details of Fourth Street bridge, Paducah, Ky.

the size of abutment required. The cost of the hinges necessary for the three-hinged arch was very considerably less than the cost of the additional steel reinforcing in the arch ring required

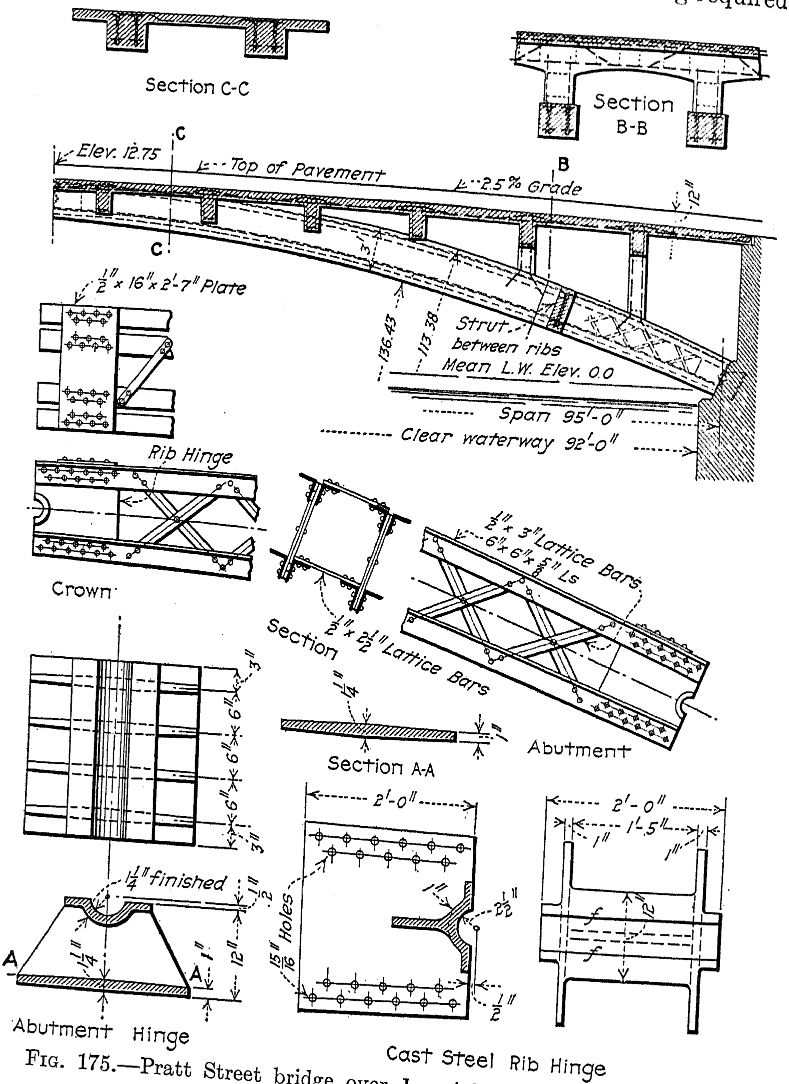


FIG. 175.—Pratt Street bridge over Jones' Falls, Baltimore, Md.

to take care of the additional bending moments in the hingeless arch. No connection whatever was needed between the ends of the cantilevers and the abutments on account of the extremely small amount of vertical motion at these points. A common type of centering was used in constructing the arch ribs in place. The cast-steel hinges were entirely encased in concrete after the centers were struck, a $\frac{1}{4}$-in. plate of sheet lead having been placed at the center of each hinge to allow the necessary motion of the arch rib under live load and temperature stresses. In this way all possibility of corrosion of the steel hinges was avoided.

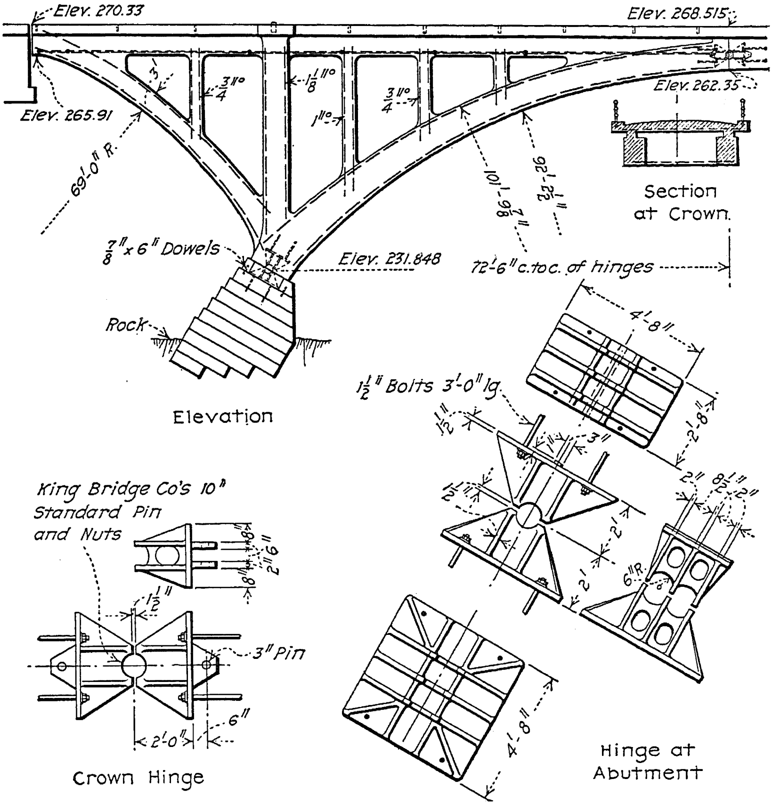


FIG. 176.—Bridge over the Vermillion River at Wakeman, Ohio

What is believed will be the longest three-hinged concrete arch ever built is shown in Fig. 177. When completed it will be one of the show bridges of the United States.

Fig. 178 gives details of the Thomas Method of arch construction already referred to in Arts. 58 and 59.

61. Design of a Three-hinged Arch.—By S. W. Bowen, M. Am. Soc. C. E., Designer of the Main Street Viaduct, Fort Worth, Texas.—To illustrate the methods employed in the design of the three-hinged arches of the Main Street viaduct, Fort Worth, Texas, the following account is given of the design of the 225-ft. arch span shown in Fig. 179. The reinforcement consisted of structural shapes which were designed to support

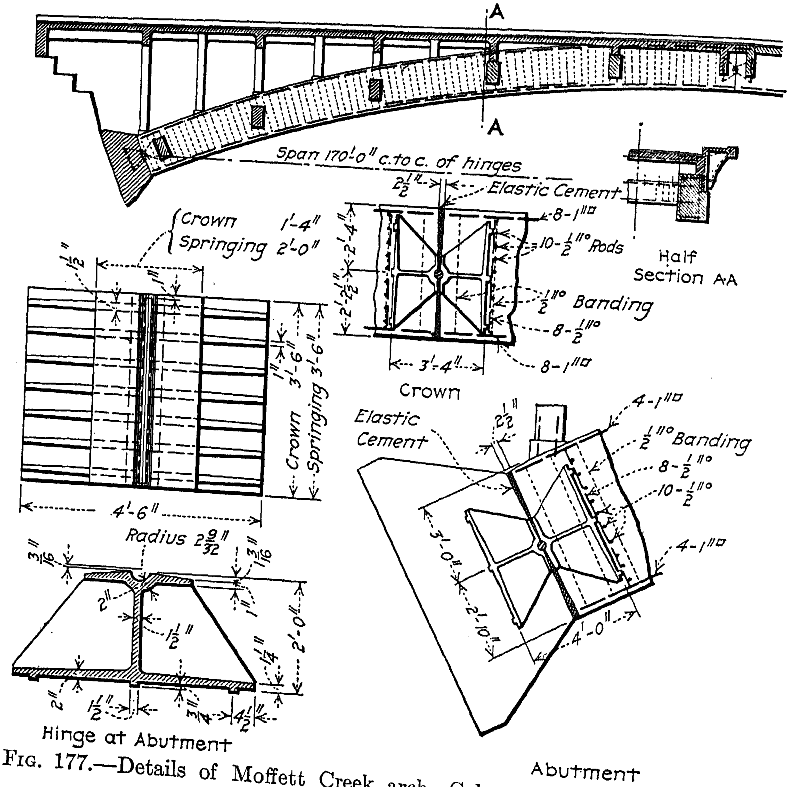
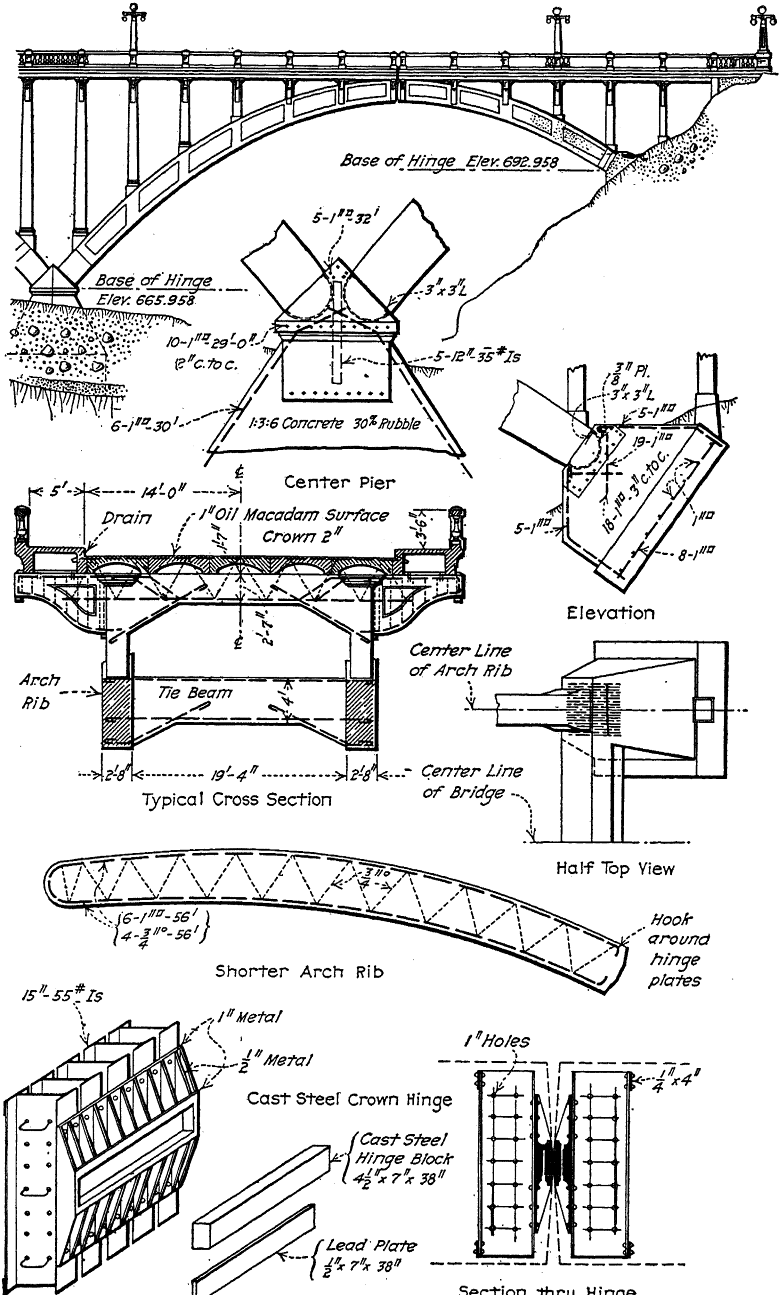


FIG. 177.—Details of Moffett Creek arch, Columbia Highway, Oregon.

the weight of the forms and plastic concrete in the arch ribs. Details of this arch are shown on Plates XL and XLI, Chapter XIII.

The entire structure, above the arch ribs, was first designed so that the dead loads coming on the ribs at each panel point could be accurately computed. The design of this portion of the work is comparatively simple, and requires no description.



Center Pier

Elevation

Typical Cross Section

Half Top View

Shorter Arch Rib

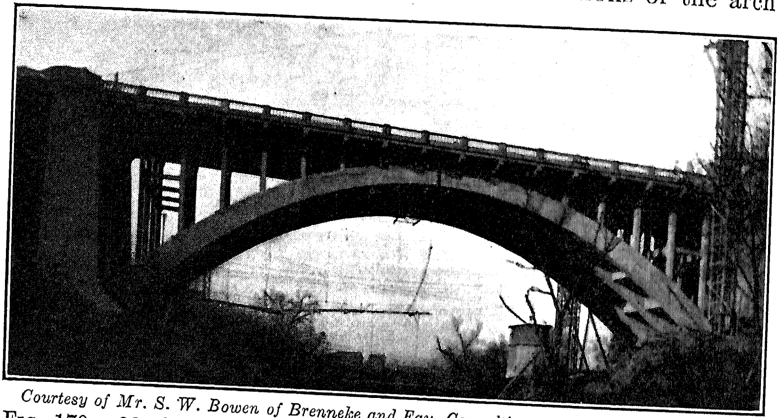
Cast Steel Crown Hinge

Section thru Hinge

Assumptions as to the dimensions of the arch ribs and braces were then made to determine the dead load from these portions of the structure.

The next step was to draw the lines of thrust in the arch ribs, for full live and dead load and for dead load combined with live load on each half of the span. This gave the position of the neutral axis, or center line of the arch rib, which was taken midway between the extreme lines of thrust, and coincided closely with the line of thrust for full live and dead load. It might be mentioned that the center line of the arch rib, thus determined, agreed closely with a parabola, with its vertex at the crown of the arch.

The general outline and approximate dimensions of the arch ribs being determined, the next step was to design the structural steel reinforcement of the ribs. This reinforcement consisted of curved lattice girders, three in each inner, and two in each outer rib. These girders were laced together in the plane of the top and bottom flanges. They were also connected under each post by vertical cross frames. The weight of the concrete in the arch ribs and braces was computed, and also the weight of the forms. An allowance was also made for the weight of the structural reinforcement. Concrete was assumed to be placed simultaneously in each half of the span, from the haunches to the crown. The stresses in the structural reinforcement were then found for each successive stage of the concreting—that is, for the forms of the ribs and braces filled with concrete up to the first, second,

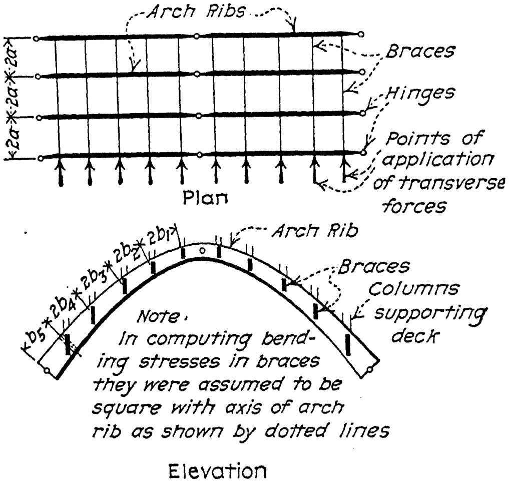


Courtesy of Mr. S. W. Bowen of Brenneke and Fay, Consulting Engineers, St. Louis.
FIG. 179.—225-ft. arch span of the Main Street viaduct, Fort Worth, Texas.

etc., panel points, counting from the haunches. All of these stresses were found graphically, and the members proportioned for the maximum conditions. The unit stresses used were 12,500 lb. per square inch in compression, and 16,000 lb. per square inch in tension. No stress whatever was assumed to be taken by the concrete, as it was unlikely that the concrete near the haunches would be sufficiently set to take stress, by the time the ribs were finished.

In the design of the arch ribs, which was next undertaken, the stresses caused by the construction of the columns and deck were considered as well as those due to live and dead loads on the finished structure. The columns were assumed to be built out from each pier, panel by panel, until the crown was reached; then the deck was assumed to be concreted; panel by panel, working from the crown toward the haunches. The stresses in the rib were found for each stage of the construction. For live-load stresses, five conditions were considered, as follows: First, live load on entire span; then each half of span loaded; next the two end quarters loaded; and finally the live load on the two middle quarters of the span. A maximum unit stress of 500 lb. per square inch was used in the extreme fiber of the rib, and sufficient reinforcement was provided to keep the stress in the concrete down to this figure. In case the structural reinforcement at any point was not sufficient for this purpose, additional metal in the shape of bars was provided. The stresses and reinforcement required were computed at each panel point, and these bars were stopped off when not required. In case the computed amount of steel at any point exceeded 5 per cent of the gross area of the rib at that point, a redesign was made. It will be seen from the above that the structural steel reinforcement takes stress in two ways: (1) that due to the weight of the plastic concrete in the ribs and braces, together with their forms; and (2) that due to the construction of the columns and deck, and to the loads on the finished structure. The first of these may be called an initial stress, which remains in the steel as the concrete sets. The second must be added to the first to get the total compressive stress in the steel. In this case the maximum possible compressive unit stress in the structural steel would be 12,500 lb., plus 15 × 500 lb., or a total of 20,000 lb. per square inch. As a matter of fact, the total unit stress was about 16,000 lb. per square inch.

Transverse forces had to be given considerable attention, because of the fact that the high-water line comes part way up on the arch ribs. The pressure due to the water was taken at 200 lb. per square foot of each arch rib and column, and that due to wind at 30 lb. per square foot of vertical projection of the structure between high-water line and a line 10 ft. above the crown of the roadway. The braces were placed in vertical planes at the foot of each column, and were made as deep as the arch ribs allowed. A rigid analysis of the stresses in these braces would be difficult and tedious, as well as unnecessarily refined, considering the assumptions which had to be made as to the transverse forces.



Note. In computing bending stresses in braces they were assumed to be square with axis of arch rib as shown by dotted lines

Fig. 180.

A method of computation was adopted which was comparatively simple, and which it is believed gives results reasonably close to the true ones. In order to be on the safe side, the braces and their reinforcement were made heavier than the calculations required; as this added but little to the cost of the structure.

Fig. 180 shows the general arrangement of ribs and braces and the points at which the transverse forces were assumed to be applied.

The transverse forces on the portion of the structure above the arch ribs was assumed to be transferred down the columns to the ribs and braces directly under them. None of these forces was assumed to be carried to the piers through the deck, because of the expansion joints in the deck at each pier and near the center of the span. There is no doubt, however, that a very considerable portion of these forces pass directly through the deck to the piers, which correspondingly relieves the bracing system.

While the arch ribs were assumed to be hinged at the haunches and crown, rods were provided at these points which tended to fix them transversely. In other words, the arch ribs are three-hinged in vertical planes but are hingeless, or practically so, transversely.

Fig. 181 shows half of the bracing system developed. The similarity to the columns and girders of an office building is apparent.

Fig. 182 shows a portion of Fig. 181 enlarged with the various forces acting on it, and the points of inflection indicated.

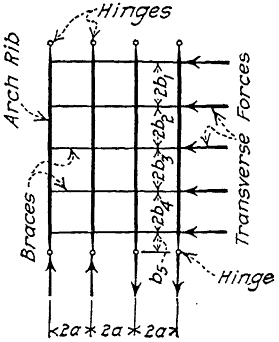


FIG. 181.

From the three conditions of equilibrium—namely: (1) the sum of the horizontal forces must equal zero, (2) the sum of the vertical forces must equal zero, and (3) the sum of the moments about any point must equal zero—and knowing the external

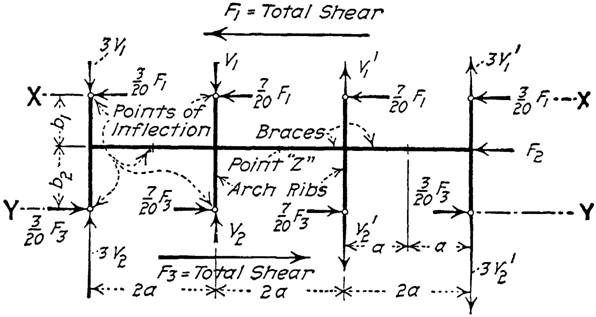


FIG. 182.

forces F_1 and F_2 and all dimensions, the thrusts, shears, and bending moments in the various members are readily calculated.

For example, the horizontal force F_1 is the total of the transverse forces acting on the structure above the plane $X - X$ where the forces F_1 are shown. F_2 is the total transverse force acting on one panel of the structure, and F_3 is the total shear at the plane $Y - Y$. Then from the first condition of equilibrium,

$$F_3 = F_1 + F_2 \tag{1}$$

The vertical forces $3V_1$, V_1, V'_1, and $3V'_1$ are those produced by the action of the horizontal forces acting above the plane $X - X$. Similarly, the forces $3V_2$, V_2, V'_2, and $3V'_2$ are those produced by the action of all horizontal forces above the plane $Y - Y$. The term "vertical" has been applied to the forces indicated in the figure as acting at right angles to the forces F_1, etc. These forces are, of course, not vertical in the structure, but act in the direction of the axis of the arch rib.

For the sake of simplicity, let $V_2 - V_1 = V_3$. Then from the second condition of equilibrium,

$$3V_3 + V_3 = V'_3 + 3V'_3$$

or

$$V_3 = V'_3 \tag{2}$$

From the third condition of equilibrium, taking moments about point Z,

$$F_1b_1 + F_3b_2 = 2 \times 3V_3 \times 3a + 2 \times V_3 \times a$$

or

$$V_3 = \frac{F_1b_1 + F_3b_2}{20a} \tag{3}$$

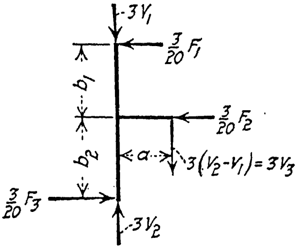


FIG. 183.

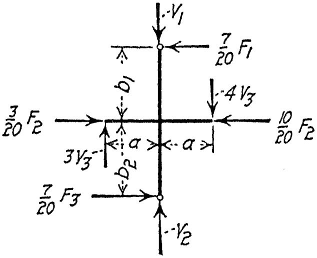


FIG. 184.

Fig. 183 shows a portion of Fig. 182. As before

$$\frac{3}{20}F_3 = \frac{3}{20}F_1 + \frac{3}{20}F_2$$

or

$$F_3 = F_1 + F_2 \tag{1'}$$

$$3V_2 - 3V_1 = 3V_3$$

or

$$V_2 - V_1 = V_3 \tag{2'}$$

$$\frac{3}{20}F_1b_1 + \frac{3}{20}F_3b_2 = 3V_3a$$

or

$$V_3 = \frac{F_1b_1 + F_3b_2}{20a} \tag{3'}$$

Fig. 184 shows the remaining portion of half of Fig. 182. The same equations apply here.

$$\frac{7}{20}F_1 + \frac{7}{20}F_2 = \frac{7}{20}F_3$$

or

$$F_3 = F_1 + F_2 \tag{1''}$$

$$3V_3 + (V_2 - V_1) = 4V_3$$

or

$$V_2 - V_1 = V_3 \tag{2''}$$

$$\frac{7}{20}F_1b_1 + \frac{7}{20}F_3b_2 = 3V_3a + 4V_3a$$

or

$$V_3 = \frac{F_1b_1 + F_3b_2}{20a} \tag{3''}$$

The maximum bending moment in the upper portion of the outer rib is $\frac{3}{20}F_1b_1$, in the lower portion $\frac{3}{20}F_3b_2$, in the upper portion of the inner rib $\frac{7}{20}F_1b_1$, and on the lower portion $\frac{7}{20}F_3b_2$. The maximum bending moment in the outer brace is $3V_3a$, and in the inner brace $4V_3a$.

The hinge castings were designed in much the same manner as a cast column base. The area in contact with the concrete was determined by dividing the total load by 800 lb. per square inch. The diameter of the ball-and-socket joint was determined by dividing the total load by 20,000 lb. per square inch, which gave the projected area of the ball.

The shears and moments on the ribs were then computed, and their thickness and that of the base determined.

In the design of the piers, several stages in the construction of the spans were considered, as well as the conditions existing in the finished structure.

For example, with the pier finished up to the roadway, the arch ribs and braces of one span (the longer) were considered as finished, but nothing was done on the adjoining span. Next the full dead load, exclusive of paving but including forms, was considered on the long span, and only the ribs and braces of the adjoining span finished. Then the finished structure was considered, first with live load on the long span only, and second with live load on both spans. In each case the resultant force was kept within the middle third of the base. The uplift of the water was considered in each case, the stage being taken as the top of the levee.

No impact was considered in the design of the piers.

Pier No. 2 was supported on timber piles driven into a mixture of sand and gravel overlying bed rock. This method was used instead of founding the pier on bed rock in order to reduce the cost, the amount of the appropriation being limited. The piles were not driven to rock in order to take advantage of the bearing of the pier base on the stratum of gravel, and also to take advantage of the friction to help resist sliding. The pier was protected against scour by sheet piling and rip-rap.

Piles were assumed to take 20,000 lb. each and the remainder of the load was assumed to be carried by bearing on the soil at about 5000 lb. per square foot.

CHAPTER XII

PATENTS

The number of patents that have been granted by the United States Government for improvements in reinforced-concrete construction has increased from a yearly average of three or four between the years 1890 and 1900 to about 150 per year at the present time—that is, not considering patents on concrete mixers, fence posts, and other minor structures. Undoubtedly some of these patents are worthless, impractical, and possibly invalid, but others unquestionably are of considerable importance. The mere granting of a patent does not establish its validity under the United States patent law, and many of these patents may be invalidated by litigation. In spite of this fact, however, the engineer has no right to consider any patent invalid unless he has positive proof to that effect.

62. Patents in General.—The Constitution of the United States contains the following clause:

> "The Congress shall have power to promote the progress of science and useful arts, by securing for a limited time for authors and inventors the exclusive right to their respective writings and discoveries."

Under this provision of the Constitution, Federal statutes have been passed authorizing the issue of a patent to the original inventor of any new and useful improvement and granting to him the exclusive right to make, to sell, or to use the patented improvement for a period of 17 years after date of issue of the patent. The patent is, in effect, an agreement between the United States Government and the inventor of a new and useful improvement, whereby the Government grants to the inventor the right to exclude others from making, using, or selling his invention for 17 years from the date of issue of the patent, in return for complete publicity of the invention. Under this agreement the inventor's reward is valuable or worthless as the invention proves valuable or worthless to others. If the invention is one desired by the general public, the exclusion of others may enable the patentee to reap a rich reward; or if the invention is worthless, the reward will be equally valueless.

By means of patents, publicity for new ideas is encouraged; for the owner of a valid patent is secure for 17 years, after that the subject of the patent becomes public property. It is for this reason that the patent consists of drawings and specifications to make a complete disclosure of the invention, so that anyone skilled in that particular industry may reproduce it. In addition to the drawings and specifications there are what are called claims. These claims define the actual limits to the patentee's rights to exclude others. Without adequate protection, there would be a tendency to suppress public information regarding new inventions, the inventor relying upon secrecy instead to gain his reward. The patent actually withdraws nothing from the public, but merely withholds for a limited time what otherwise might have been kept secret and perhaps lost forever. It must add to the sum of human knowledge, for in so far as it does not do that, it is to that extent invalid. All that is necessary to establish invalidity is to prove that nothing has been added to the sum of human knowledge by the patent. If it is plainly apparent to an engineer that a patent is invalid, the engineer should be able to so convince the court.

Patents pertaining to reinforced concrete may be divided into two classes: (1) those that apply to improvements in manufactured details, such as reinforcements, and which have been described in Art. 18 of Volume I and in Chapter V of Volume II; and (2) those which may more properly be called engineering patents and which relate to improvements in the structure itself as, for example, the *location* of reinforcement, or the processes of erection. Patents on manufactured details develop a field for the sale of special types of reinforcement, the royalty being included in an enhanced selling price. In other words, this class of patents is used to promote the business of the manufacturer rather than that of the engineer. The second class above mentioned is used more often by the engineer as a support for specialization in engineering, and the royalty is ordinarily

CHAPTER XII

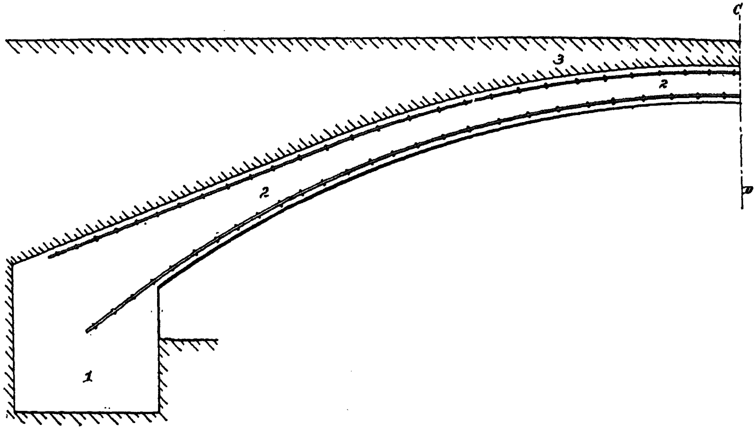
PATENTS

patents that have been granted by the United ent for improvements in reinforced-concrete increased from a yearly average of three or four rs 1890 and 1900 to about 150 per year at the at is, not considering patents on concrete mixers, d other minor structures. Undoubtedly some are worthless, impractical, and possibly invalid, questionably are of considerable importance. ting of a patent does not establish its validity ed States patent law, and many of these patents ated by litigation. In spite of this fact, however, s no right to consider any patent invalid unless proof to that effect.

in General.—The Constitution of the United the following clause:

s shall have power to promote the progress of science y securing for a limited time for authors and inventors ht to their respective writings and discoveries."

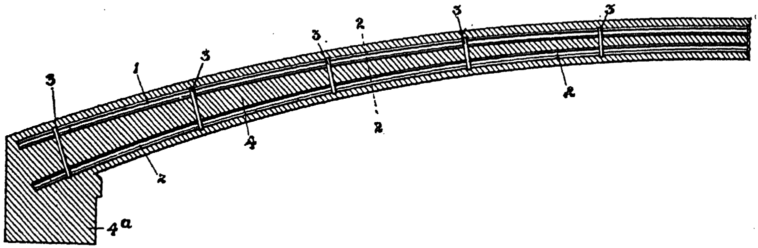
rovision of the Constitution, Federal statutes have uthorizing the issue of a patent to the original y new and useful improvement and granting to ive right to make, to sell, or to use the patented for a period of 17 years after date of issue of he patent is, in effect, an agreement between the overnment and the inventor of a new and useful whereby the Government grants to the inventor xclude others from making, using, or selling his the patent drawings and include more than is covered by the claims cited. The reason for this is due to the fact that a claim is for an improvement while the patent drawings must show how to construct a complete working structure.)



CLAIM 3.—The combination with abutments, and a concrete arch spanning the space between the abutments, of a series of metal bars in pairs, one bar of each pair above the other bar, near the extrados and intrados of the arch, each bar of a pair being independent of the other, and one bar of each pair extending well into the abutment, substantially as described.

FIG. 185.—Thacher patent.

A somewhat similar reinforcement is shown in the patent to Arnold, No. 749,771, issued, Jan. 19, 1904, on an application filed Sept. 13, 1902. The bars are arranged in pairs, but bonded together by metallic straps as indicated in Fig. 186, thus differing from the Thacher patent in which the bars are claimed as independent of one another.

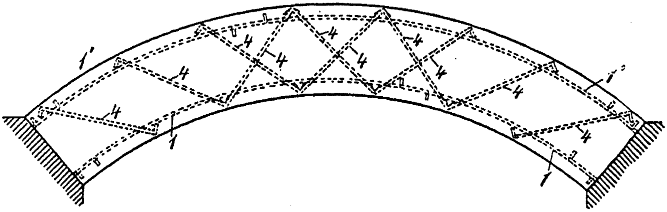


CLAIM 2.—In a bridge, an arch of concrete or other masonry, having embedded therein ribs, arranged one above but separated from each other and by the masonry and composed of railroad rails or other iron, said ribs being bonded together by metallic straps wound around them and at intervals of the length thereof, substantially as described.

FIG. 186.—Arnold patent.

Another modification of the Thacher patent is shown in Melber, No. 660,518, issued Oct. 23, 1900, on an application filed August 24, 1899, in which shear members cross diagonally from one reinforcing member to another (Fig. 187).

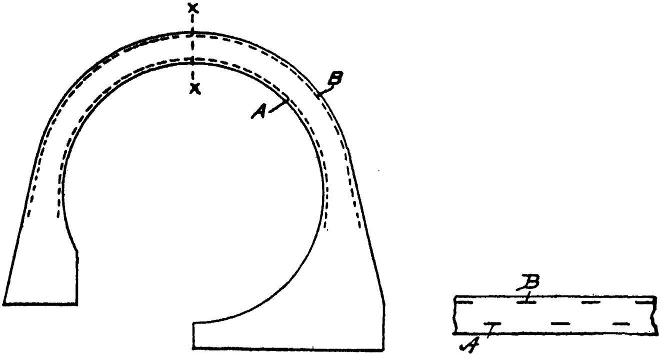
The Parmley patent, No. 696,838, issued April 1, 1902, on an application filed June 4, 1901, differs from the Thacher patent in having the bars near intrados and extrados alternating with one another instead of being arranged in pairs (Fig. 188).



CLAIM 1.—In cement or other concrete construction metal reinforcing bars, unattached at their ends to other metal reinforcing bars, embedded therein transverse to the calculated shearing strains.

FIG. 187.—Melber patent.

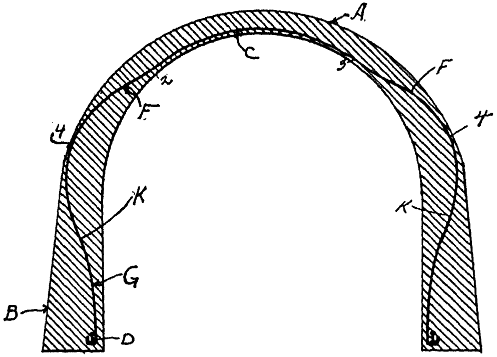
A materially different form of arch reinforcement is shown in the patent to Luten, No. 1,009,676, Nov. 21, 1911; application filed April 29, 1901. This patent shows the arch reinforced with but a single series of reinforcing members (Fig. 189) instead of the double row of the Thacher, Arnold, Melber, and Parmley patents. A modification of this device is shown in another patent to Luten, No. 818,386, April 17, 1906; application filed May 17,



CLAIM 3.—A concrete arch having bars embedded therein, both in intrados and extrados at the regions of tension and extending continuously through the regions of tension and sufficiently into the abutments to obtain a secure anchorage, the bars in the extrados being opposite the spaces between those in the intrados.

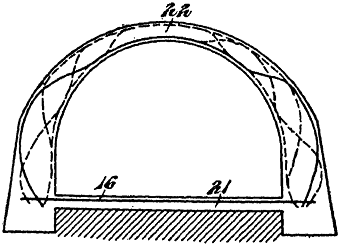
FIG. 188.—Parmley patent.

1902. In this patent the points of crossing the arch ring for various rods of the single series have been displaced laterally along the arch ring, so that the arch is reinforced on both sides through a short region where there is doubt as to whether the bending moments will be positive or negative (Fig. 190). The reason for placing the reinforcement in this way is based on the theory that tension will occur at alternately opposite regions of an arch in limited regions only, and that steel is located in those regions and extends continuously from one to the other for convenience in placing. A similar object may be accomplished with non-continuous rods as shown in patent to Luten No. 852,971, May 7, 1907; application filed June 30, 1906. (See Fig. 191.) The difficulty of placing this form of reinforcement probably offsets the saving in metal.



Claim 2.—In an arched structure of concrete the combination with a curved or arched member and abutments to support the same, of a series of tension members embedded in said curved or arched member, and passing close to the interior face of the abutments.

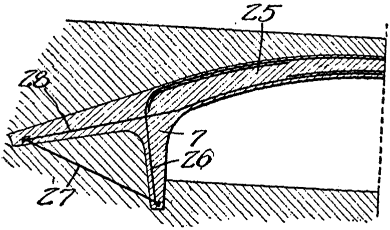
Fig. 189.—Luten patent.



Claim 7.—An arch having embedded therein rods, bars, or other tension members in two or more series following one face of the arch rib, thence across and following the other face of the rib, the points of crossing for the different series being angularly or laterally displaced with respect to each other, substantially as described.

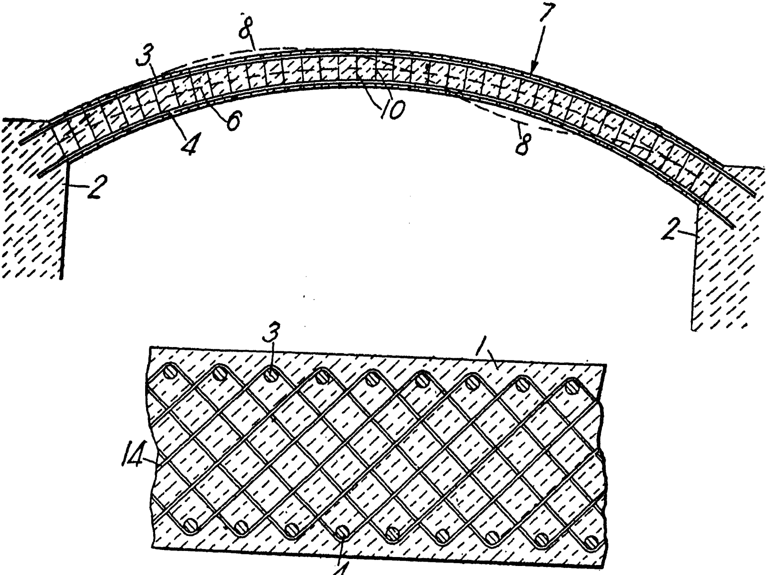
Fig. 190.—Luten patent.

The Cummings patent, No. 978,361, issued Dec. 13, 1910, on an application filed June 11, 1909, relates entirely to transverse reinforcement in the arch ring or arch rib. A typical claim is given in Fig. 192.



CLAIM 9.—A concrete arch having reinforcement near both surfaces alternately heavy and light on opposite sides of points of calculated change of moment.

FIG. 191.—Luten patent.



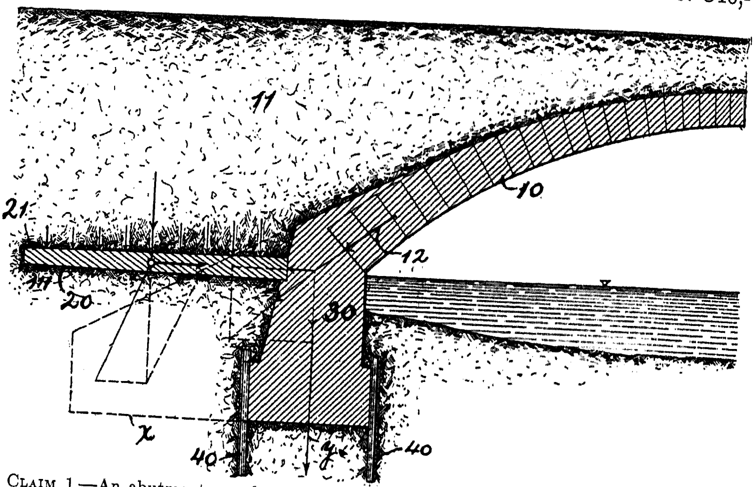
CLAIM 4.—A reinforced concrete arch or girder comprising longitudinal reinforcement bars arranged near the upper and lower surfaces thereof, a series of open metal sheet or mesh members arranged close together and all parts of each lying in substantially a single plane substantially normal to the neutral equilibrium curve of the arch or girder and extending from top to bottom and side to side thereof and comprising a substantial portion of the cross-sectional area thereof, whereby said reinforcement members resist vertical tension stresses throughout the width of the arch or girder, and cementitious material enveloping and embedding said longitudinal and transverse reinforcement members.

FIG. 192.—Cummings patent.

In any of the above patents the reinforcement may be almost any type patented or non-patented, but usually consists of round rods. However, there is no reason why any of the types of reinforcement described in Art. 18 of Vol. I and in Chapter V of Vol. II may not be applied in accordance with the claims of the above patents which, as far as the writer knows, are all the patents that apply specifically to arch ribs.

A patent that has nothing to do with reinforcement is that to Moller, No. 776,252, Nov. 29, 1904; application filed July 6, 1904. This patent pertains to an abutment for an arch, in which a horizontal plate is used to resist the horizontal thrust (Fig. 193).

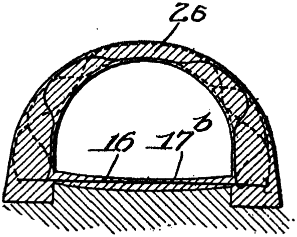
The converse of this is shown in the patent to Luten No. 840,224, issued Jan. 1, 1907; on an application filed Aug. 23, 1905. In this patent the horizontal thrust of the arch on the abutment is resisted by a metal tie from abutment to abutment embedded in a concrete pavement across the bed of the stream (Fig. 194). This pavement also serves as a support for the falsework in erecting the bridge and is intended to render the bridge flood proof by preventing scour under the foundations.



CLAIM 1.—An abutment member for arches, consisting of a plate placed in a substantially horizontal position in contact with the end of the arch adjacent to its pressure-line.

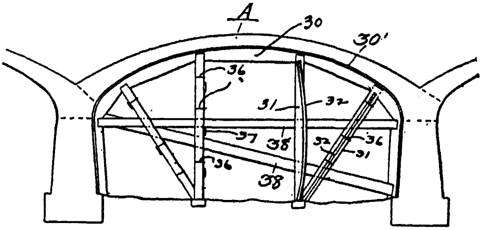
FIG. 193.—Moller patent.

The application of process patents to arches is illustrated in a typical manner in the patent to Luten No. 923,058, issued March 25, 1909, on an application filed April 1, 1907, for a method of building a bridge in which the stiffening effect of the spandrel is applied to produce what is considered a more efficient structure with elimination of possible settlement cracks. The old method was to remove the centers from beneath an arch before the spandrels, fill, and railings were added, the arch thus assuming its settlement stresses before their addition; but the erection of spandrels or fill after centers are removed is dangerous unless the arch ring is made excessively heavy. The process of the Luten patent referred to covers the building of the arch and *part* of the spandrel *before* striking centers, and the addition of the coping and railings *after* striking centers. The addition of the lower part of the spandrel walls before striking centers permits the addition of the earth fill supported by these walls and this applies to the arch its normal loading at the time the centers are struck. The part spandrel serves to stiffen the arch and render it secure, making possible the use of a light arch ring, yet with elimination of settlement cracks. One of the broader claims (claim 14) is as follows:



Claim 9.—An arch or bridge having the bed of stream paved with concrete with tension members embedded transverse to the course of the stream.

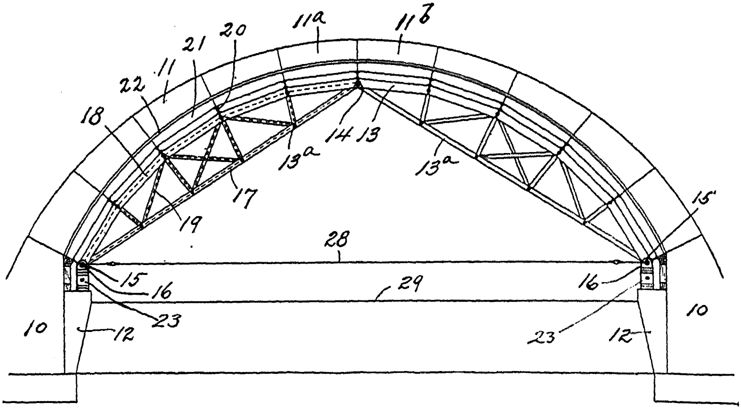
Fig. 194.—Luten patent.



Claim 4.—A falsework centering containing compound compression members for supporting a load and each composed of a plurality of pieces having major and minor dimensions transverse to their lengths, the major dimension of one piece being arranged transversely to the major dimension of another piece, and a withdrawable connection to unite the pieces of each compression member.

Fig. 195.—Luten patent.

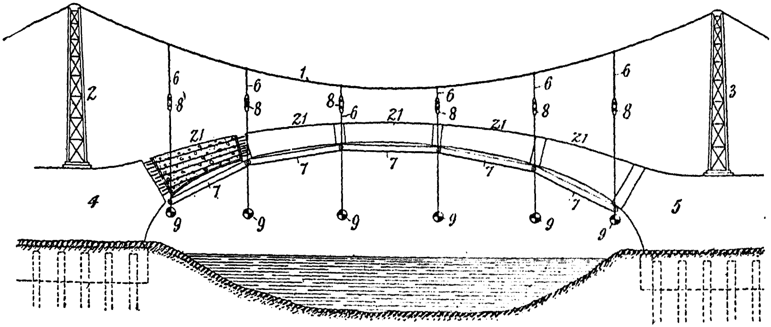
"That improvement in the art of building an arch or girder comprising erecting the arch or girder together with part of the spandrel or superstructure on centers, then lowering the centers and subsequently adding coping or railing."



CLAIM.—A centering or temporary falsework for masonry, or concrete arches comprising a three-point pinned arch composed of two sections supported solely at their lower ends and connected together at their upper ends at the center plane of the arch by a single pin through which stresses are transmitted from one section to the other, supports at the lower ends of the two sections each connected to the corresponding section by a single pin.

FIG. 196.—Watson patent.

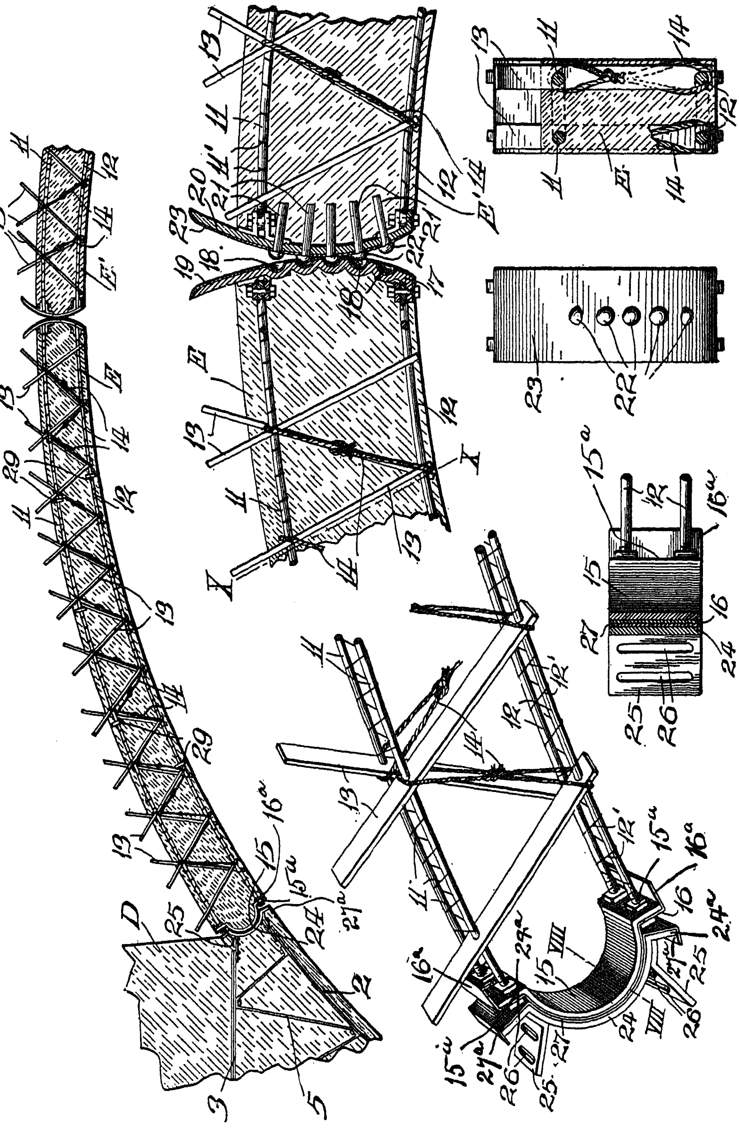
An improved form of centering is disclosed in Luten patent No. 1,106,880, Aug. 11, 1914; application filed Nov. 1, 1906. The uprights consist of compound T-columns fastened together with wires (Fig. 195) and consisting usually of 2 in. by 6-in. or 2 in. by 8-in. timbers. No wedges or sand boxes are used for striking centers, but by clipping the wires, the compound columns are resolved into their simple elements, which buckle, lowering the center, and the arch thus gradually assumes its load.



CLAIM 2.—In a system of concrete construction, the combination of supporting cables, arranged over a space to be spanned, a centering comprising a plurality of flexibly jointed sections, and hangers suspending the said sections from the cables.

FIG. 197.—Aylett Patent.

A three-pin type of centering is shown in the patent to Watson, No. 1,071,118, issued Aug. 26, 1913, on an application filed



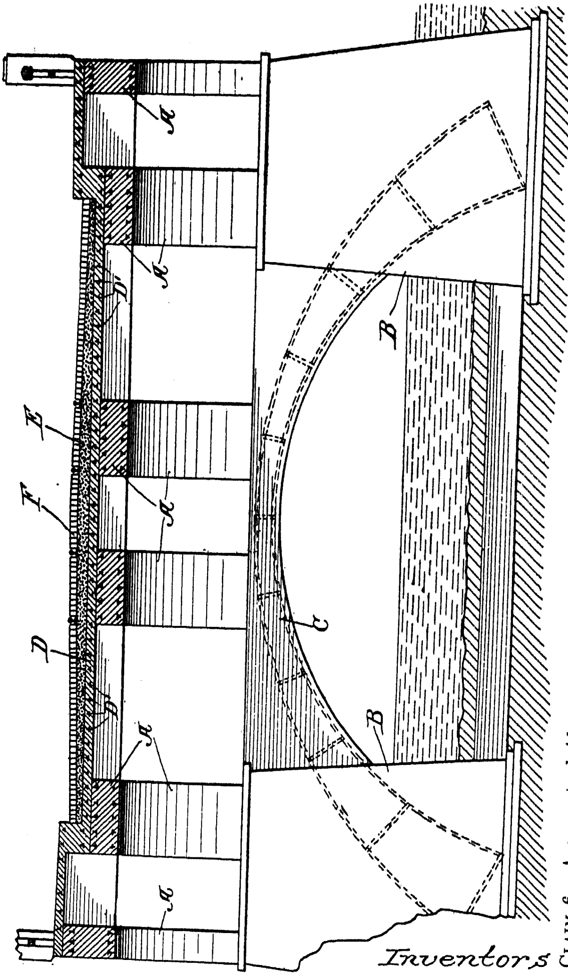
Claim 7.—In a bridge construction, a pair of mating concrete arch sections having hinge plates at their crown ends the hinge plate of one of the sections being provided with pockets and the hinge plate of the other section being provided with pins having heads adapted to enter the pockets in the mating hinge plate, substantially as set forth.

Claim 11.—A bridge construction comprising a concrete abutment formed with a cantilever arm having reinforcing framework, segmental shaped socket plates having wings connected to the framework of the cantilever arm, and an arch section having semi-circular hinge members fitted to the socket plates.

Fig. 198.—Thomas patent.

Jan. 27, 1911. One of the objects of the invention is to provide a centering which can be used repeatedly, which has a minimum deflection when the load is applied, and which is well adapted for construction of arch bridges where obstruction to traffic under the bridge must be avoided. (See Fig. 196.)

A suspended system of arch construction is described in the Aylett patent No. 965,358, July 26, 1910; application filed Oct. 26, 1907. (See Fig. 197.) The Aylett patents also cover the method of attaching the main cables directly to the voussoirs without any centering whatever—or what might be called a suspended pre-casted voussoir method.



CLAIM 6.—A concrete bridge comprising a series of piers separated longitudinally and transversely, a superstructure supported upon said piers and comprising a series of longitudinal ribs connected by a floor integral therewith and having a less thickness than the ribs.

FIG. 199.—Strauss and Collins patent.

A three-hinged arch patent is disclosed in the patent to Thomas, No. 915,316, Mar. 16, 1909; application filed Jan. 30, 1908. (See Fig. 198.)

The Strauss and Collins patent No. 762,361, issued June 14, 1904, on an application filed Jan. 25, 1904, describes a special type of arch bridge, as indicated in Fig. 199.

64. Patent Litigation.—There has been much activity in litigation of arch patents. Of the suits won, many have been concluded by consent decrees, but none have thoroughly contested the validity of the patents. Many of the pending suits are being hotly contested at the present time and will undoubtedly result in decrees that will definitely determine the validity of some of the patents. A single favorable opinion in a thoroughly contested case will go a long way to fix the standing of the patents before the public. An unfavorable opinion on the other hand will not be conclusive, because it will determine the validity of only a few claims, and the Luten patents alone contain hundreds of claims. Any one suit might invalidate a possible half-dozen of these claims but would not be conclusive as to the patents themselves or the other claims that were not involved. Hence no early conclusion of litigation under these various patents can be expected.

Since a patent is a Federal grant, litigation for violation of the patent itself must always be in the Federal court. Property rights in patents and contracts under patents may come within the jurisdiction of the State courts, but infringement of a patent is a matter for correction exclusively in the Federal court. A patent is infringed by anyone who makes, sells, or uses any device that includes all of the elements named in any one claim of the patent. If any element of the claim be absent from the device the patent is not infringed. But the presence of more elements than called for by the claim does not avoid infringement. The drawings and specifications have very little to do with the question of infringement, being principally useful to the court in determining any ambiguity that may arise as to the claim. In the Thacher patent illustrated in Fig. 185 the reinforcement is shown consisting of deformed bars, but the claims are not thus limited. Hence smooth rods or bars or their equivalent would infringe the patent if arranged as claimed.

The remedy for infringement may be a suit at law before a jury to collect damages, or it may be a suit in equity before a judge for an injunction which, if granted, carries with it an accounting for damages and profits. Nearly all infringement suits are consequently suits in equity, having the injunction against future infringement as their primary object. An injunction to prevent infringement may be granted either before or after there has been infringement, a past infringement being construed by the court as a threat to infringe again. If one claim of a valid patent is shown to be infringed, the court will grant an injunction forever restraining the future infringement of that patent. The suit may be against any party violating the patent. Thus the builder and the purchaser are both liable and neither can relieve the other. The designer may also be held liable under certain conditions, even though he has not actually made, used, or sold the infringing structure itself. The courts have held that there may be contributory infringement by one who assists others to make, sell, or use patented devices, as in the case of an engineer designing and supervising an infringing structure.

There are many possible defenses to a suit for infringement, but none of them are ordinarily of much practical value, except the proof that the patent was not granted to the original inventor in this country. The most convincing test of the validity of a patent is: Did it add anything to the sum of human knowledge? If it did, it is to just that extent valid. One or more claims of a patent may be found invalid and the remaining claims of the patent still remain valid. And the patent itself is not void until all of its claims have been proved invalid.

Suits for infringements are begun in district courts of the United States. From this court there may be an appeal to the Circuit Court of Appeals of the Circuit, and its decision is final. There are nine Federal Circuits in the United States and nine courts of appeal, and the decision of one of these courts is not binding upon any other, although it is persuasive. In case of conflicting decisions a remedy may be had in the Supreme Court of the United States, but this is possible only in case such court consents to a hearing. Consequently a patent may be valid in one district and invalid in another, and adverse decisions are necessary in nine district courts to completely invalidate it throughout the United States. Usually one or two thoroughly contested decisions are considered conclusive.

The damages that may be secured in a suit for infringement are usually measured by the customary royalty charged for the use of the patent. In case the infringement was done knowingly and wilfully, treble damages may be awarded at the pleasure of the court. But even so the cost of litigation usually far exceeds the damages secured and the primary object of a suit for infringement must be the injunction to stop further infringement. Under the old rules of Federal court procedure the cost of a patent suit not infrequently ran into tens of thousands of dollars. The total cost of the suit that established the validity of the Cameron Septic Tank patent is said to have exceeded $50,000, and the Warren Bitulithic Paving patent, $60,000. But on Feb. 1, 1913, the new rules of Federal court procedure went into effect and infringement suits may now be tried in open court with but little delay, so that lower court decisions may now be had in 1 or 2 years, where formerly 5 years was not unusual, and Mr. Edison is said to have contested one suit through 30 years of litigation.

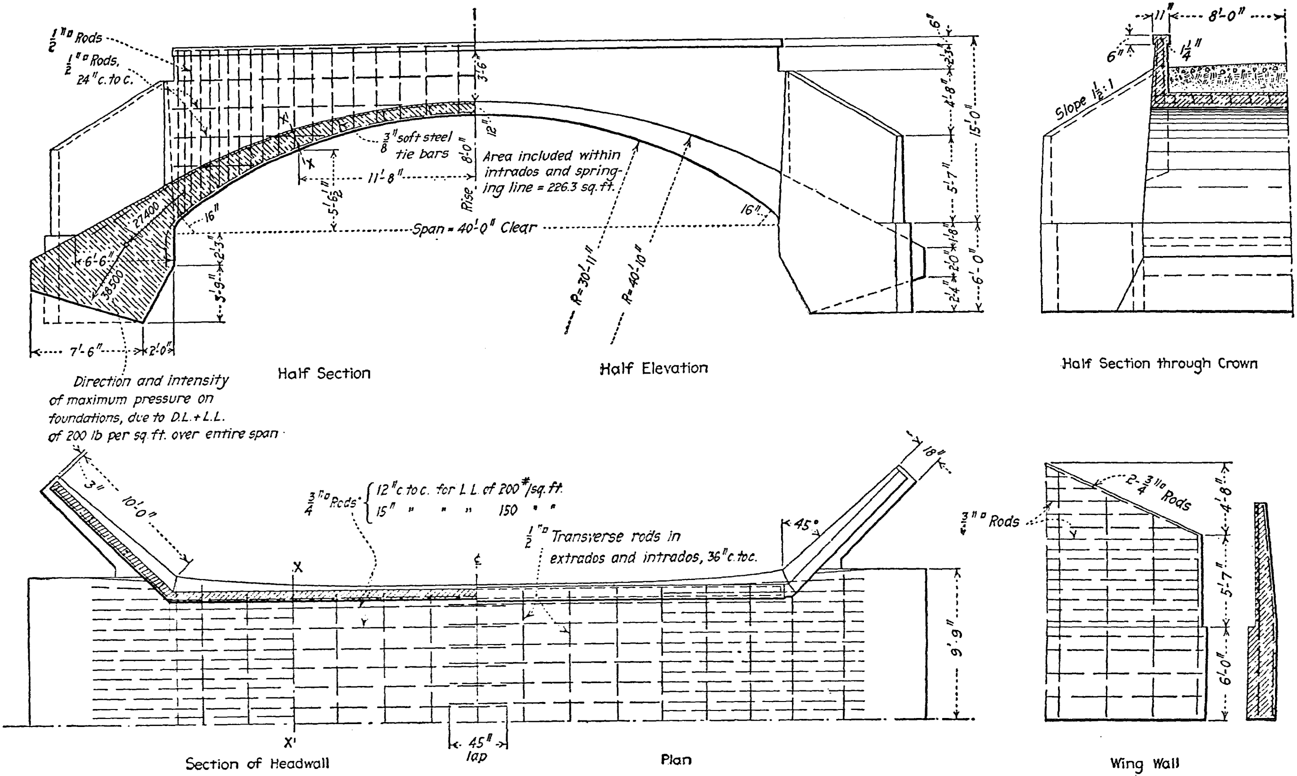
CHAPTER XIII

TYPICAL DESIGNS OF THE VARIOUS TYPES OF ARCH BRIDGES

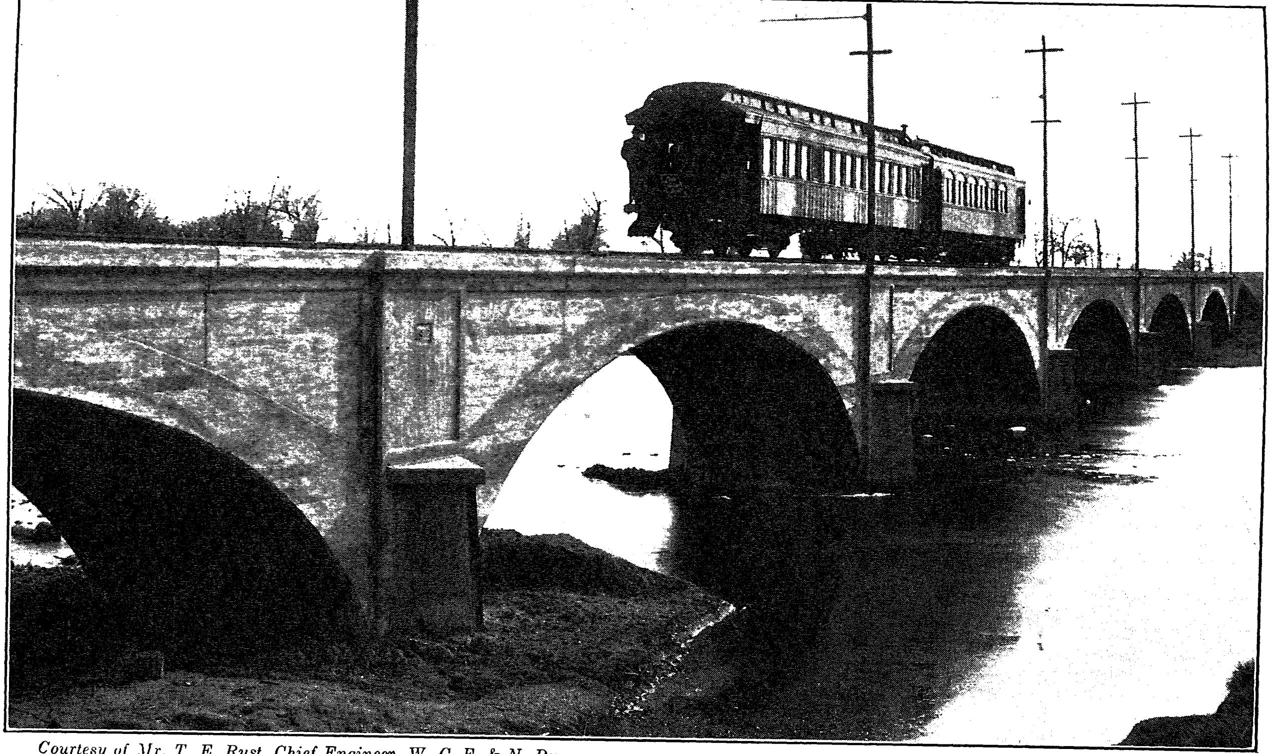
The bridges illustrated in this chapter have been selected as representative of the various types of arch bridges built in this country, and much can be gained, the writer believes, by a careful study of the details shown. In fact, it is thought that the practical designing of concrete arch bridges can be taught in no other way than by requiring a well-directed study of a representative collection of typical detailed designs, such as those shown, in addition to the usual problems of analysis. The construction views included should prove instructive.

The construction in detail of the Yardley bridge (Plates IV, V, and VI) may be found in Chapter XXXI. The Third Street bridge at Logansport, Ind. (Plate VII) is a bridge of Luten Design—a patented type of bridge referred to in Chapters X and XII. The lattice girders set across the arch ring in the Atherton Avenue bridge (Fig. 207 and Plate IX) should be especially noted as they are somewhat unusual, being employed to provide positive spacing and locking together of the arch reinforcement. The analysis of the Carondelet Park bridge (Plates XII, XIII, and XIV) and also of the Main Street viaduct (Plates XL and XLI) may be found in Arts. 47 and 61 respectively. The Latah Creek bridge (Figs. 232 and 233) was included to show one method of providing rigidity and at the same time reducing the number of ribs showing on the arch soffit—this being done by connecting each outer rib with the adjacent inner one by a thin slab at the intrados. The highway bridge at Danville, Va. (Plate XXXVIII) shows a type of bridge with the arch ribs extending up to the roadway and the reinforcement so placed that the greater part of rib may be considered as effective arch depth. The Benson Street bridge at Lockland, Ohio (Plate XXXIX), is a through arch bridge with the horizontal component of the arch thrust taken by steel rods extending between the skewbacks and fastened to the steel in the ribs.

PLATE II

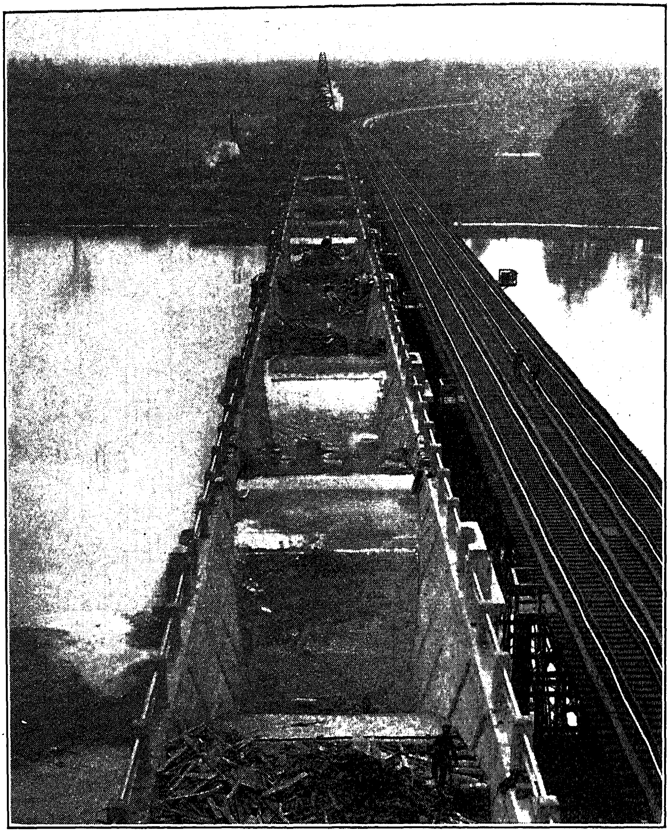


Standard 40-ft. arch, State of Missouri Highway Department.



Courtesy of Mr. T. E. Rust, Chief Engineer, W. C. F. & N. Ry.

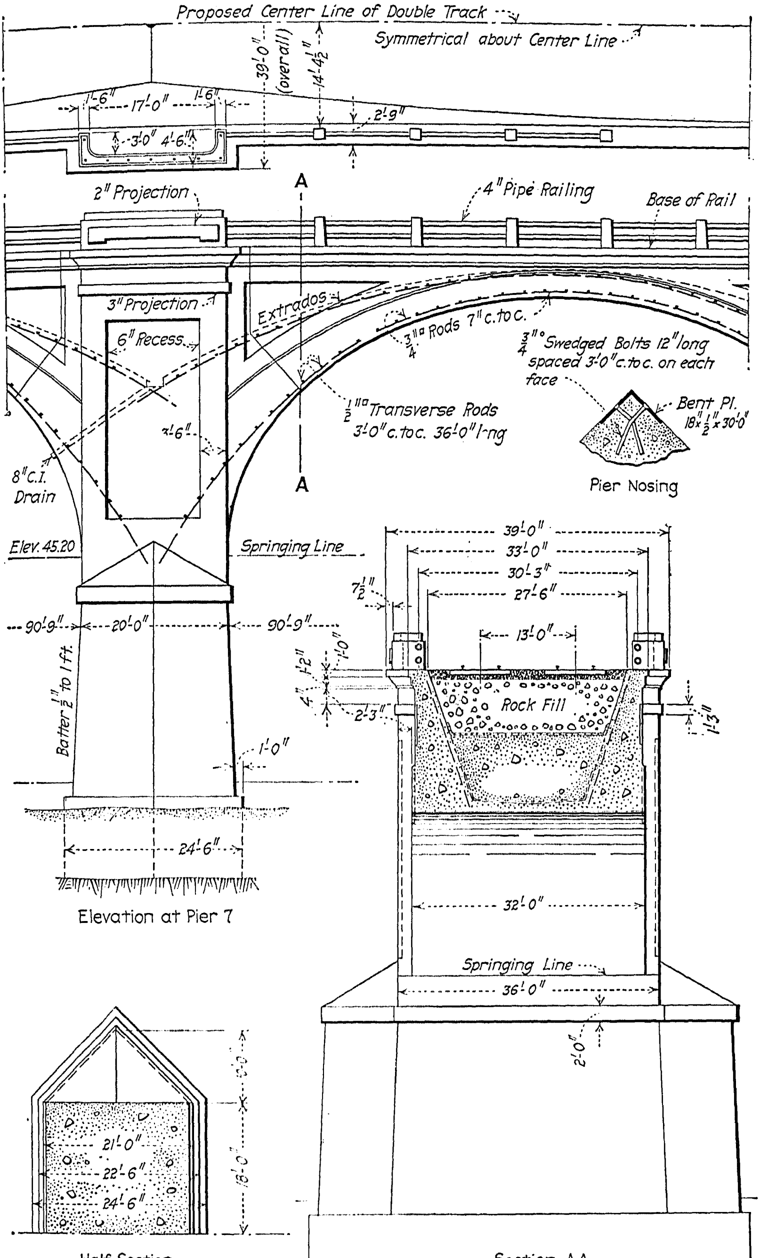
FIG. 200.—Elk Run bridge across Cedar River, Waterloo, Cedar Falls & Northern Ry. Co.



Courtesy of Mr. William Hunter, late Chief Engineer, P. & R. Ry. Co.

FIG. 202.—Yardley bridge, Philadelphia & Reading Ry. Co.

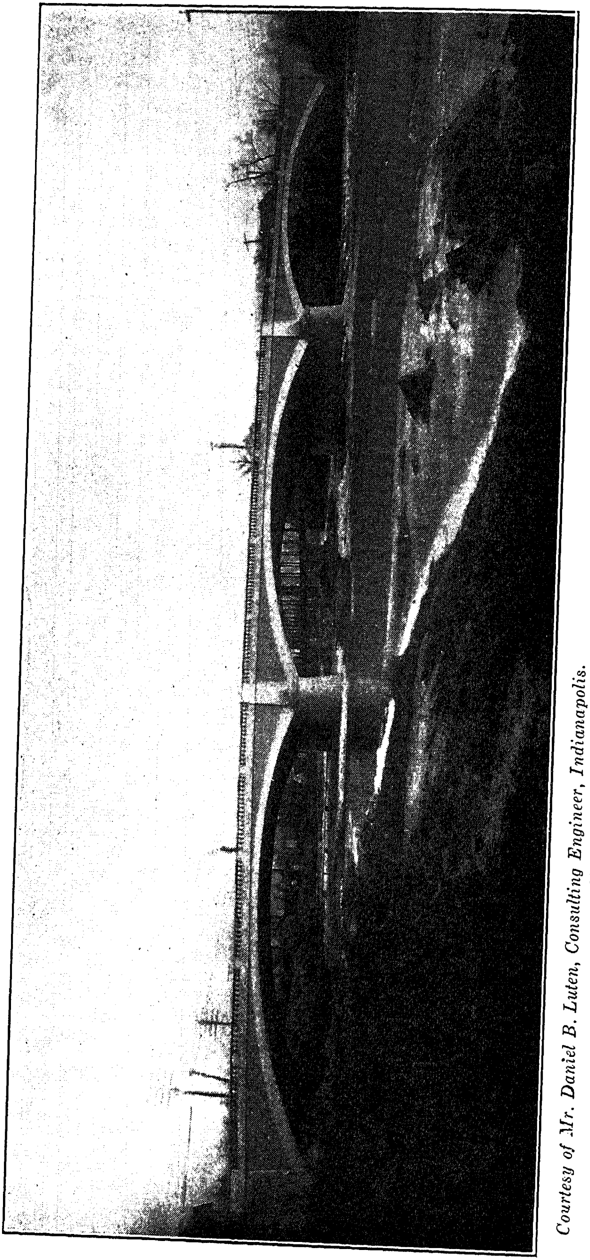
PLATE IV



Elevation at Pier 7

Half Section at Springing Line

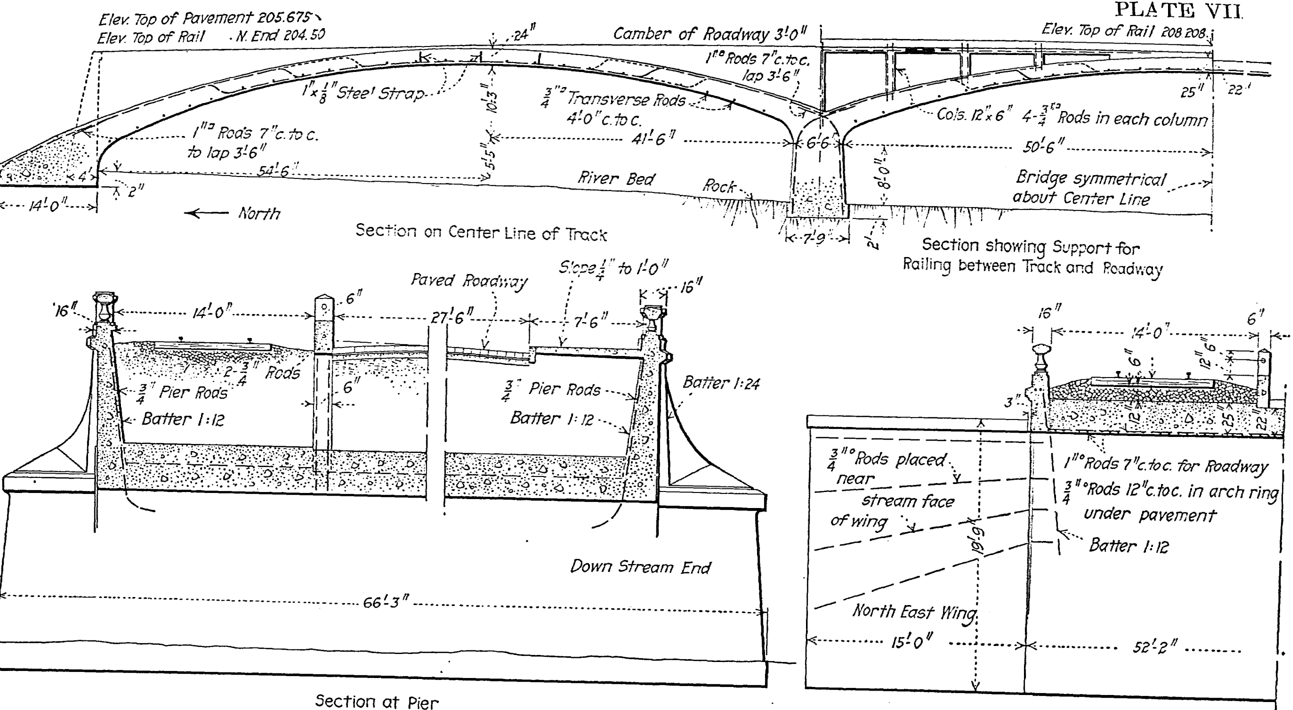
Section A-A



Courtesy of Mr. Daniel B. Luten, Consulting Engineer, Indianapolis.

FIG. 203.—Third Street bridge, Logansport, Ind.

PLATE VII



Section on Center Line of Track

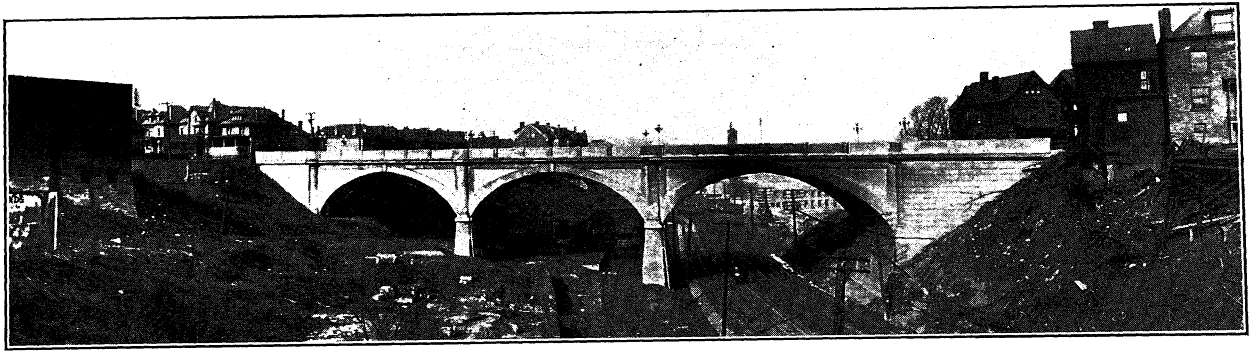
Section showing Support for Railing between Track and Roadway

Section at Pier

Section at Crown

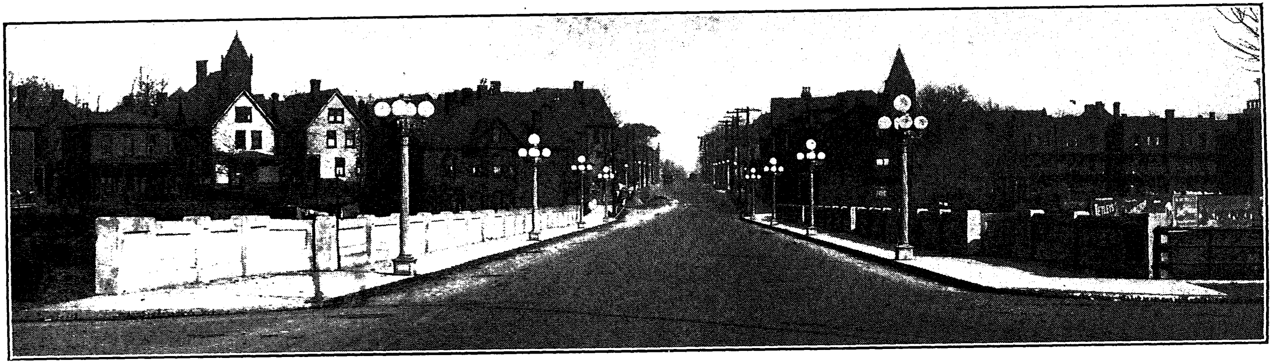
Elev. Top of Pavement 205.675
Elev. Top of Rail . N. End 204.50
24"
Camber of Roadway 3'0"
Elev. Top of Rail 208.208
1" Rods 7" c. to c. lap 3'6"
1"×1/8" Steel Strap
3/4" Transverse Rods 4'0" c. to c.
Cols. 12"×6" 4-3/4" Rods in each column
25"
22'
10'3"
1" Rods 7" c. to c. to lap 3'6"
41'6"
6'6"
50'6"
5'5"
4'
54'6"
8'0"
River Bed
Rock
Bridge symmetrical about Center Line
2"
14'0"
North
7'9"
2'
Paved Roadway
Slope 1/4" to 1'0"
16"
16"
14'0"
6"
27'6"
7'6"
2-3/4" Rods
3/4" Pier Rods
Batter 1:12
6"
3/4" Pier Rods
Batter 1:12
Batter 1:24
Down Stream End
66'3"
16"
14'0"
6"
6"
12"
12"
25
22
3"
3/4" Rods placed near stream face of wing
1" Rods 7" c. to c. for Roadway
3/4" Rods 12" c. to c. in arch ring under pavement
Batter 1:12
19'6"
North East Wing
15'0"
52'2"

Third Street bridge, Logansport, Ind.



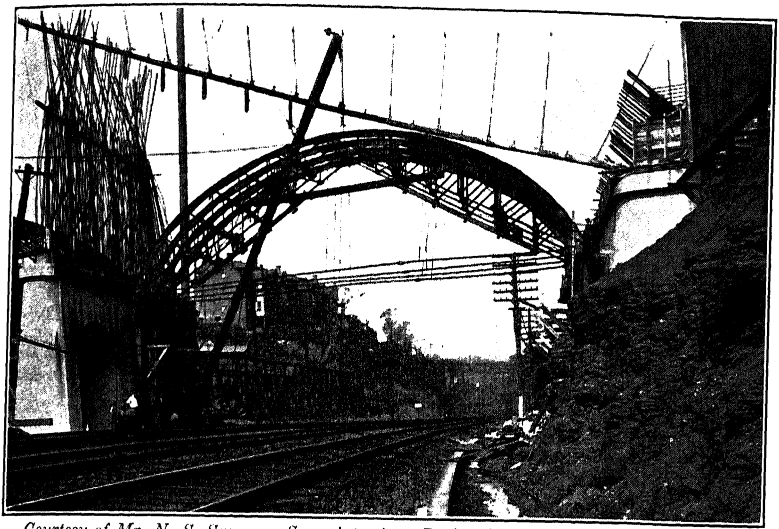
Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

FIG. 204.—Atherton Avenue bridge over P. R. R., City of Pittsburgh.



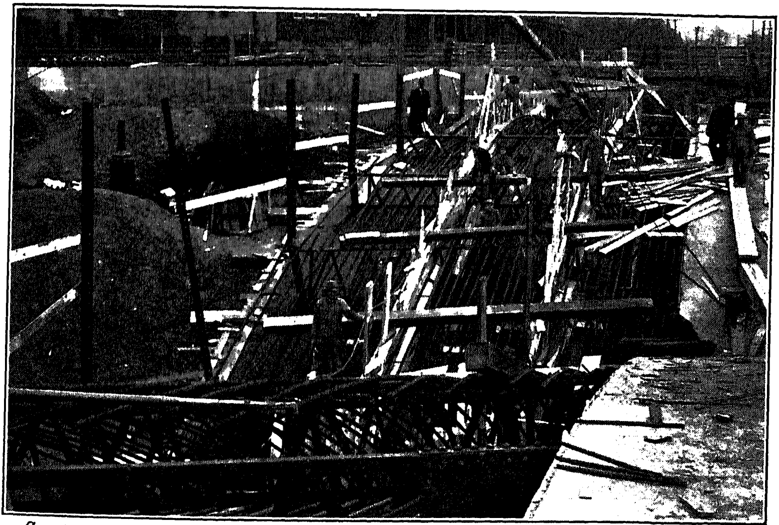
Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

FIG. 205.—Roadway of Atherton Avenue bridge over P. R. R., City of Pittsburgh.



Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

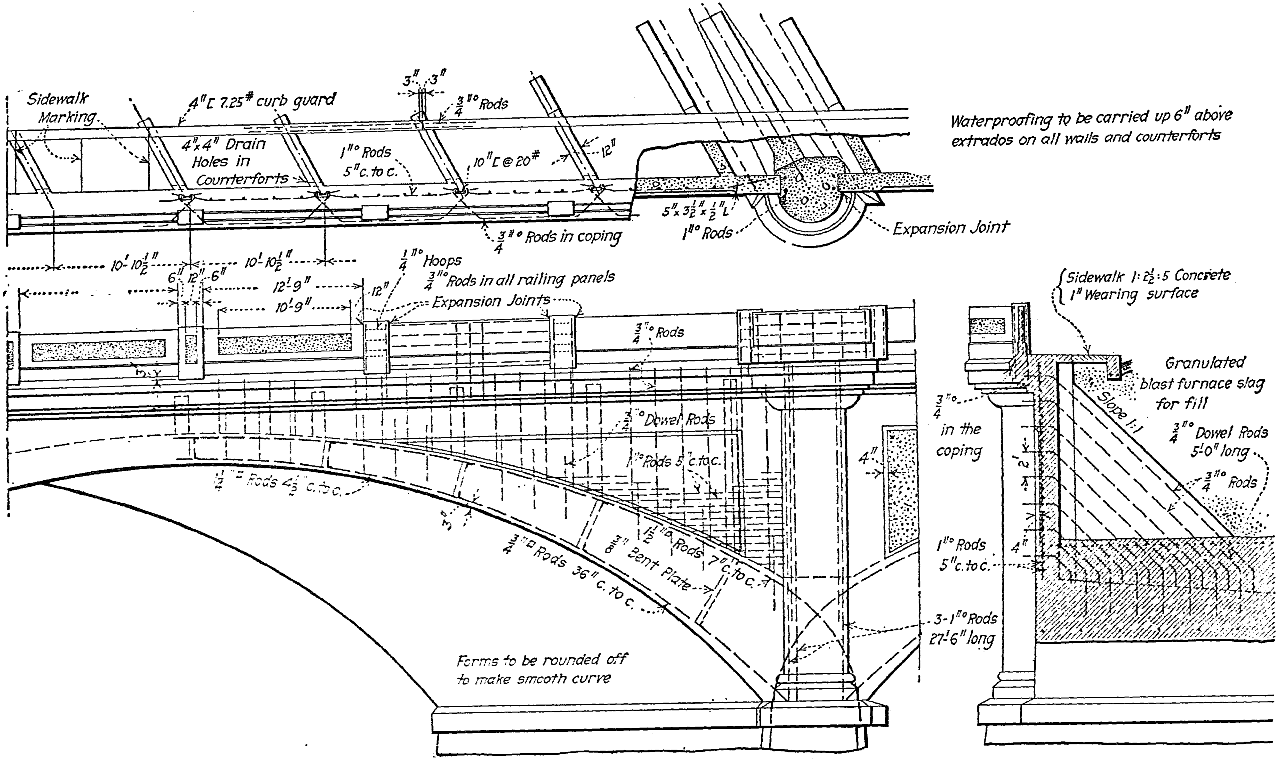
FIG. 206.—Steel centers—Atherton Avenue bridge, Pittsburgh.



Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

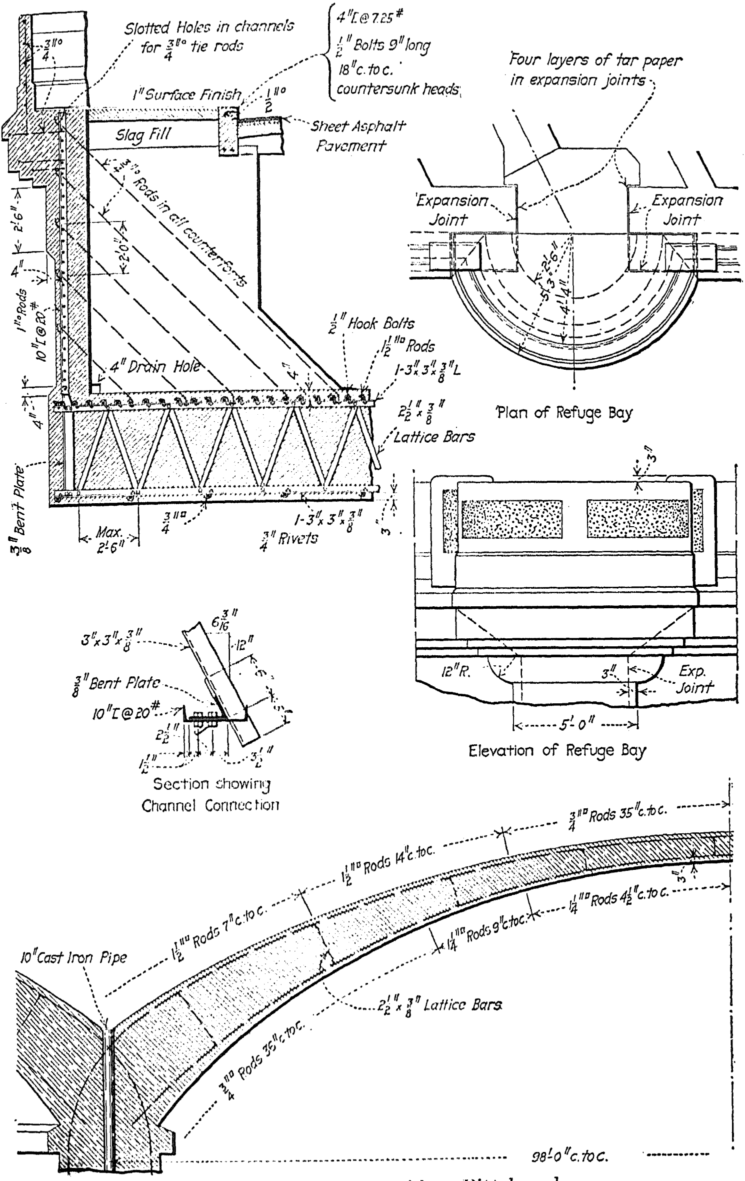
FIG. 207.—Construction view—Atherton Avenue bridge, Pittsburgh.

PLATE VIII

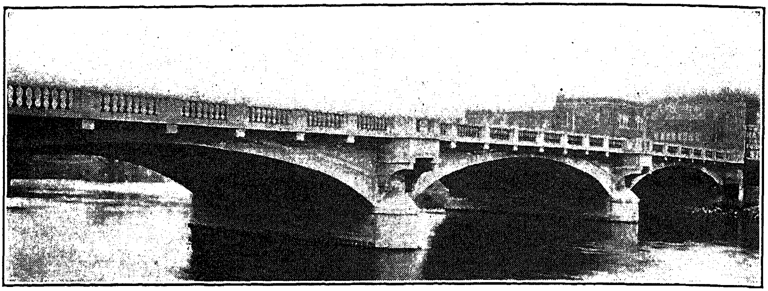


Atherton Avenue bridge, Pittsburgh.

PLATE IX

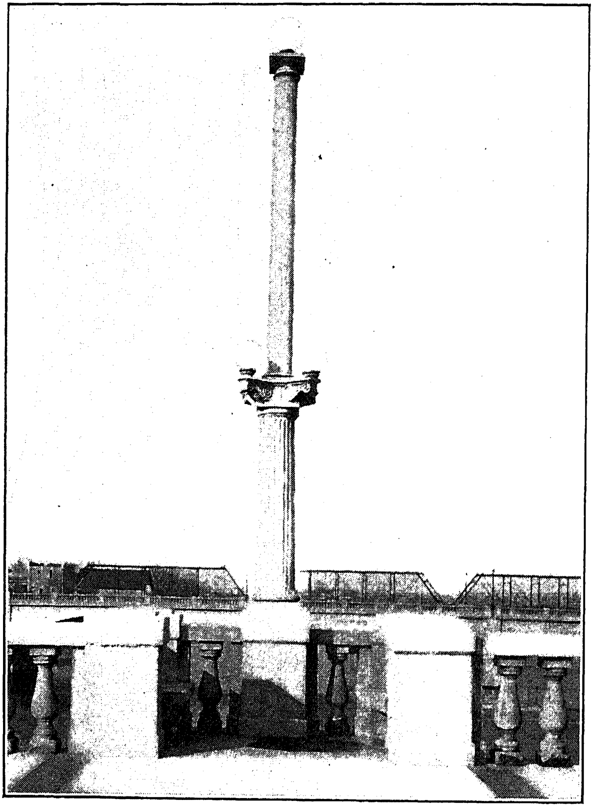

Slotted Holes in channels for $\frac{3}{4}$″ø tie rods
$\frac{3}{4}$″ø
4″I @ 7.25#
$\frac{1}{2}$″ Bolts 9″ long
18″ c. to c.
countersunk heads
1″ Surface Finish
$\frac{1}{2}$″ø
Slag Fill
Sheet Asphalt Pavement
$\frac{3}{4}$″ø Rods in all counterforts
2′-0″
2′-6″
4″
1″ø Rods
10″I @ 20#
4″ Drain Hole
4″
$\frac{1}{2}$″ Hook Bolts
$1\frac{1}{2}$″ø Rods
1-3″×3″×$\frac{3}{8}$″ L
$2\frac{1}{2}$″×$\frac{3}{8}$″
Lattice Bars
4″
$\frac{3}{8}$″ Bent Plate
Max. 2′-6″
$\frac{3}{4}$″ø
1-3″×3″×$\frac{3}{8}$″
$\frac{3}{4}$″ Rivets
3″
Four layers of tar paper in expansion joints
Expansion Joint
Expansion Joint
5′-3″
2′-6″
4′-4″
Plan of Refuge Bay
3″
12″R.
3″
Exp. Joint
5′-0″
Elevation of Refuge Bay
3″×3″×$\frac{3}{8}$″
$6\frac{3}{16}$″
12″
$\frac{3}{8}$″ Bent Plate
10″I @ 20#
$2\frac{1}{2}$″
$1\frac{1}{2}$″
$3\frac{1}{2}$″
Section showing Channel Connection
$\frac{3}{4}$″ø Rods 35″ c. to c.
$1\frac{1}{2}$″ø Rods 14″ c. to c.
$1\frac{1}{2}$″ø Rods 7″ c. to c.
$1\frac{1}{4}$″ø Rods $4\frac{1}{2}$″ c. to c.
$1\frac{1}{4}$″ø Rods 9″ c. to c.
3″
10″ Cast Iron Pipe
$2\frac{1}{2}$″×$\frac{3}{8}$″ Lattice Bars
$\frac{3}{4}$″ø Rods 35″ c. to c.
98′-0″ c. to c.

Atherton Avenue bridge, Pittsburgh.



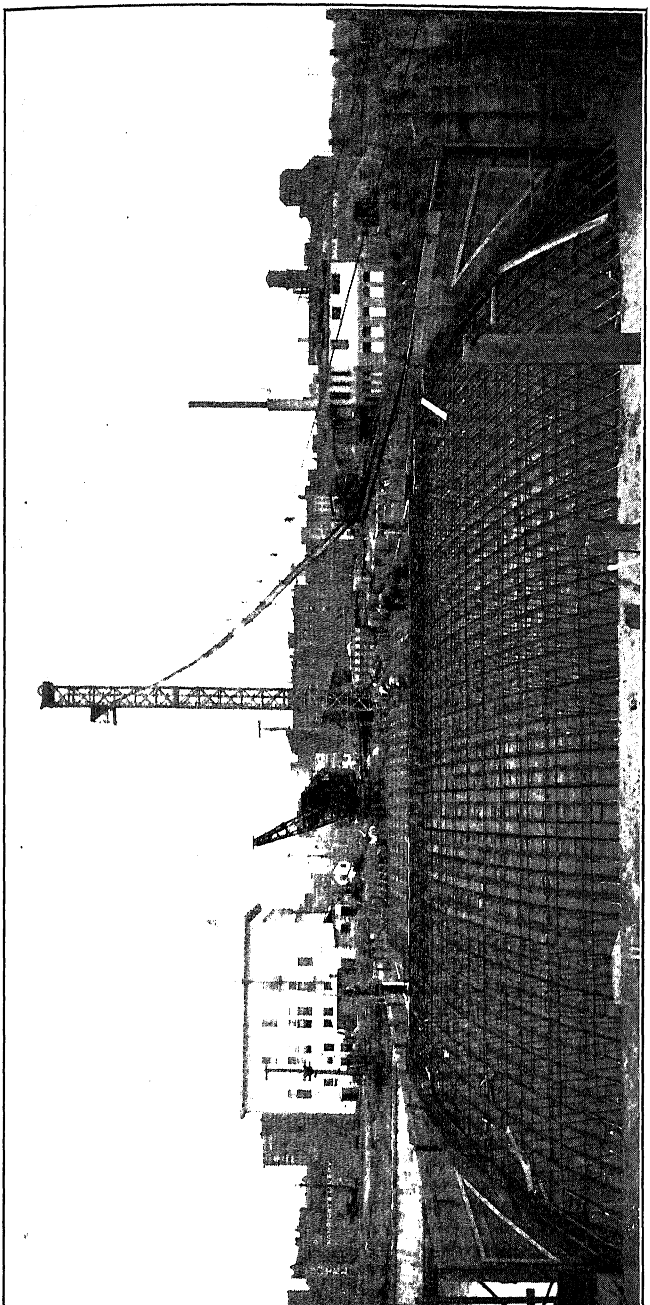
Courtesy of Engineering and Contracting.

FIG. 208.—Third Avenue bridge over Cedar River, Cedar Rapids, Iowa.



Courtesy of Engineering and Contracting.

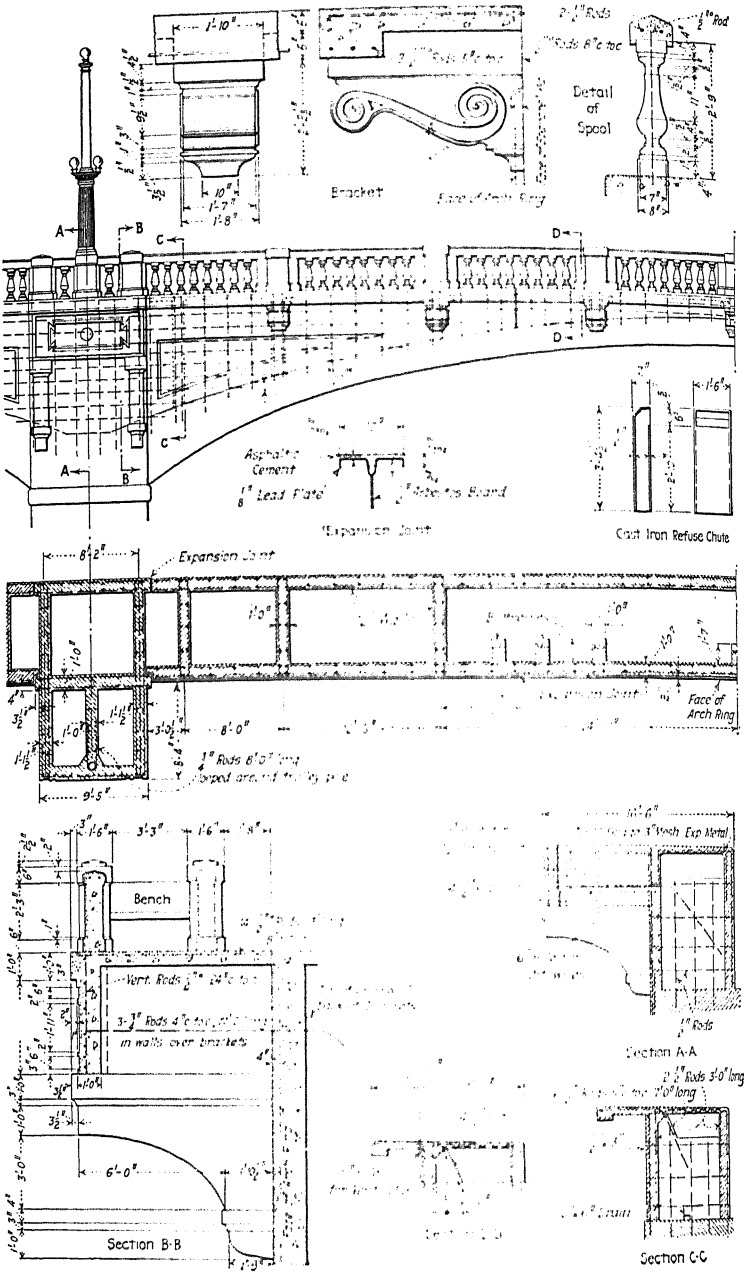
FIG. 209.—Refuge bay of Third Avenue bridge, Cedar Rapids, Iowa.



Courtesy of Engineering and Contracting.

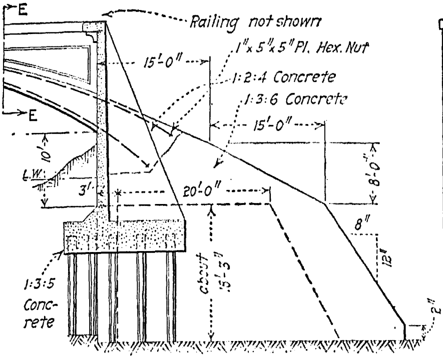
FIG. 210.—Construction view of Third Avenue bridge, Cedar Rapids, Iowa.

PLATE X

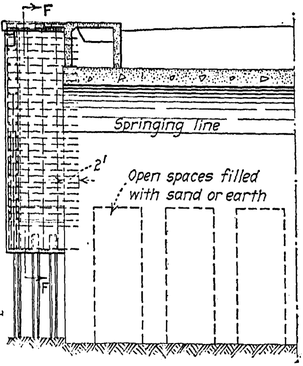


1'-10"
Bracket
Face of Arch Ring
2-1/2" Rods
1/2" Rod
Detail of Spool
2'-9"
7"
8"
A
B
C
D
Asphaltic Cement
1/8" Lead Plate
1/2" Asbestos Board
Expansion Joint
1'-6"
Cast Iron Refuse Chute
8'-2"
Expansion Joint
1'-0"
4"
3 1/2"
1'-1 1/2"
3'-0 1/2"
8'-0"
1/4" Rods 8'-0" long looped around
9'-5"
Face of Arch Ring
Bench
Vert. Rods 1/2" 24" c to c
3-3/4" Rods 4" c to c
in walls over brackets
6'-0"
Section B-B
10'-6"
3" Mesh Exp Metal
1/2" Rods
Section A-A
2-1/2" Rods 3'-0" long
7'-0" long
Section C-C

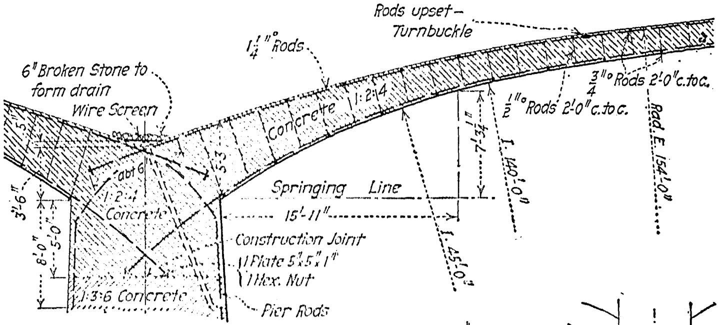
PLATE XI

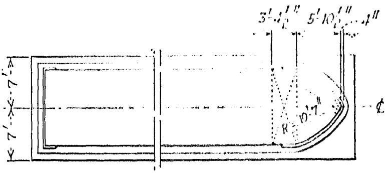


Section F-F through Retaining Wall

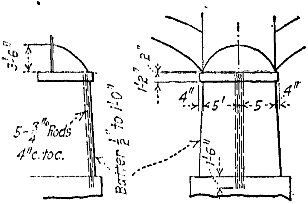


Section E-E

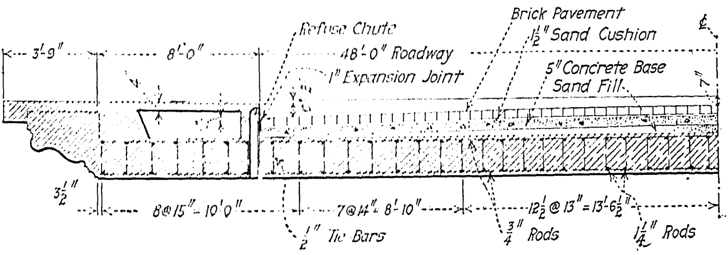




Pier



Pier



Half Section through Crown

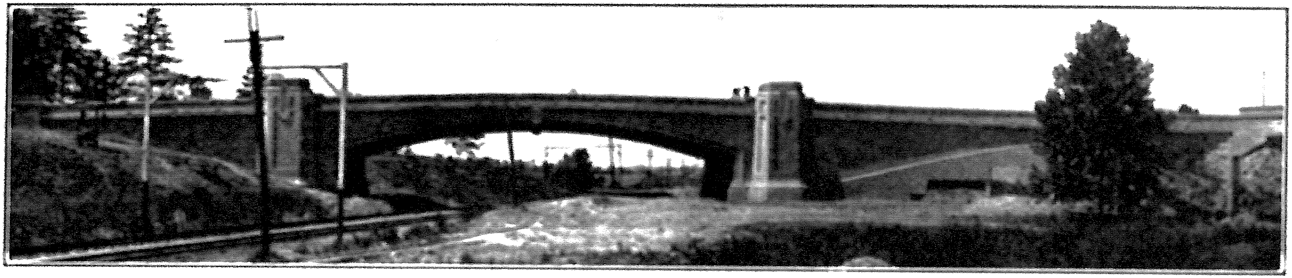


FIG. 211.—Carondelet Park bridge, St. Louis.

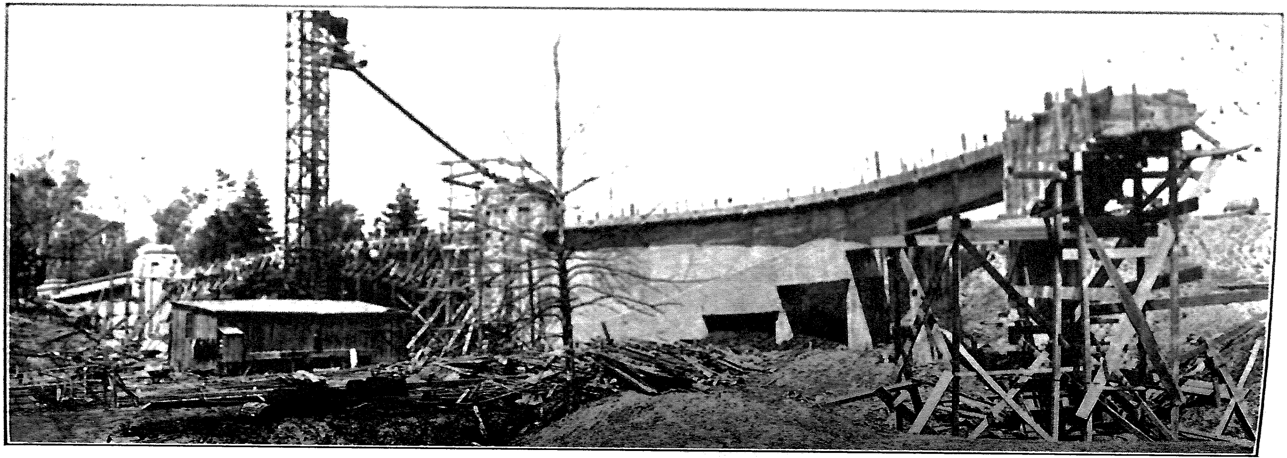


FIG. 212.—Construction view of Carondelet Park bridge, St. Louis.

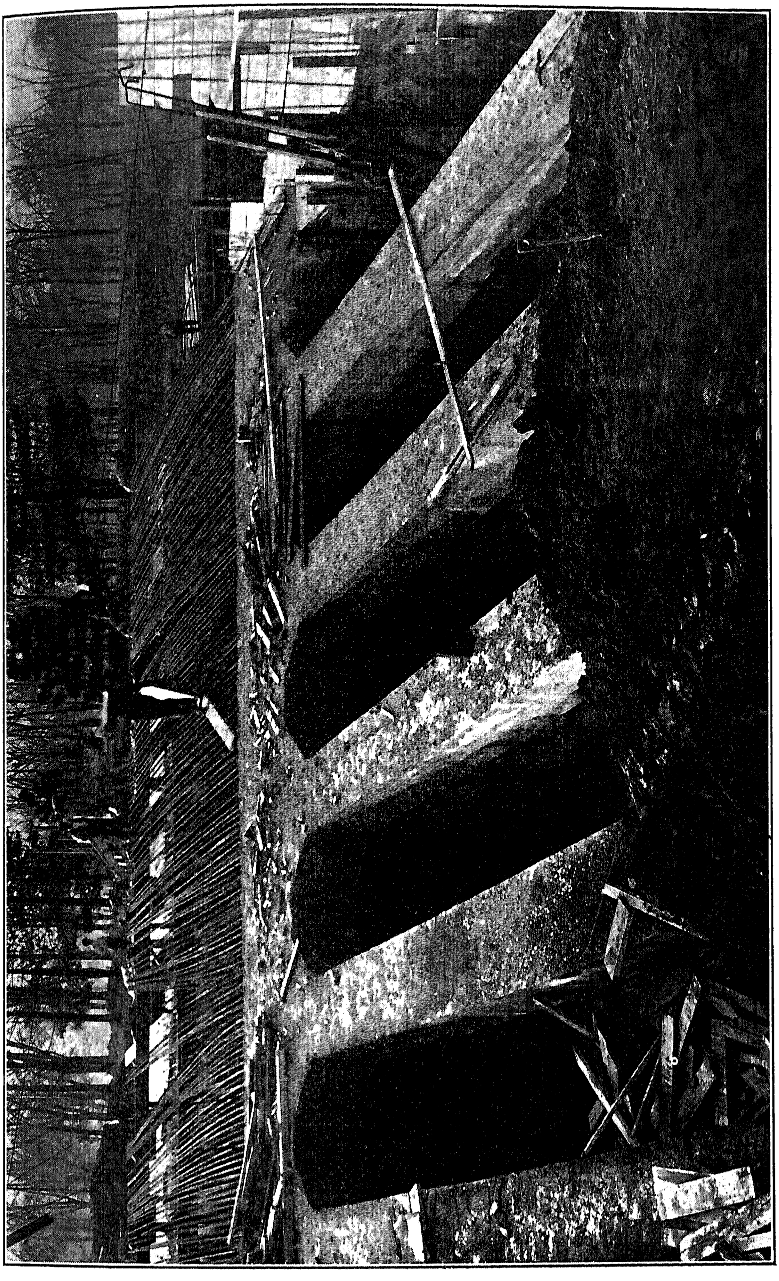
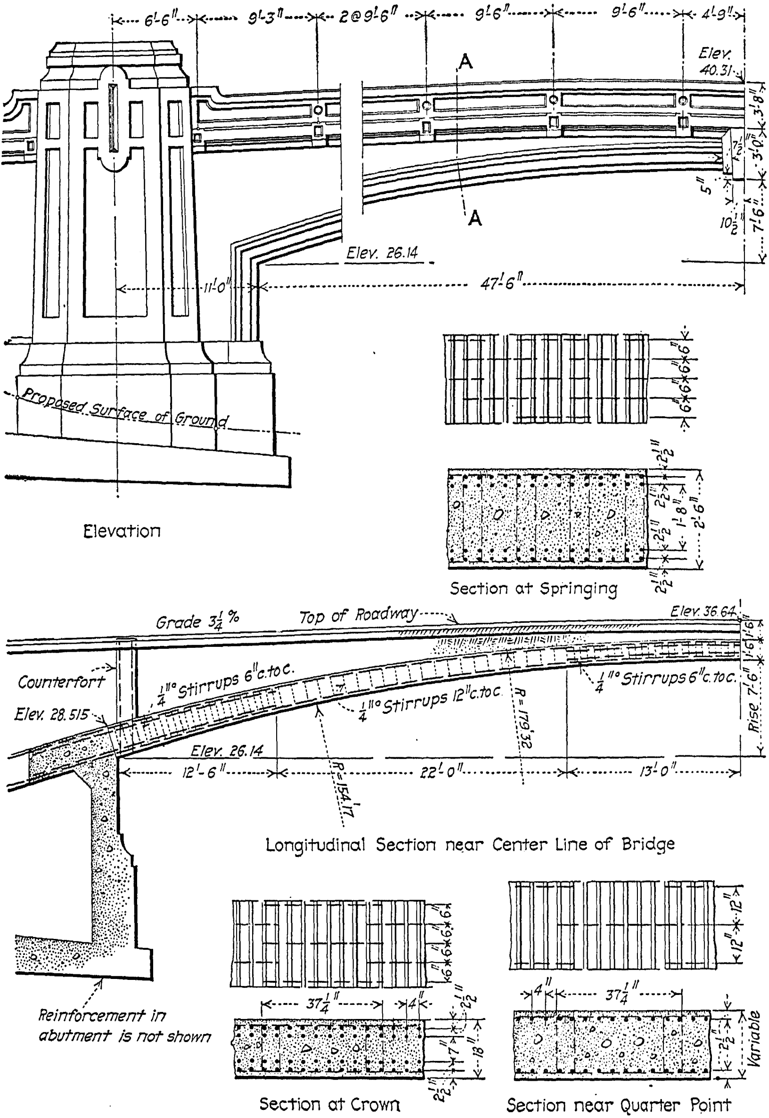


FIG. 213.—Abutments of Carondelet Park bridge, St. Louis.

PLATE XII



Elevation

Section at Springing

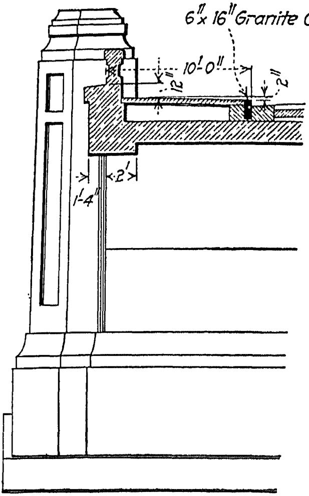
Longitudinal Section near Center Line of Bridge

Section at Crown

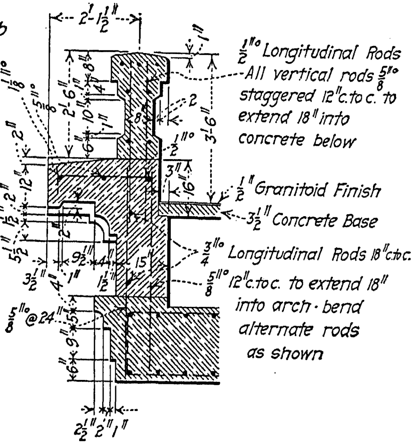
Section near Quarter Point

Carondelet Park bridge, St. Louis.

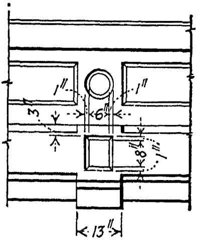
PLATE XIII



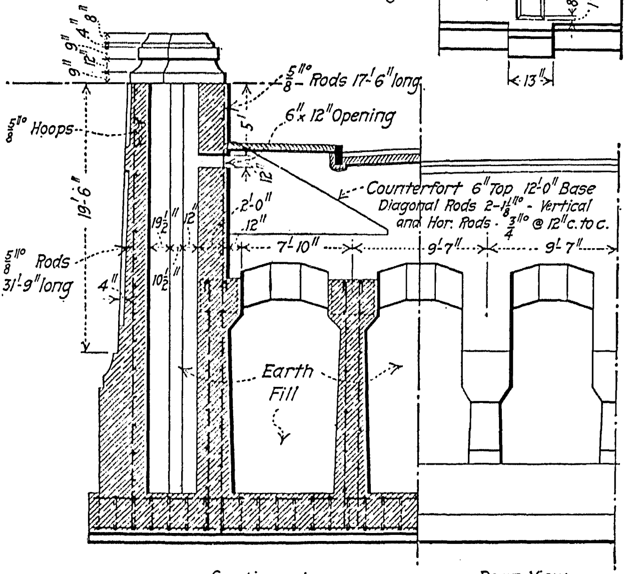
Transverse Section at Center


2'-1½"
½"° Longitudinal Rods
All vertical rods ⅝"° staggered 12" c. to c. to extend 18" into concrete below
½" Granitoid Finish
3½" Concrete Base
¾"° Longitudinal Rods 18" c. to c.
⅝"° 12" c. to c. to extend 18" into arch - bend alternate rods as shown
⅝"° @ 24"
1"
3'-6"
2'-6"
10"
4"
8"
6"
1"
2"
12"
16"
3"
15"
9½"
4"
1½"
3½"
5½"
2½" 2" 1"

Section A-A
(See other page)



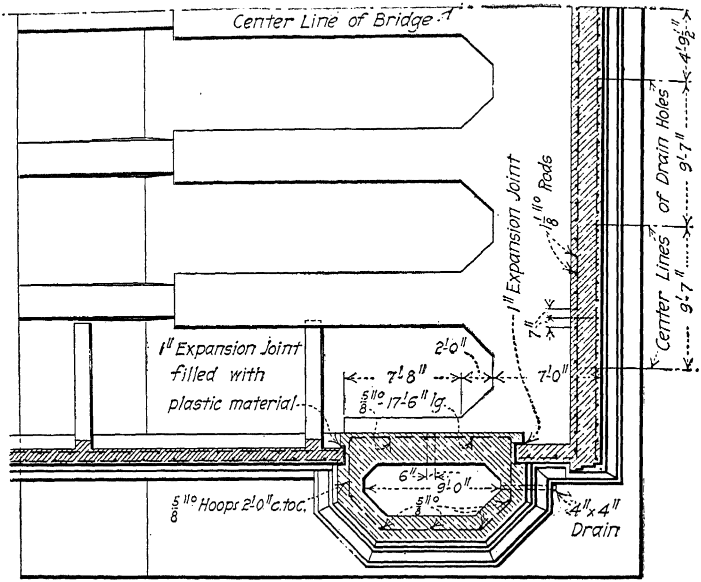
Elevation of Railing



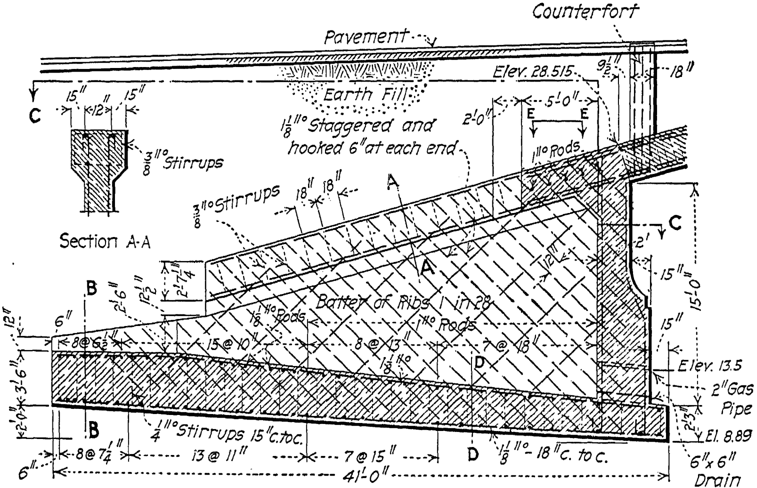
Section at Center Line of Monument

Rear View

PLATE XIV

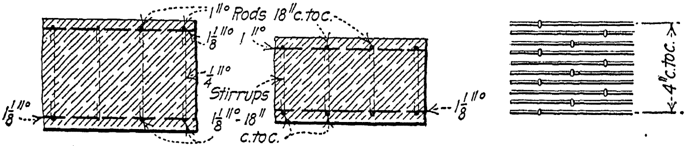


Section C-C



Section A-A

Longitudinal Section through West Pier.



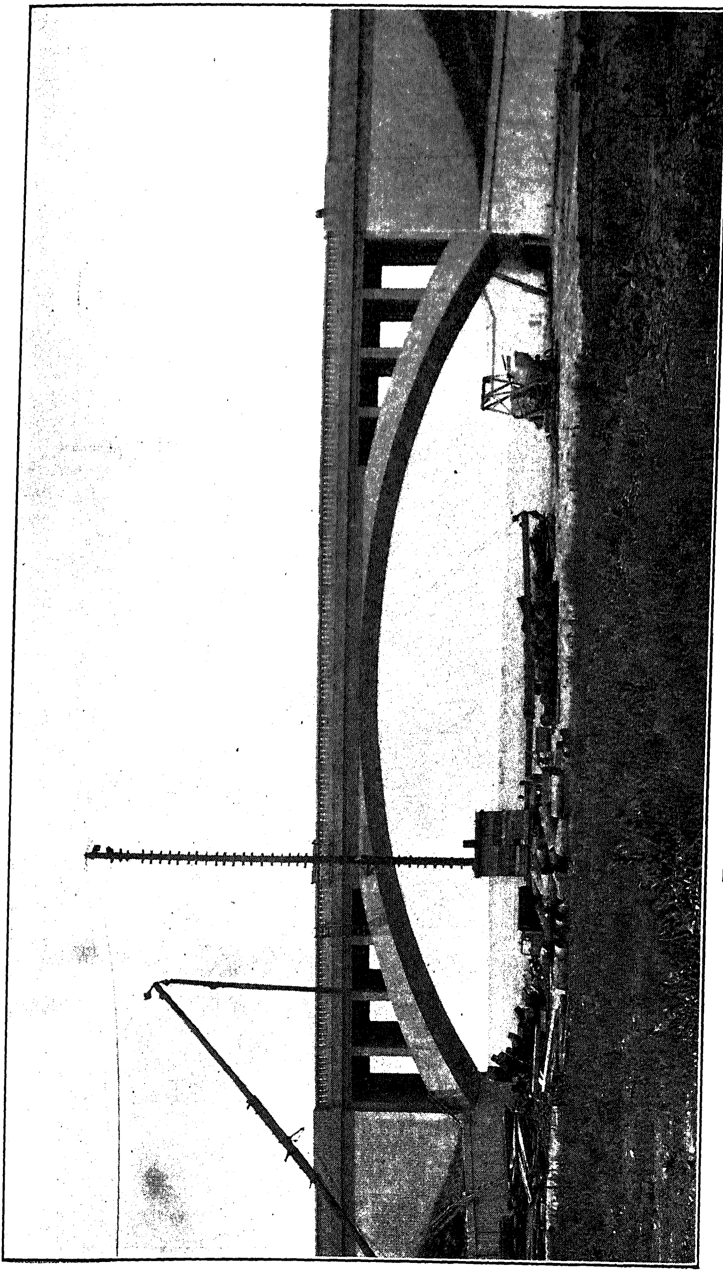


FIG. 214.—Ashokan Reservoir Spillway bridge.

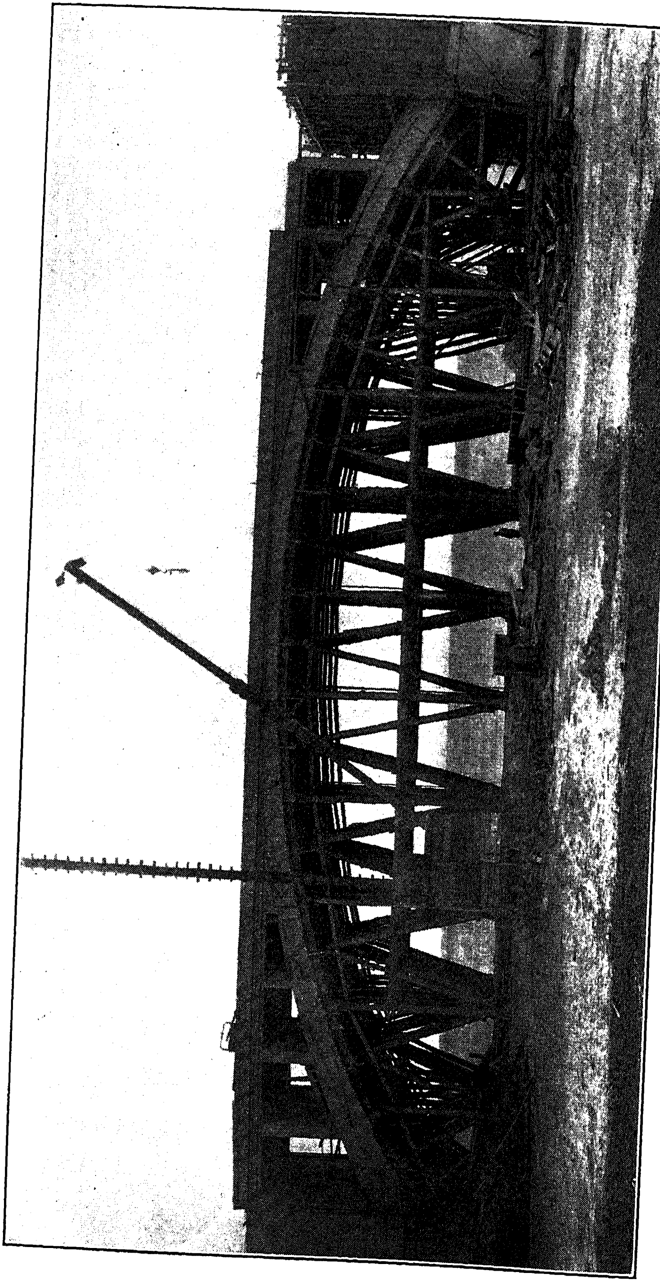
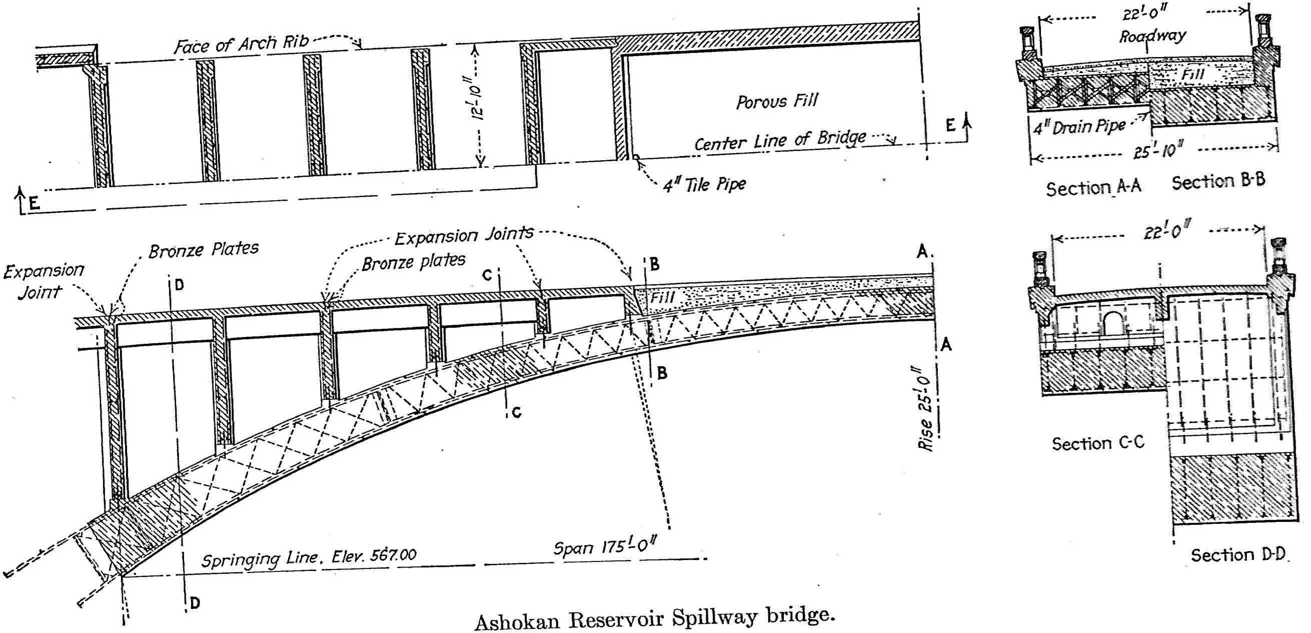


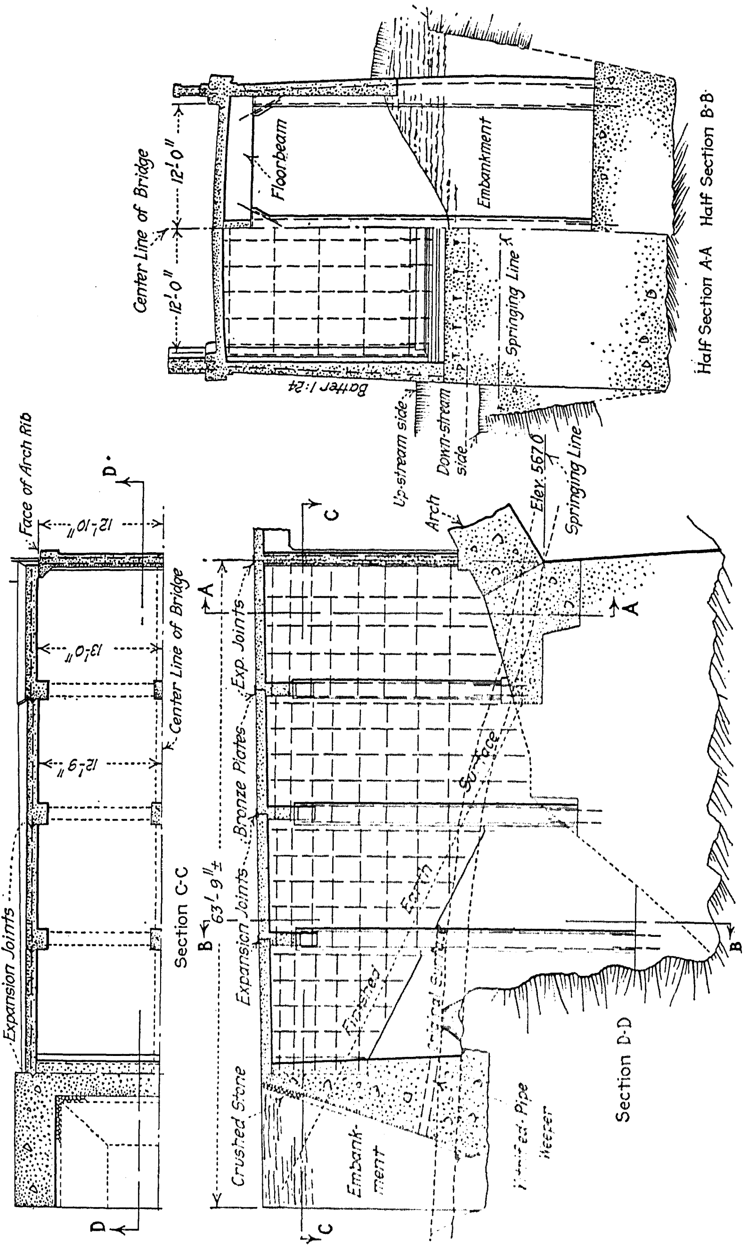
FIG. 215.—Falsework for Ashokan Reservoir Spillway bridge.

PLATE XV

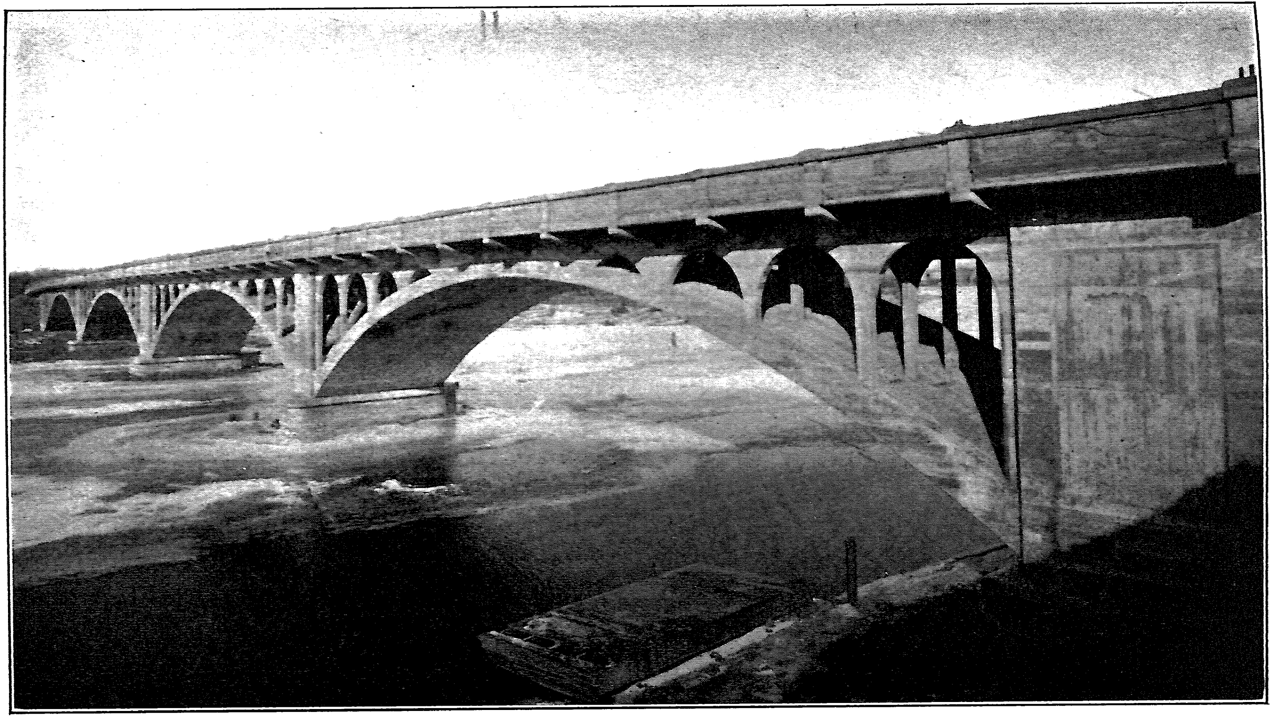


Ashokan Reservoir Spillway bridge.

PLATE XVI

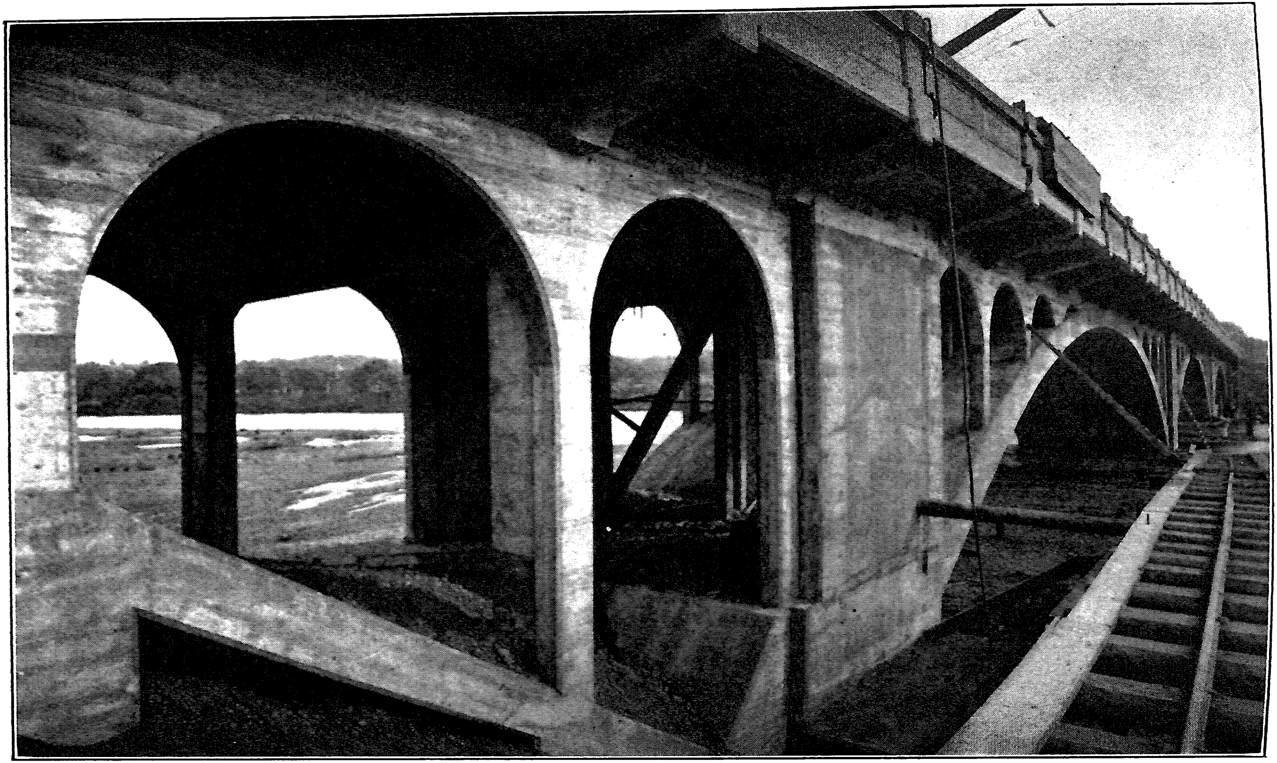


Ashokan Reservoir Spillway bridge.



Courtesy of Mr. Louis P. Wolff, Consulting Engineer, St. Paul, Minn.

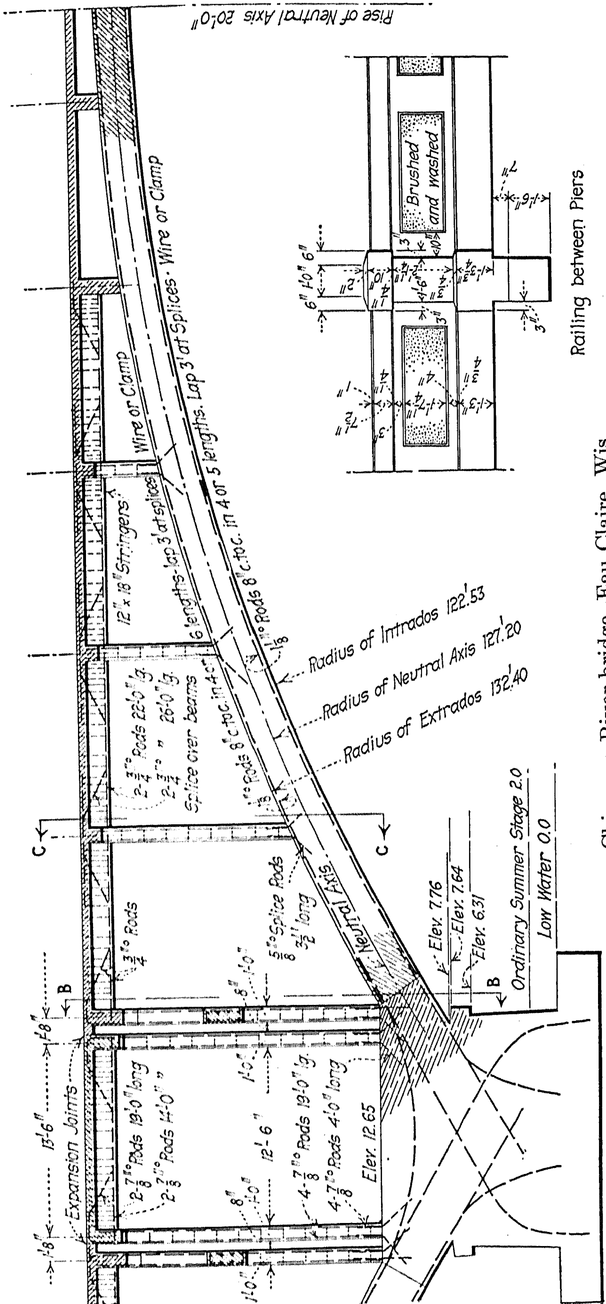
FIG. 216.—Chippewa River bridge, Eau Claire, Wis.



Courtesy of Sandusky Portland Cement Co.

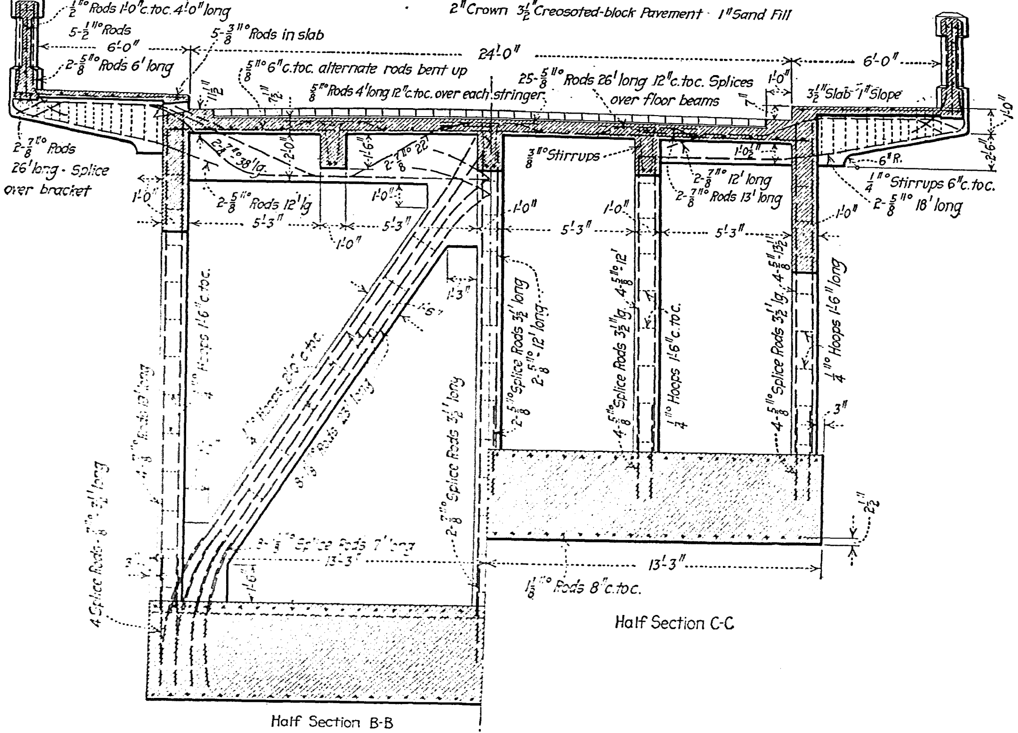
FIG. 217.—Chippewa River bridge, Eau Claire, Wis.

PLATE XVII



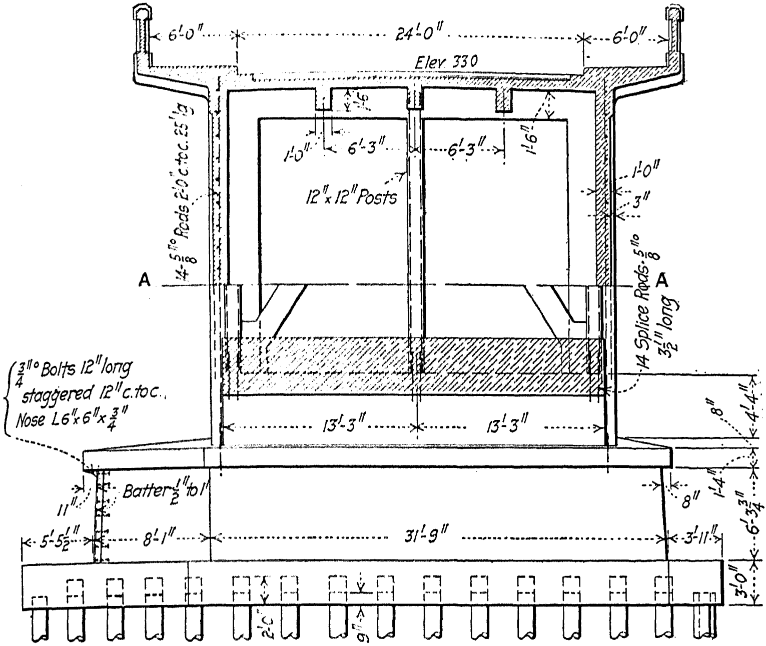
Chippewa River bridge, Eau Claire, Wis.

PLATE XVIII

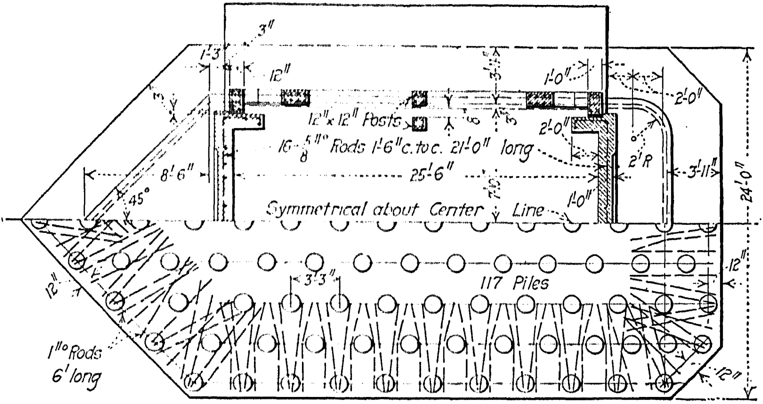

1/2"° Rods 1'-0" c.to c. 4'-0" long
5-1/2"° Rods
6'-0"
2-5/8"° Rods 6' long
5-3/8"° Rods in slab
2" Crown 3 1/2" Creosoted-block Pavement · 1" Sand Fill
24'-0"
6'-0"
5/8"° 6" c.to c. alternate rods bent up
5/8"° Rods 4' long 12" c.to c. over each stringer
25-5/8"° Rods 26' long 12" c.to c. Splices over floor beams
3 1/2" Slab 1" Slope
2-7/8"° Rods 26' long - Splice over bracket
2-7/8"° 38' lg.
2-7/8"° 22'
2-5/8"° Rods 12' lg
3/8"° Stirrups
2-7/8"° 12' long
2-7/8"° Rods 13' long
1/4"° Stirrups 6" c.to c.
2-5/8"° 18' long
6" R.
5'-3"
1'-0"
1'-6"
1'-3"
1/4"° Hoops 1'-6" c.to c.
2-5/8"° Splice Rods 3 1/2' long
2-5/8"° 12' long
4-5/8"° Splice Rods 3 1/2' lg. 4-5/8"° 12'
1/4"° Hoops 1'-6" c.to c.
4-5/8"° Splice Rods 3 1/2' lg. 4-5/8"° 13 1/2'
1/4"° Hoops 1'-6" long
2-7/8"° Splice Rods 3 1/2' long
8-7/8"° Rods 23' long
1/4"° Hoops 2'-0" c.to c.
4-7/8"° Rods 13' long
4 Splice Rods - 7/8"° 3 1/2' long
8-7/8"° Splice Rods 7' long
13'-3"
13'-3"
1 1/8"° Rods 8" c.to c.
2 1/2"
3"
Half Section B-B
Half Section C-C

Chippewa River bridge, Eau Claire, Wis.

PLATE XIX



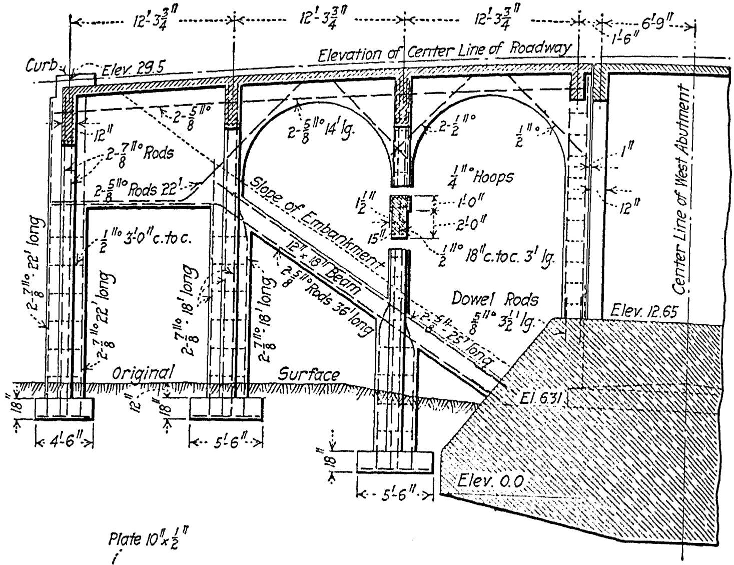
Center Pier



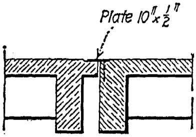
Half Sectional Plan A-A and Half Foundation Plan showing Piles

Chippewa River bridge, Eau Claire, Wis.

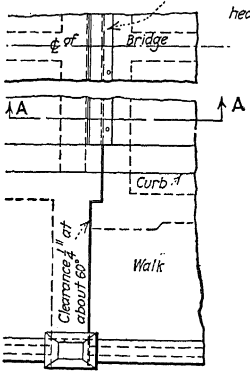
PLATE XX



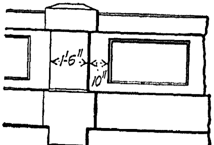
Longitudinal Section of West Approach

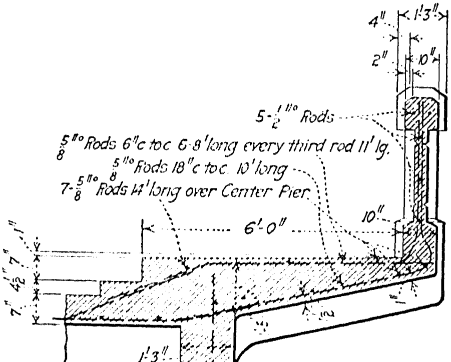


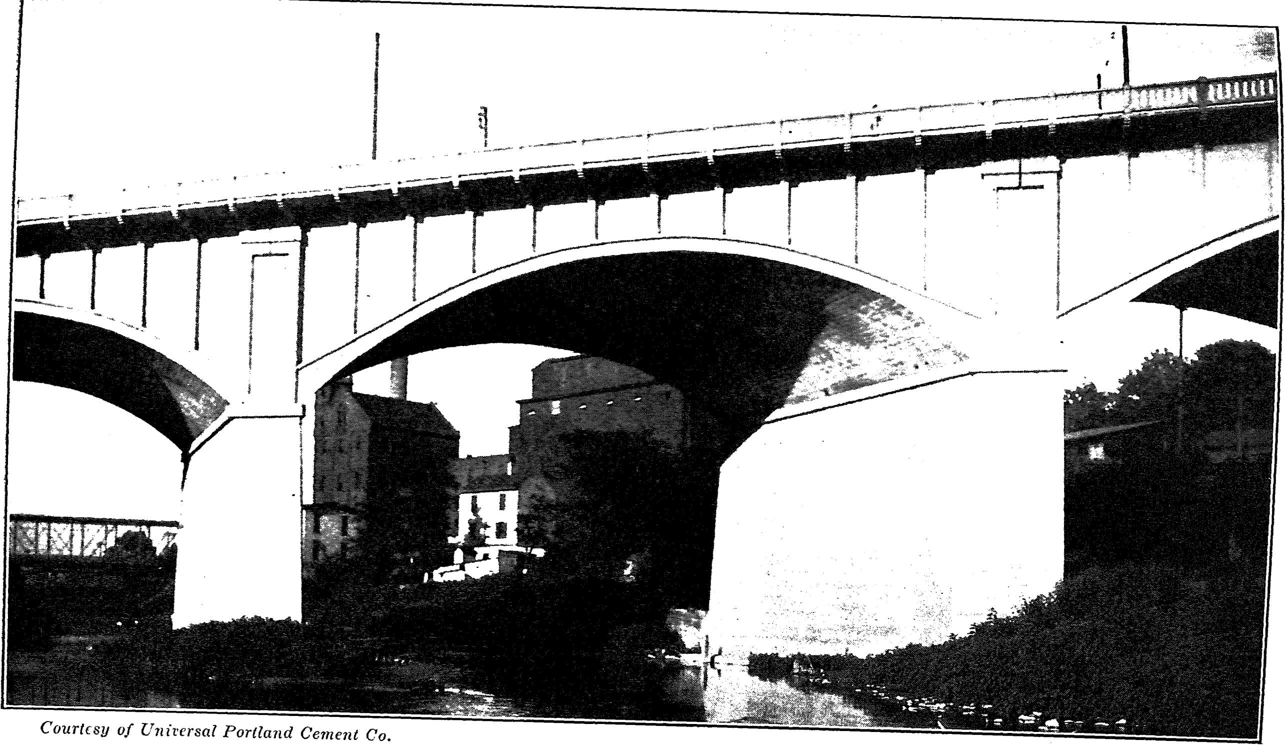
Section A-A


10"×½"×21'-10" Plate over Expansion Joint anchored with ⅝"° Bolts with countersunk heads spaced about 2'-0" c. to c.
℄ of Bridge
A
A
Curb
Clearance ¼" at about 60°
Walk

Plan



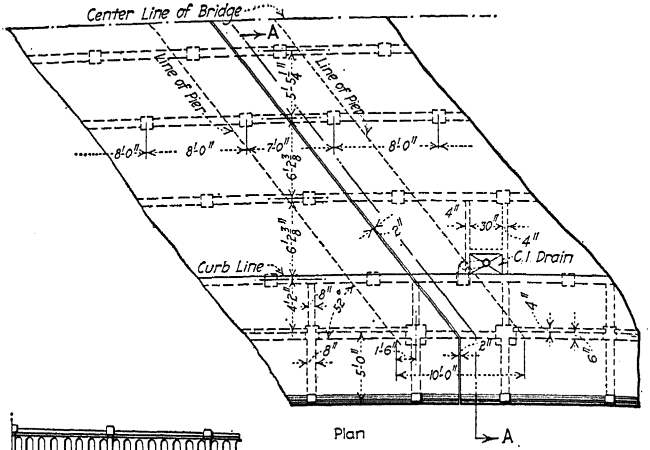

4"
1'-3"
2"
10"
5-½"° Rods
⅝"° Rods 6" c. to c. 6-8' long every third rod 11' lg.
⅝"° Rods 18" c. to c. 10' long
7-⅝"° Rods 14' long over Center Pier
10"
6'-0"
7"
4½"
7"
1'-3"



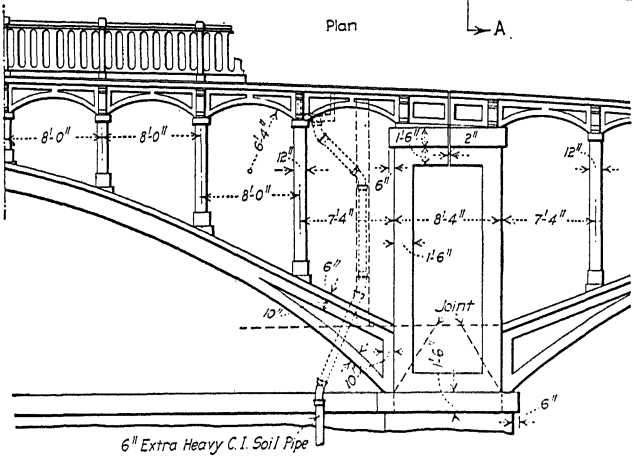
Courtesy of Universal Portland Cement Co.

FIG. 218.—Ludlow Avenue viaduct, Cincinnati, Ohio.

PLATE XXI



Plan



Elevation

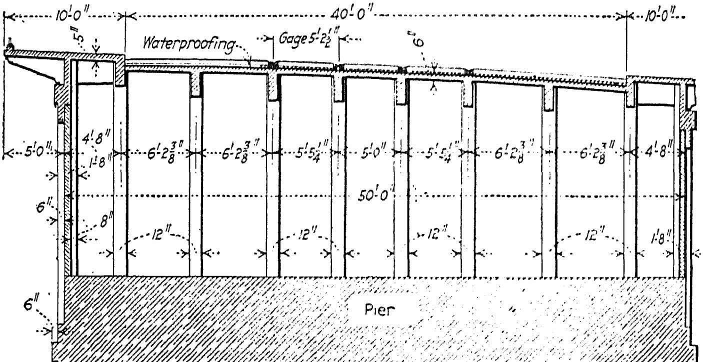
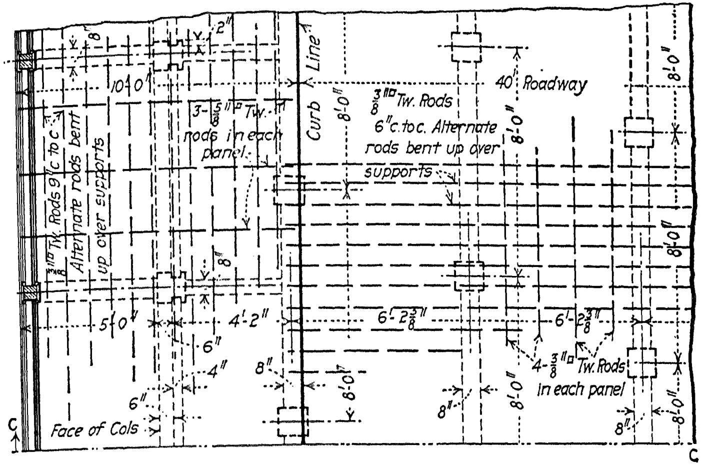
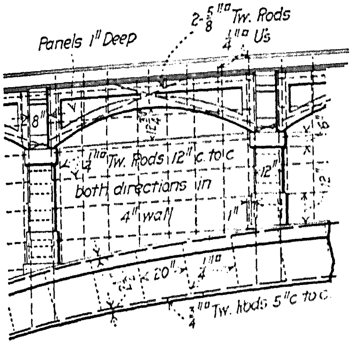


PLATE XXII

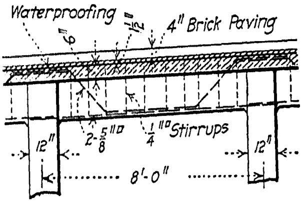


2"
8"
10'-0"
3-5/8"ø Tw. rods in each panel
Curb Line
3/8"ø Tw. Rods 6" c. to c. Alternate rods bent up over supports
40' Roadway
8'-0"
3/8"ø Tw. Rods 9" c. to c. Alternate rods bent up over supports
8"
5'-0"
6"
4'-2"
6'-2 3/8"
6'-2 3/8"
4-3/8"ø Tw. Rods in each panel
4"
8"
6"
Face of Cols
C
C

Part Plan of Roadway over Arches



Panels 1" Deep
2-5/8"ø Tw. Rods
1/4"ø U's
8"
1/4"ø Tw. Rods 12" c. to c both directions in 4" wall
12"
1"
20"
1/4"ø
3/4"ø Tw. Rods 5" c. to c.



Section B-B

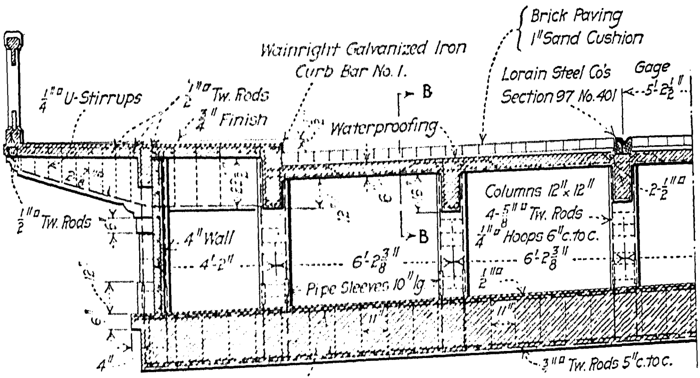
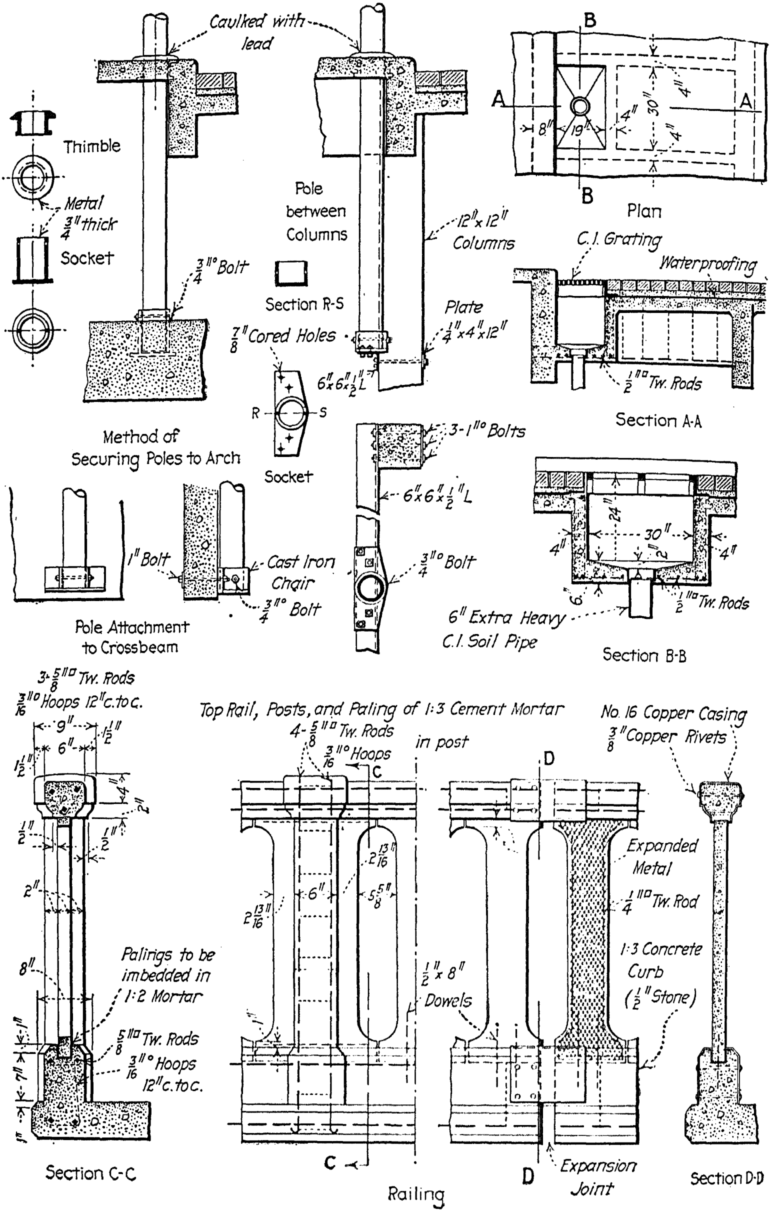
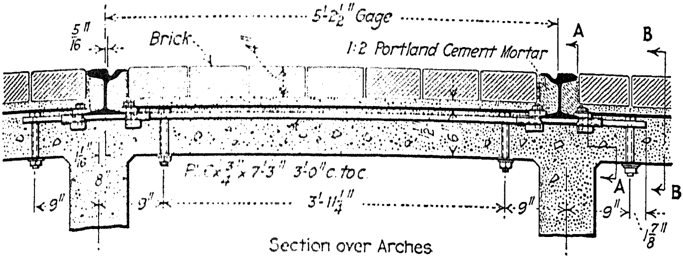


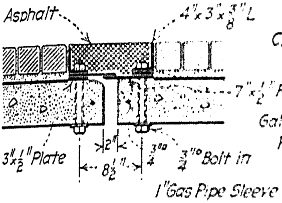
PLATE XXIII



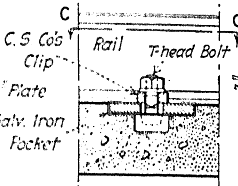
Ludlow Avenue viaduct, Cincinnati, Ohio.



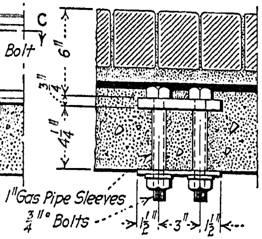
Section over Arches



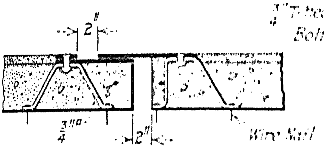
Detail of Expansion Joint in Roadway



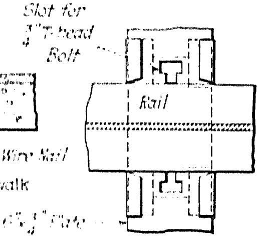
Section A-A



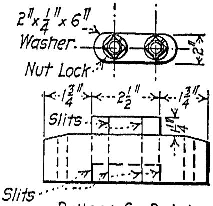
Section B-B



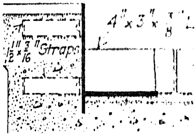
Detail of Expansion Joint in Sidewalk



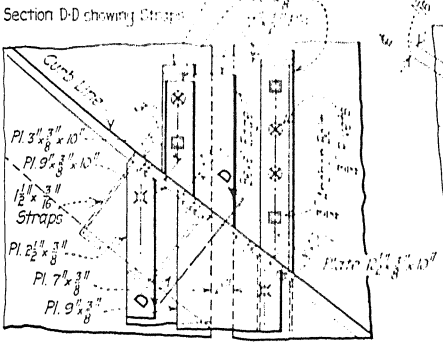
Section C-C



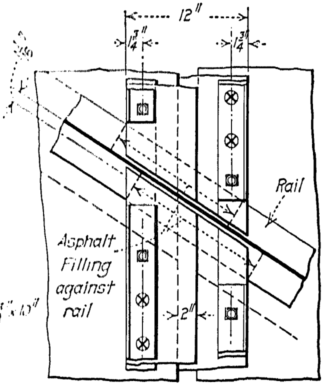
Pattern for Pockets (No. 20 B.W.G. Galv. Iron)



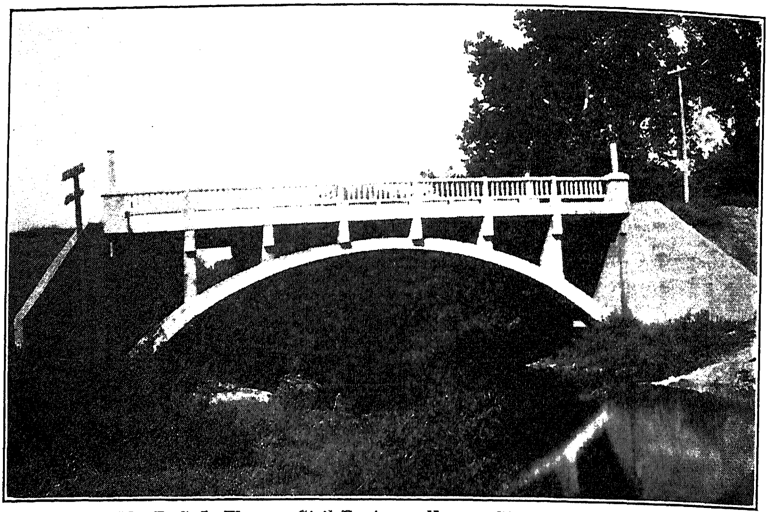
Section D-D showing Straps



Expansion Joint Detail at Curb Line

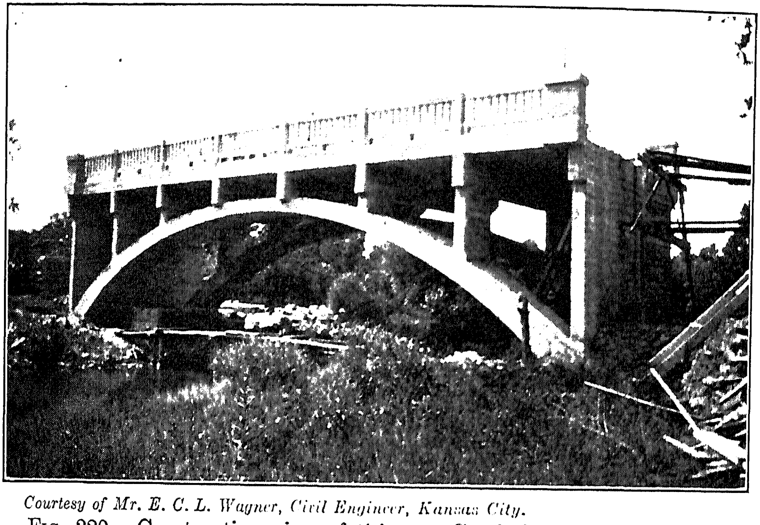


Plan of Expansion Joint in Roadway



Courtesy of Mr. E. C. L. Wagner, Civil Engineer, Kansas City.

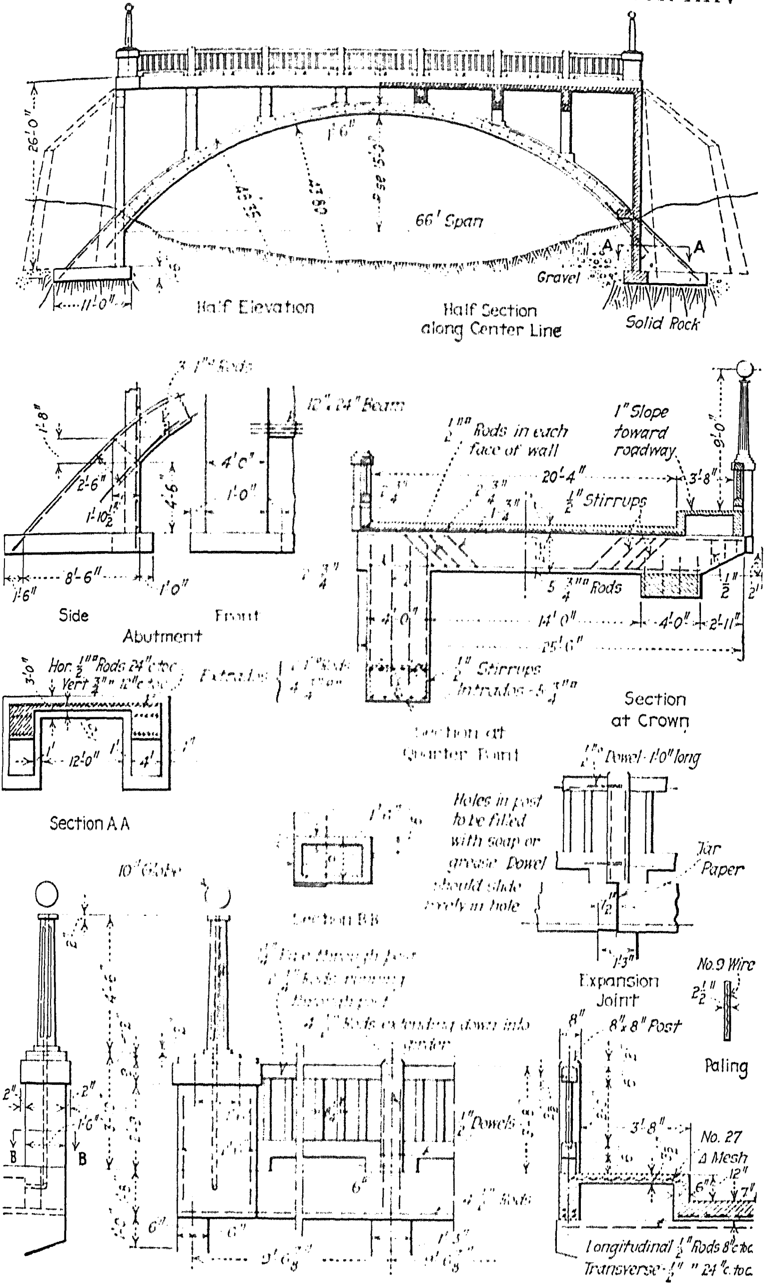
FIG. 219.—Stimson Creek bridge, Fulton, Mo.

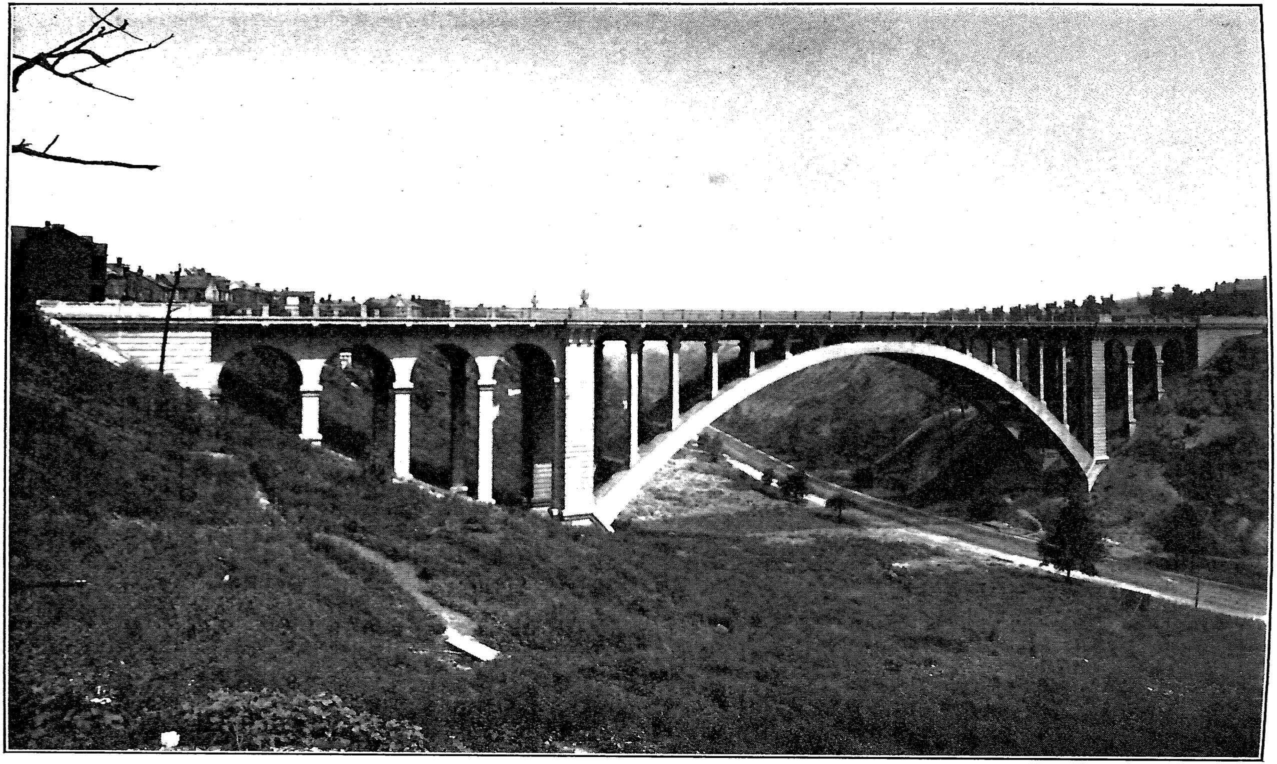


Courtesy of Mr. E. C. L. Wagner, Civil Engineer, Kansas City.

FIG. 220.—Construction view of Stimson Creek bridge, Fulton, Mo.

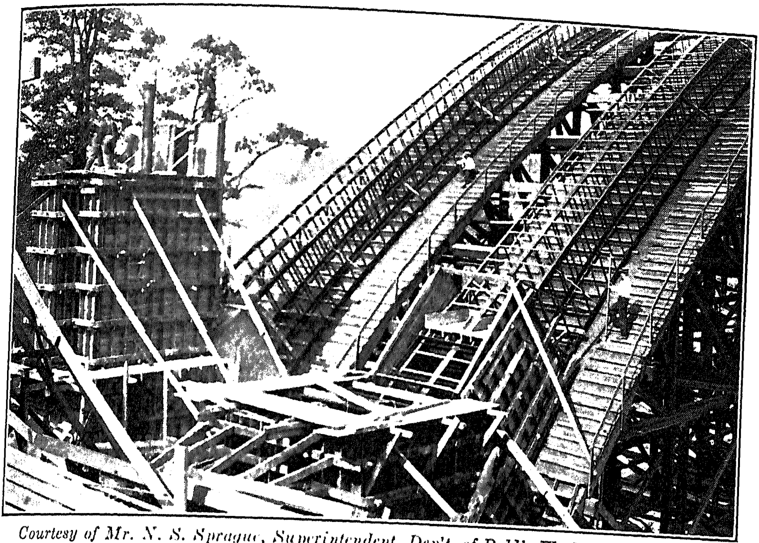
PLATE XXV


26'-0"
1'-6"
48.55
49.60
55.2
66' Span
A
A
Gravel
3'-0"
11'-0"
Half Elevation
Half Section along Center Line
Solid Rock
3-1" Rods
12" x 24" Beam
1'-8"
2'-6"
1'-10½"
4'-6"
4'-0"
1'-0"
8'-6"
1'-6"
1'-0"
Side
Front
Abutment
½" Rods in each face of wall
1" Slope toward roadway
9'-0"
20'-4"
3'-8"
½" Stirrups
5 ¾" Rods
½"
2"
4'-0"
14'-0"
4'-0"
2'-11"
25'-6"
½" Stirrups
Intrados-5 ¾"
Extrados
Section at Quarter Point
Section at Crown
3'-0"
Hor. ½" Rods 24" c. to c.
Vert ¾" " 12" c. to c.
1'
12'-0"
1'
4'
Section A A
½" Dowel-1'-0" long
Holes in post to be filled with soap or grease. Dowel should slide freely in hole
Tar Paper
½"
1'3"
Expansion Joint
Section B B
10" Globe
Paling
No. 9 Wire
2½"
8"
8" x 8" Post
½" Dowels
½" Rods
3'-8"
No. 27 Δ Mesh
12"
6"
7"
2"
2"
1'-6"
B
B
6"
6"
6"
1'-3"
9'-6⅝"
9'-6⅝"
Longitudinal ½" Rods 8" c. to c.
Transverse ½" " 24" c. to c.



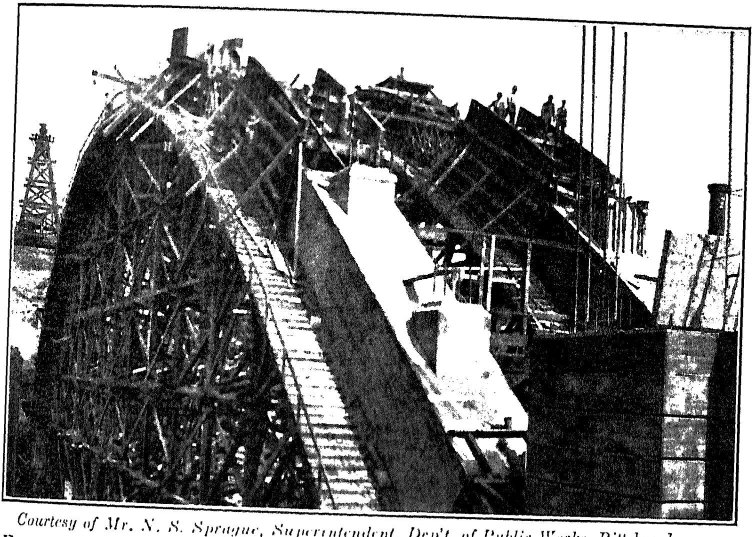
Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

FIG. 221.—Larimer Avenue bridge, Pittsburgh, Pa.



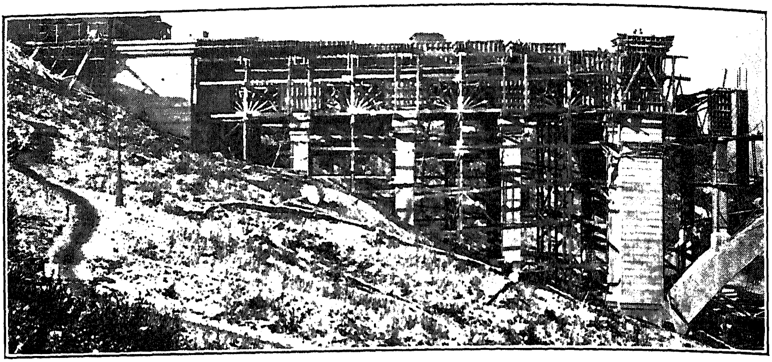
Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

FIG. 222.—Arch-rib reinforcement in place, Larimer Avenue bridge, Pittsburgh.



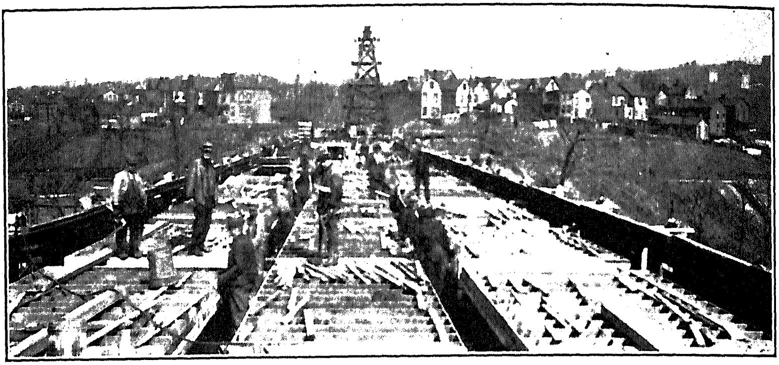
Courtesy of Mr. N. S. Sprague, Superintendent, Dep't. of Public Works, Pittsburgh.

FIG. 223.—Concreting of arch ribs, Larimer Avenue bridge, Pittsburgh.



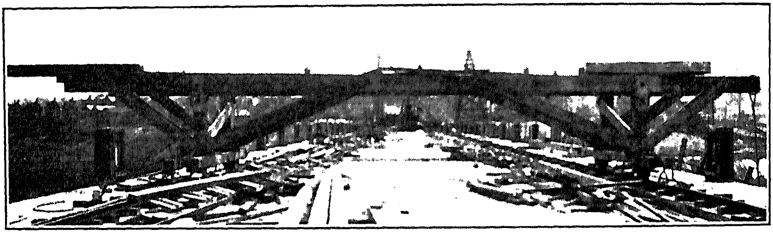
Courtesy of Engineering and Contracting.

FIG. 224.—Construction view of west approach, Larimer Avenue bridge, Pittsburgh.



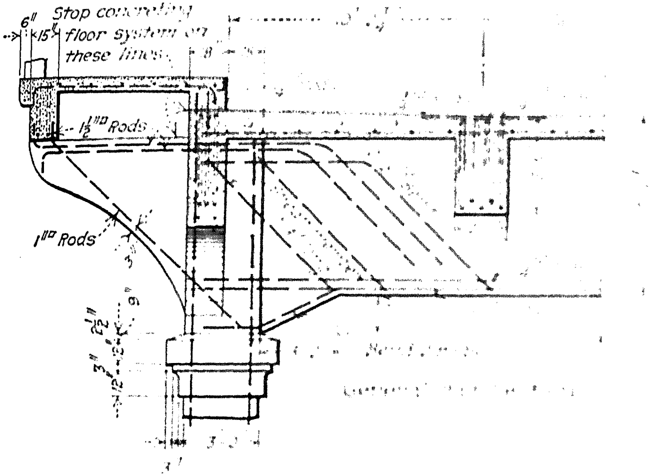
Courtesy of Engineering and Contracting.

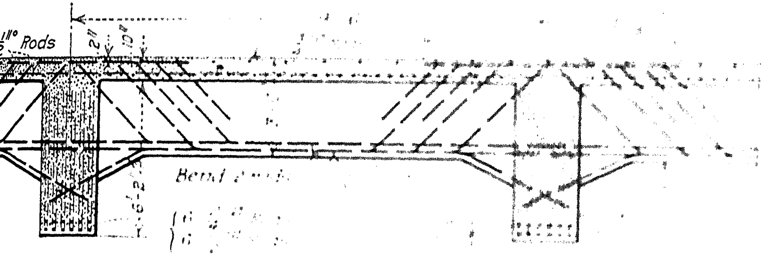
FIG. 225.—Forms for floor system over main arch, Larimer Avenue bridge, Pittsburgh.



Courtesy of Engineering and Contracting.

FIG. 226.—View of traveler for suspending working scaffolds for bushhammer work, Larimer Avenue bridge, Pittsburgh.


6"
15"
Stop concreting floor system on these lines
8"
$1\frac{1}{2}$" □ Rods
1" □ Rods
3"
9"
$2\frac{1}{2}$"
3"
12"
3'-2"
3'


1" □ Rods
2"
10"
Bend [illegible]
6'-2"
Cross Se[illegible]

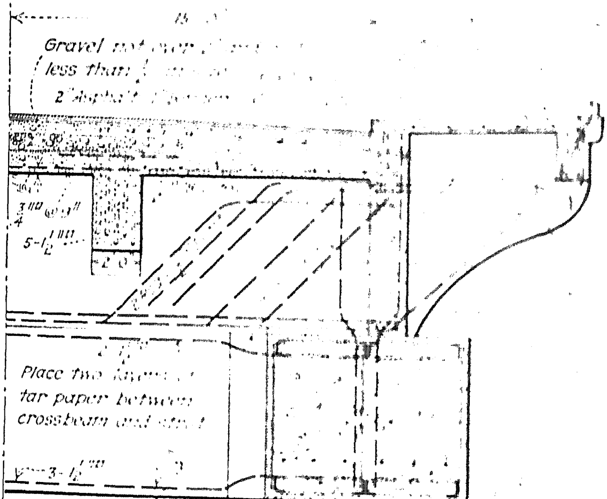
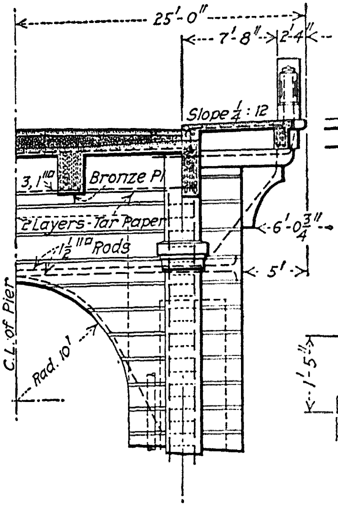
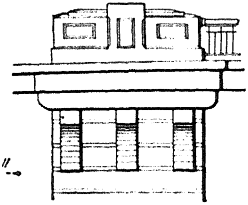
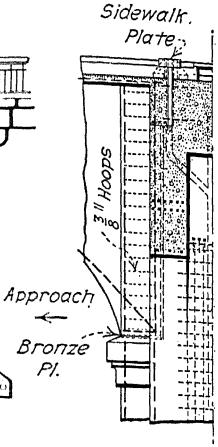

Gravel not over [illegible]
less than [illegible]
2" asphalt [illegible]
$\frac{3}{4}$" □ @ 9"
5-$1\frac{1}{2}$" □
2'-0"
Place two layers of tar paper between crossbeam and strut
3-$1\frac{1}{2}$" □

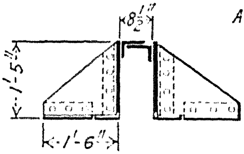
PLATE XXVII



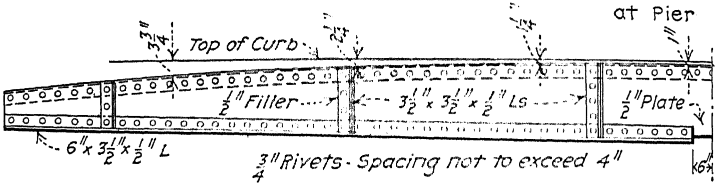




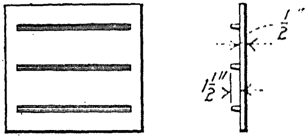
Expansion Joint at Pier



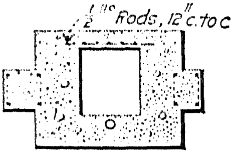
Section of Expansion Plates



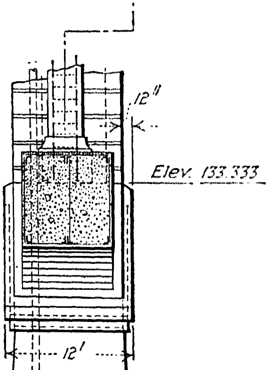
Roadway Expansion Joint



Bronze Plate



Section



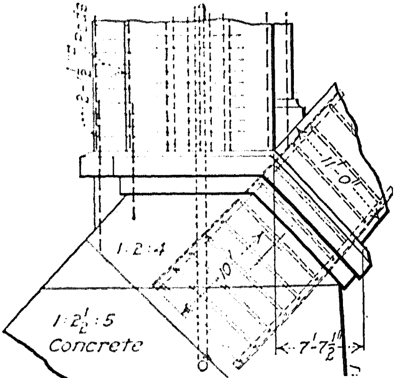
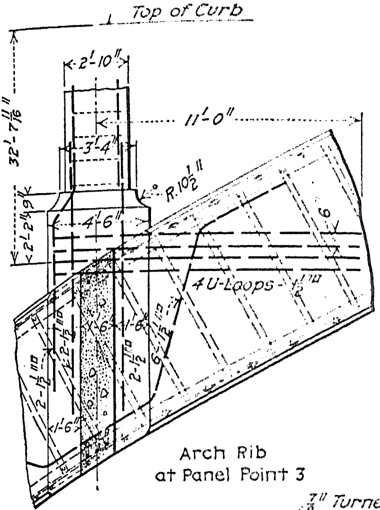
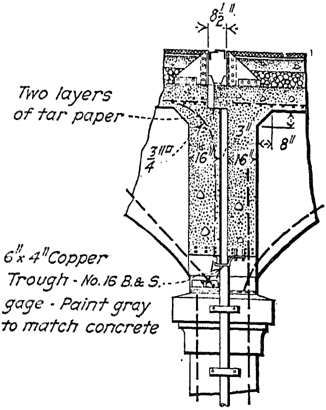


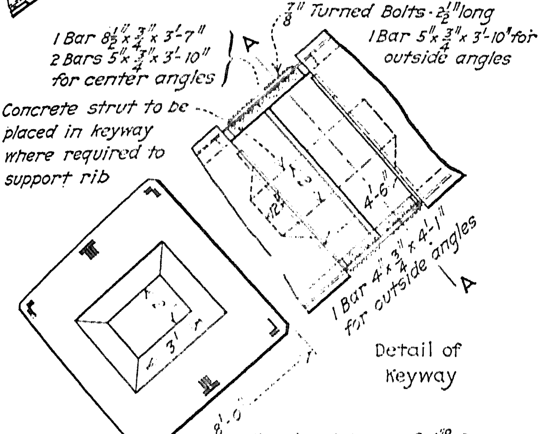
PLATE XXVIII



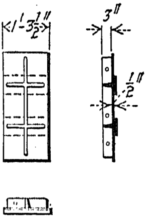
Arch Rib
at Panel Point 3


8 1/2"
Two layers of tar paper
3"
8"
3/4"
16"
16"
6"x 4" Copper Trough - No. 16 B.& S. gage - Paint gray to match concrete

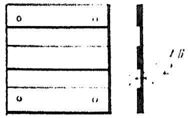
Section
showing
Double Floorbeam


7/8" Turned Bolts - 2 1/2" long
1 Bar 8 1/2" x 3/4" x 3'-7"
2 Bars 5" x 3/4" x 3'-10"
for center angles
A
1 Bar 5" x 3/4" x 3'-10" for outside angles
Concrete strut to be placed in keyway where required to support rib
12"
4'-6"
1 Bar 4" x 3/4" x 4'-1" for outside angles
A
3'
8'-0"

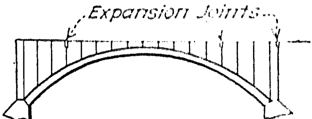
Detail of
Keyway



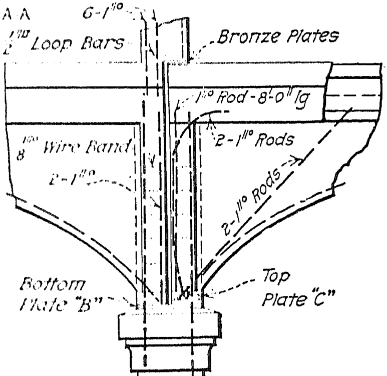
Top Plate "C"

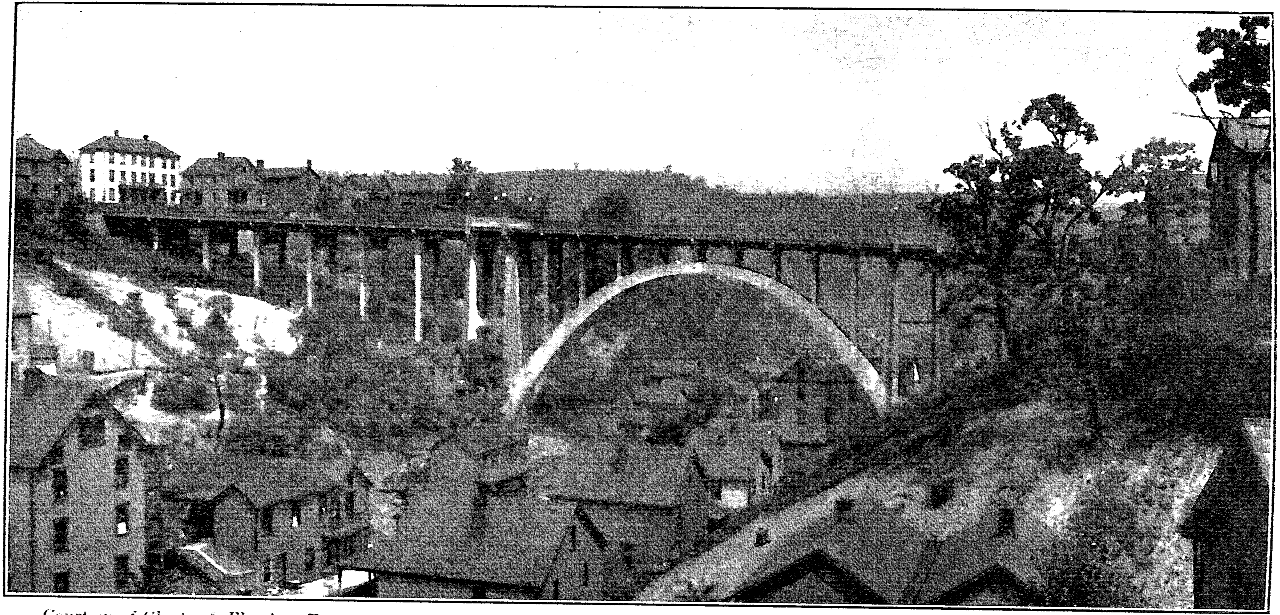


Bottom Plate "B"



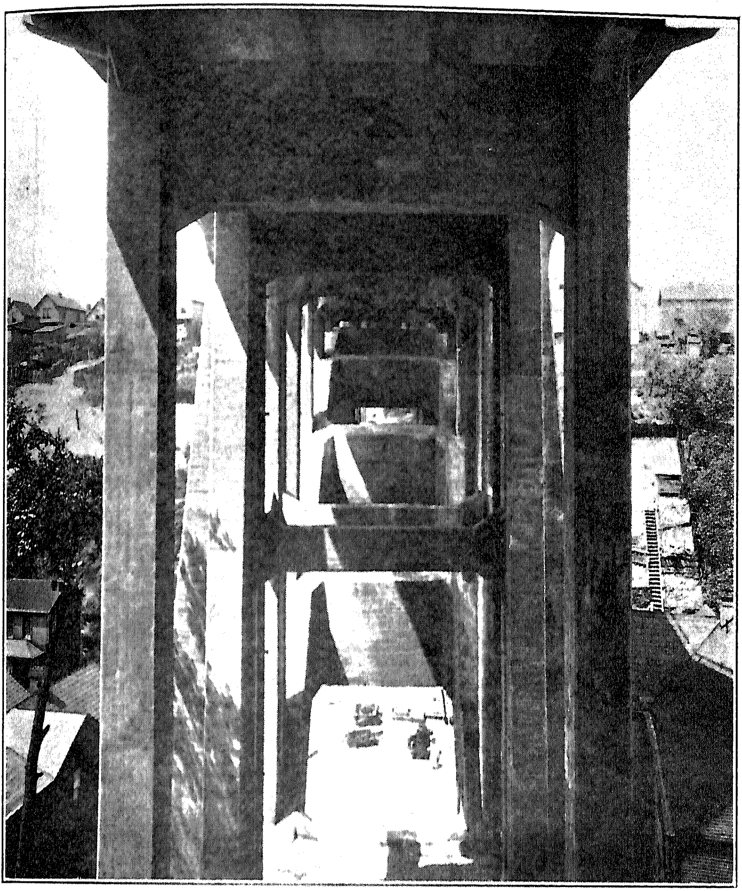
Section A-A





Courtesy of Chester & Fleming, Engineers, Pittsburgh.

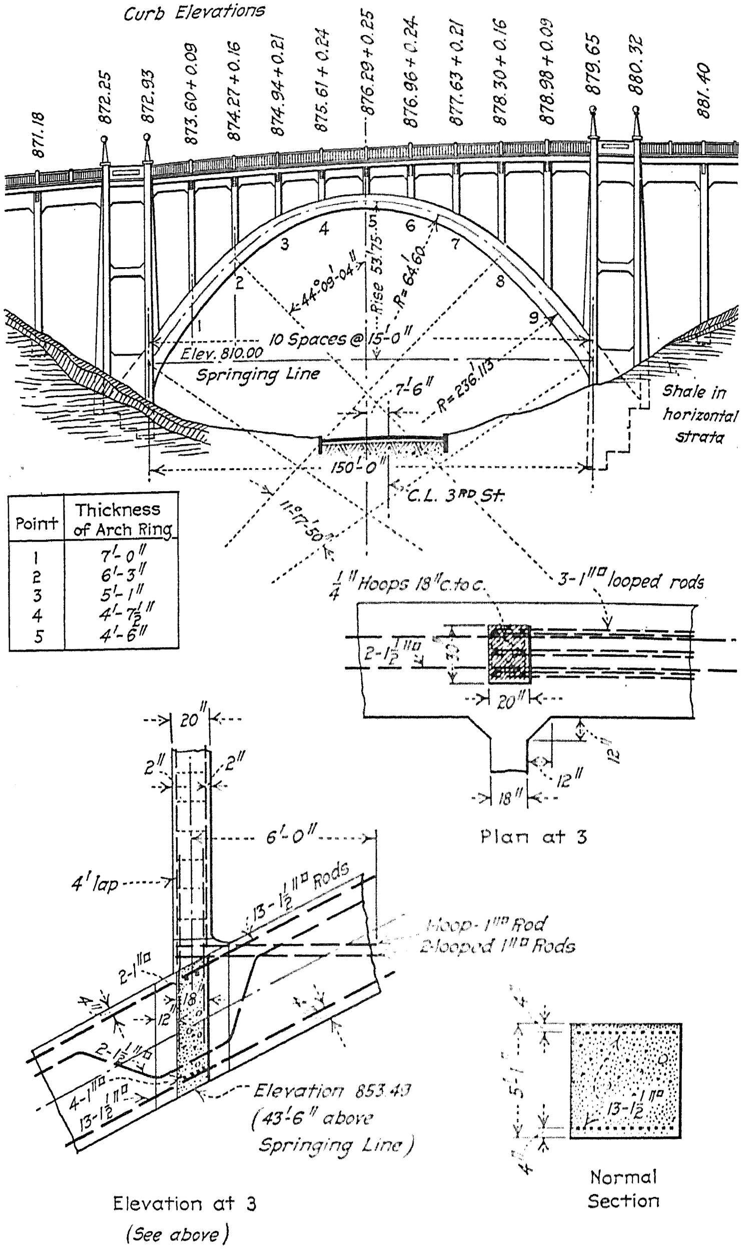
FIG. 227.—Monessen viaduct, Monessen, Pa.



Courtesy of Chester & Fleming, Engineers, Pittsburgh.

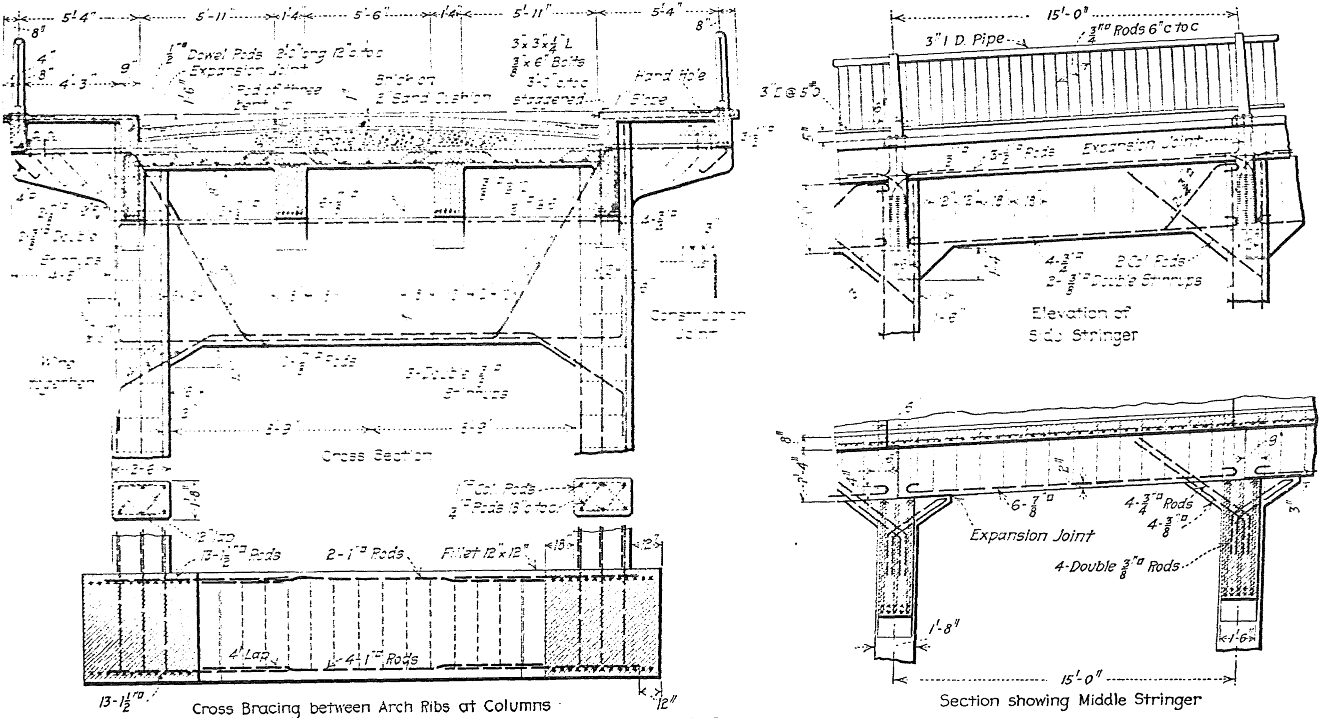
FIG. 228.—Monessen viaduct, Monessen, Pa.

PLATE XXIX



Point	Thickness of Arch Ring
1	7′-0″
2	6′-3″
3	5′-1″
4	4′-7½″
5	4′-6″

Monessen viaduct



Cross Section

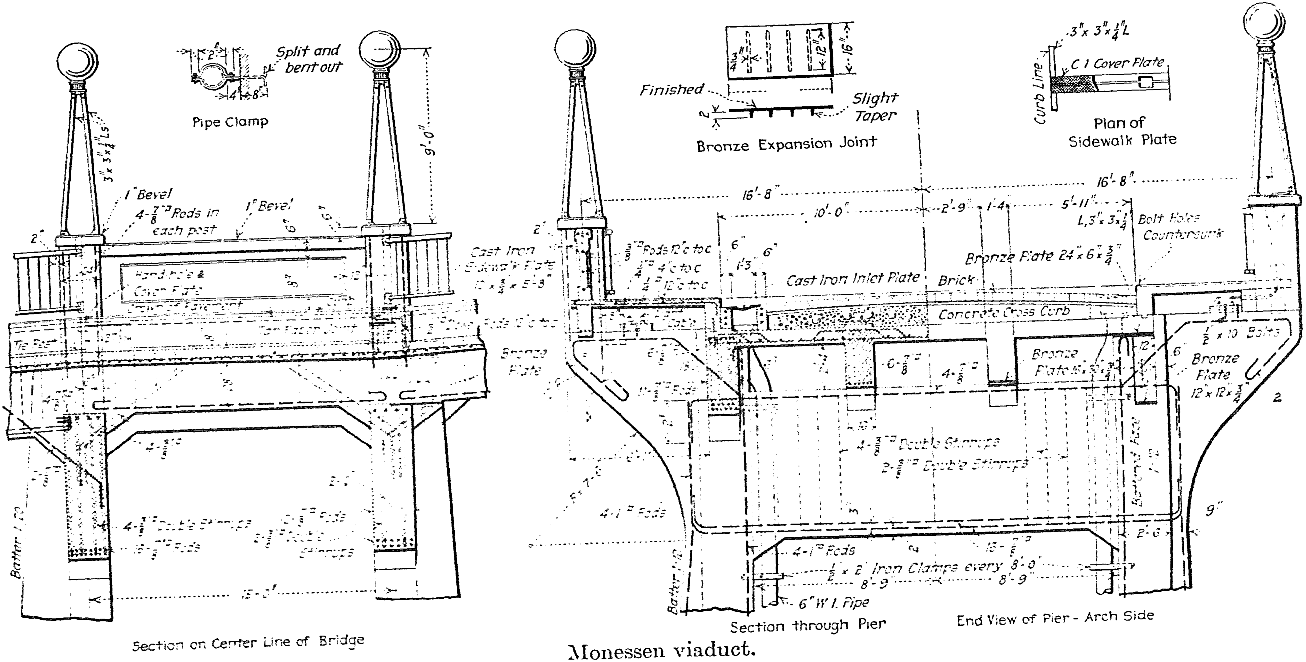
Cross Bracing between Arch Ribs at Columns

Elevation of Side Stringer

Section showing Middle Stringer

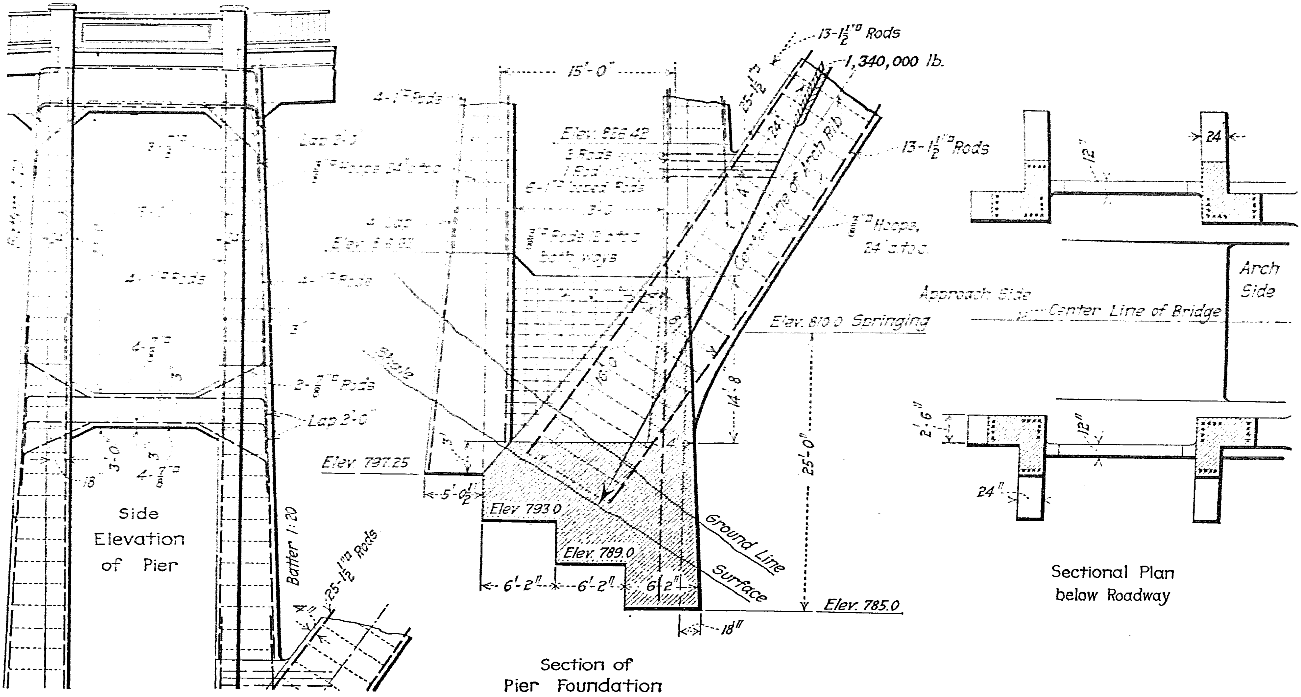
Monessen viaduct.

PLATE XXXI



Monessen viaduct.

PLATE XXXII



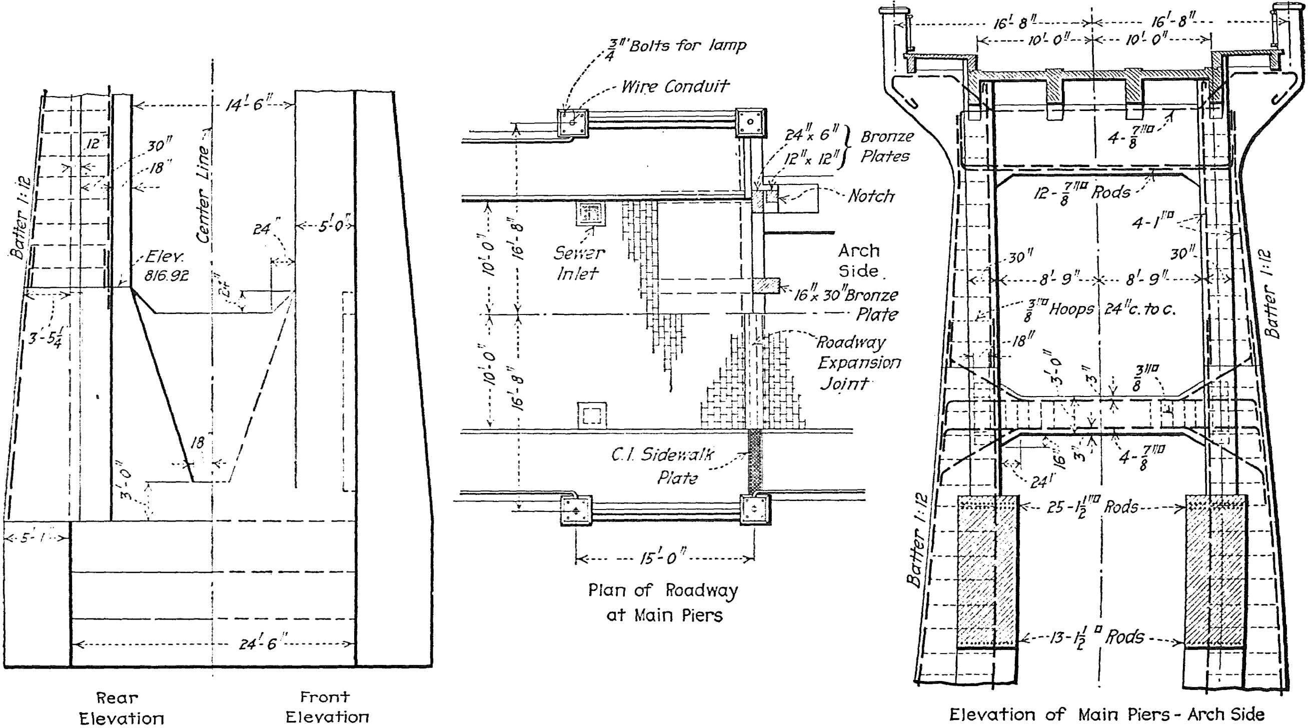
Side Elevation of Pier

Section of Pier Foundation

Sectional Plan below Roadway

Monessen viaduct.

PLATE XXXIII



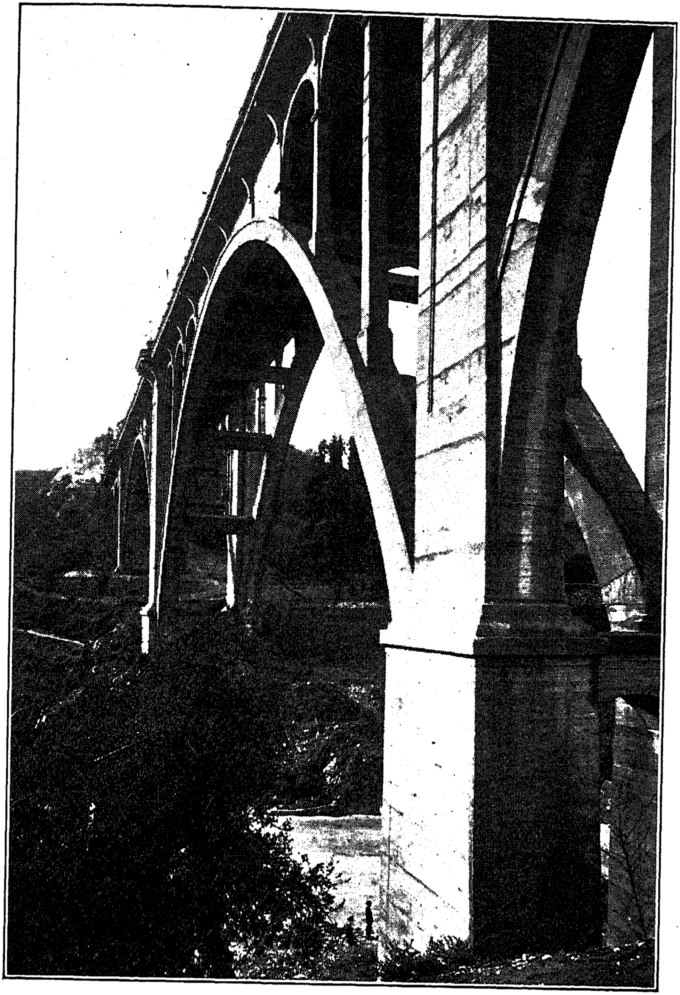
Rear Elevation

Front Elevation

Plan of Roadway at Main Piers

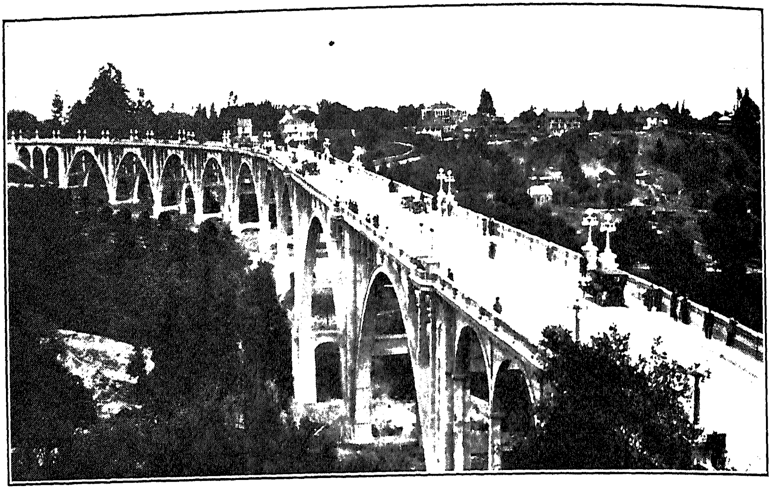
Elevation of Main Piers - Arch Side

Monessen viaduct.



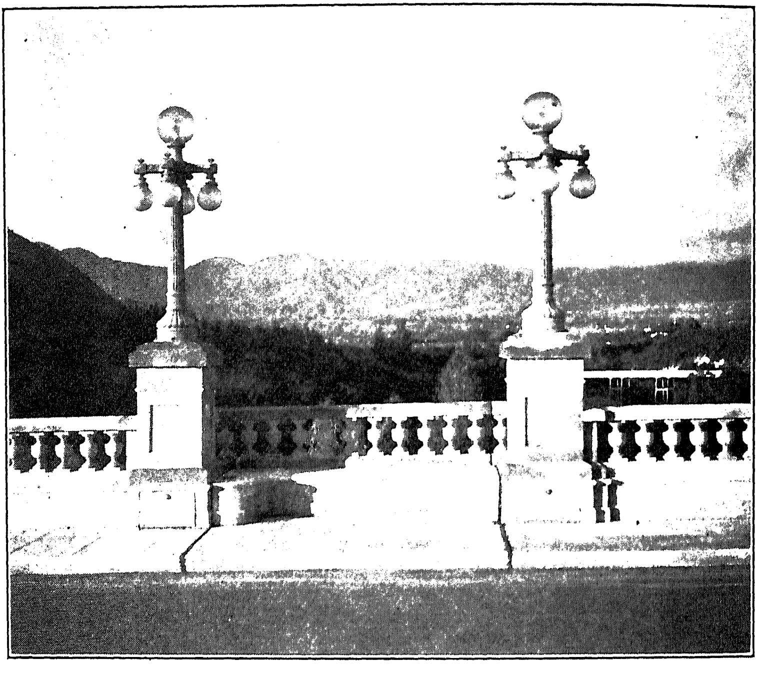
Courtesy of Concrete-Cement Age.

FIG. 229.—Bridge over the Arroyo Seco, Pasadena, Cal.



Courtesy of Mr. Charles A. Byers, Los Angeles, Cal.

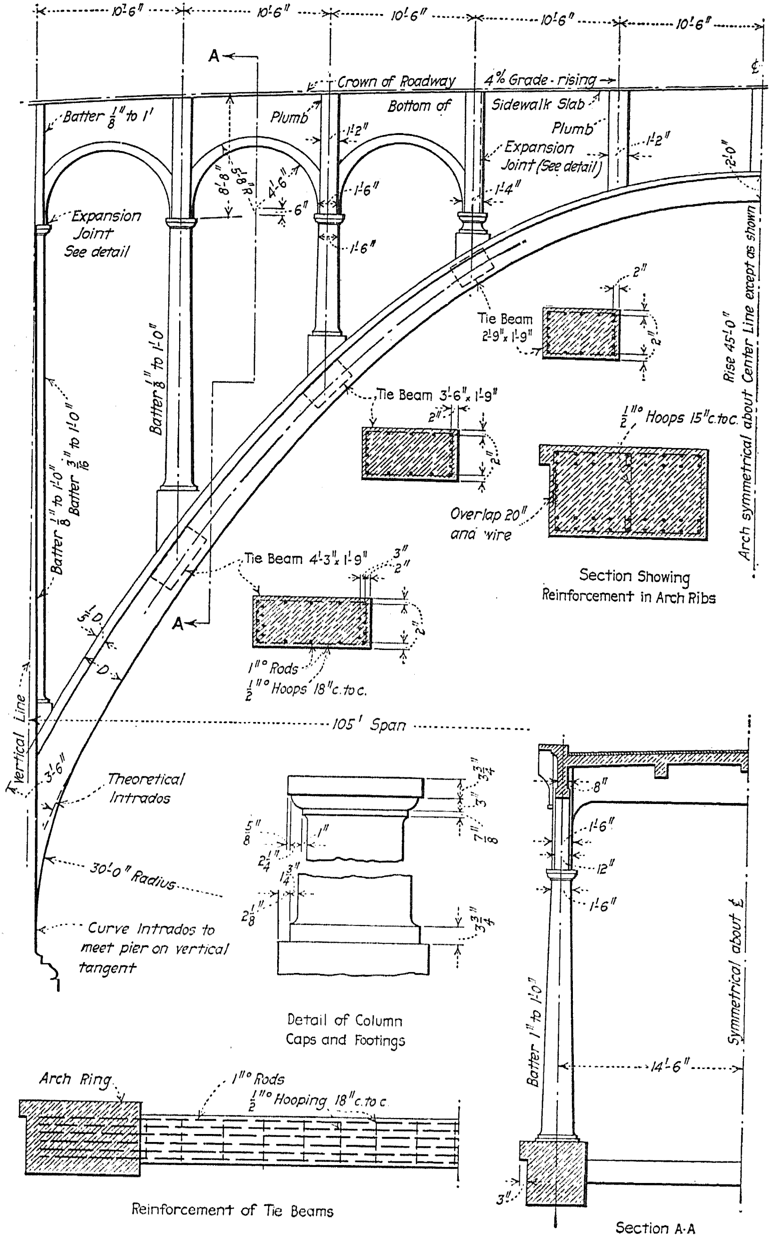
FIG. 230.—General view of bridge over the Arroyo Seco, Pasadena, Cal.



Courtesy of Mr. Charles A. Byers, Los Angeles, Cal.

FIG. 231.—Refuge bay on bridge over the Arroyo Seco, Pasadena, Cal.

PLATE XXXIV



Section Showing Reinforcement in Arch Ribs

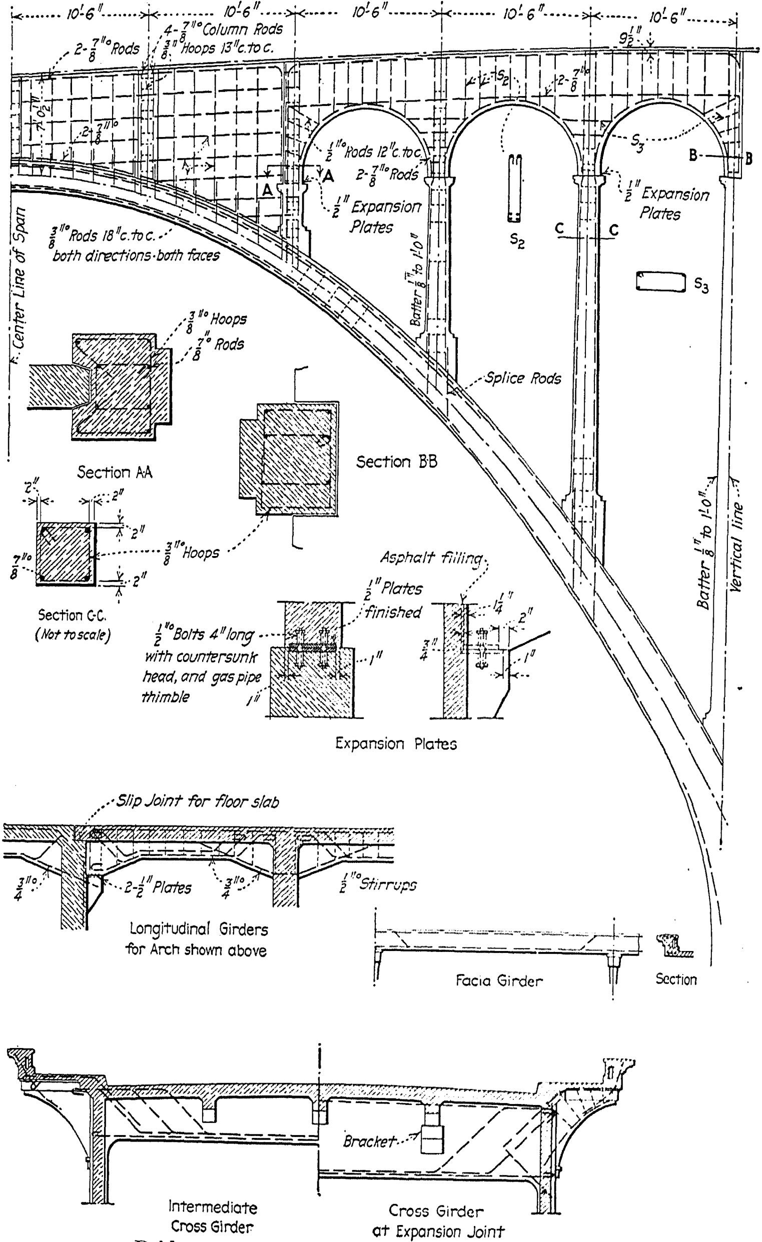
Detail of Column Caps and Footings

Reinforcement of Tie Beams

Section A-A

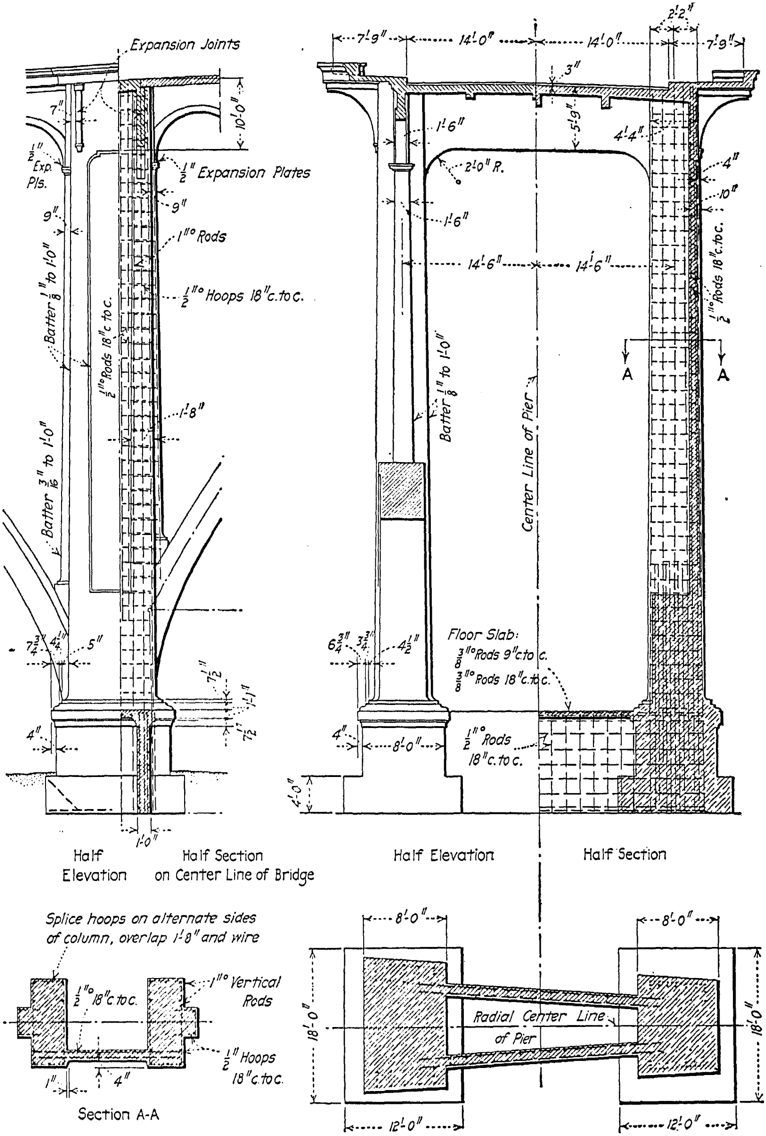
Bridge over the Arroyo Seco, Pasadena, Cal.

PLATE XXXV



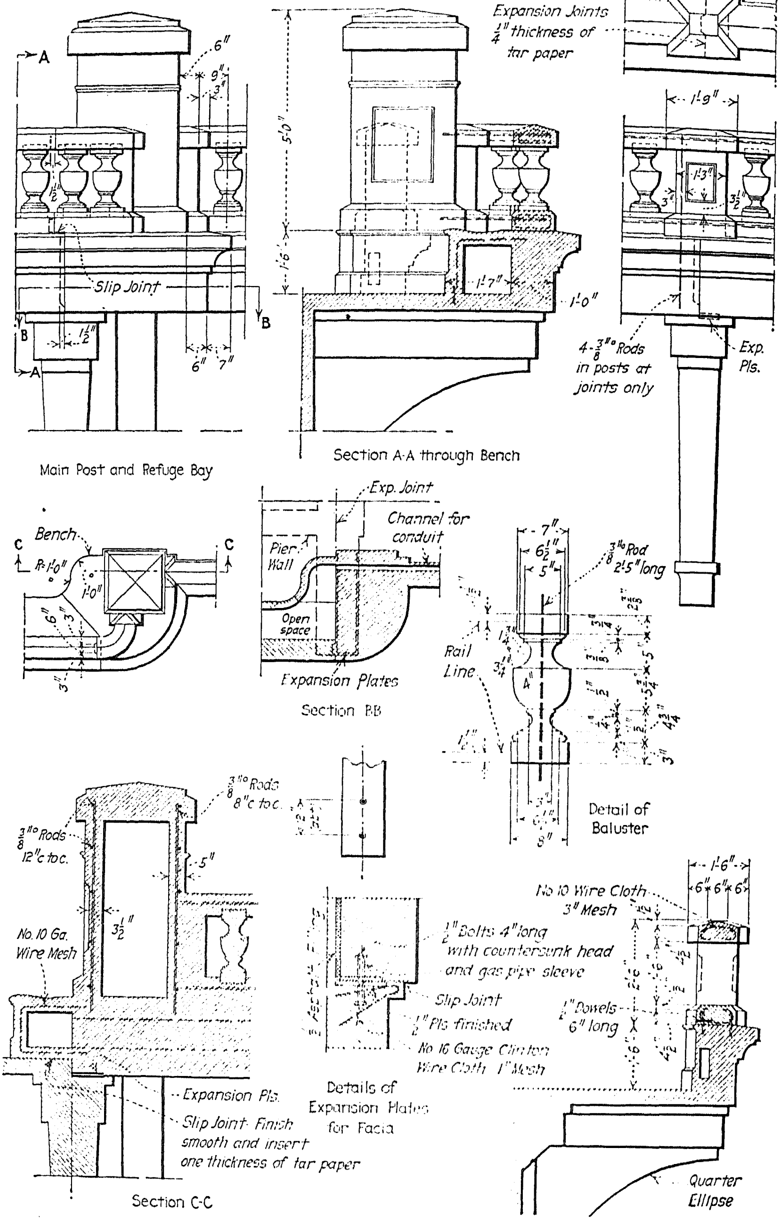
Bridge over the Arroyo Seco, Pasadena, Cal.

PLATE XXXVI


Expansion Joints
10'-0"
7"
$\frac{1}{2}$" Exp. Pls.
$\frac{1}{2}$" Expansion Plates
9"
9"
1"° Rods
Batter $\frac{1}{8}$" to 1'-0"
$\frac{1}{2}$"° Rods 18" c. to c.
$\frac{1}{2}$"° Hoops 18" c. to c.
1'-8"
Batter $\frac{3}{16}$" to 1'-0"
$7\frac{3}{4}$"
$4\frac{1}{4}$"
5"
$7\frac{1}{2}$"
1'-1"
$7\frac{1}{2}$"
4"
1'-0"
Half Elevation
Half Section on Center Line of Bridge
7'-9"
14'-0"
14'-0"
7'-9"
2'-2"
3"
1'-6"
5'-9"
4'-4"
2'-0" R.
4"
10"
1'-6"
14'-6"
14'-6"
$\frac{1}{2}$"° Rods 18" c. to c.
A
A
Batter $\frac{1}{8}$" to 1'-0"
Center Line of Pier
Floor Slab:
$\frac{3}{8}$"° Rods 9" c. to c.
$\frac{3}{8}$"° Rods 18" c. to c.
$6\frac{3}{4}$"
$3\frac{3}{4}$"
$4\frac{1}{2}$"
4"
8'-0"
$\frac{1}{2}$"° Rods 18" c. to c.
4'-0"
Half Elevation
Half Section
Splice hoops on alternate sides of column, overlap 1'-8" and wire
$\frac{1}{2}$"° 18" c. to c.
1"° Vertical Rods
$\frac{1}{2}$" Hoops 18" c. to c.
1"
4"
Section A-A
8'-0"
8'-0"
18'-0"
18'-0"
Radial Center Line of Pier
12'-0"
12'-0"

Bridge over the Arroyo Seco, Pasadena, Cal.

PLATE XXXVII



Main Post and Refuge Bay

Section A-A through Bench

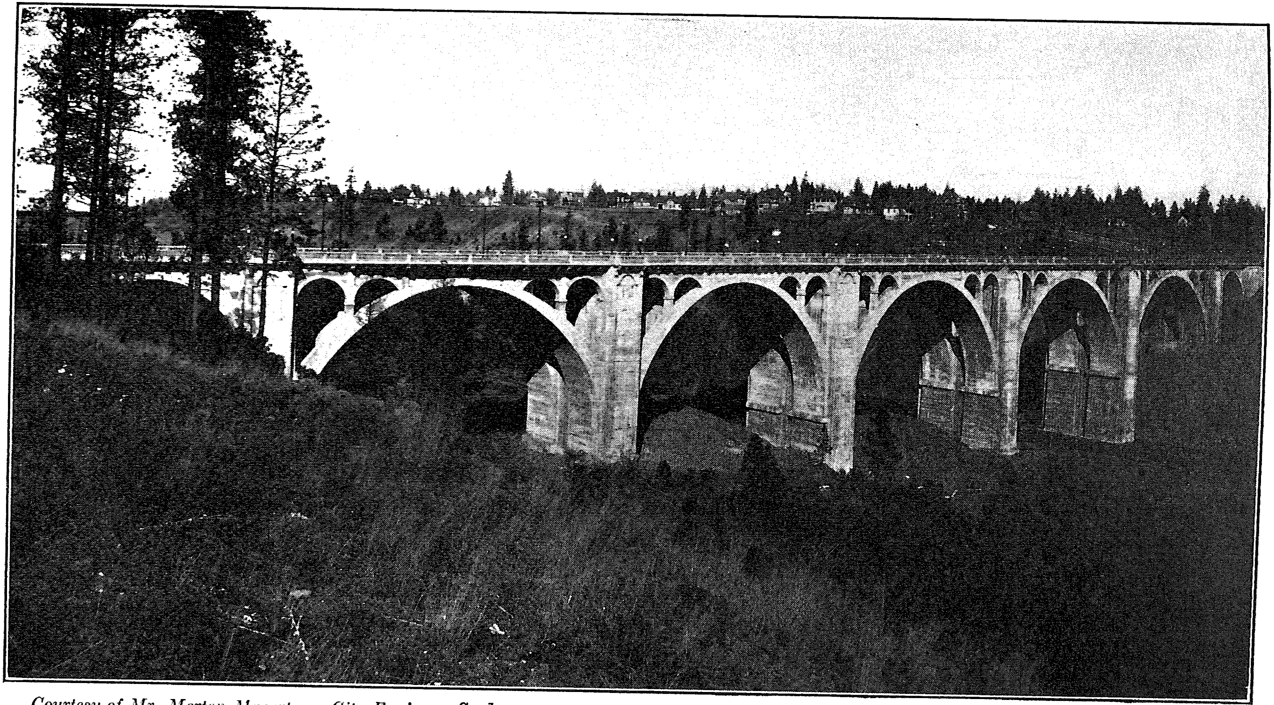
Section BB

Detail of Baluster

Section C-C

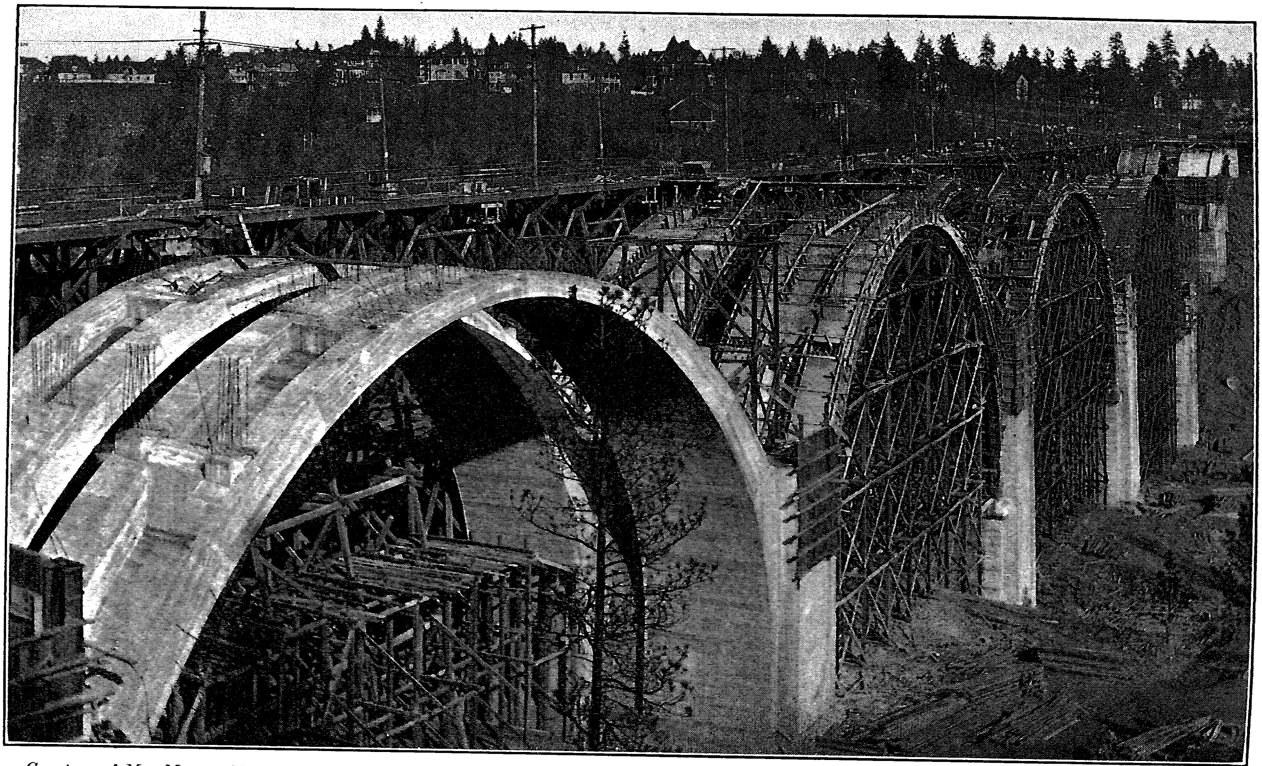
Details of Expansion Plates for Facia

Bridge over the Arroyo Seco, Pasadena, Cal.



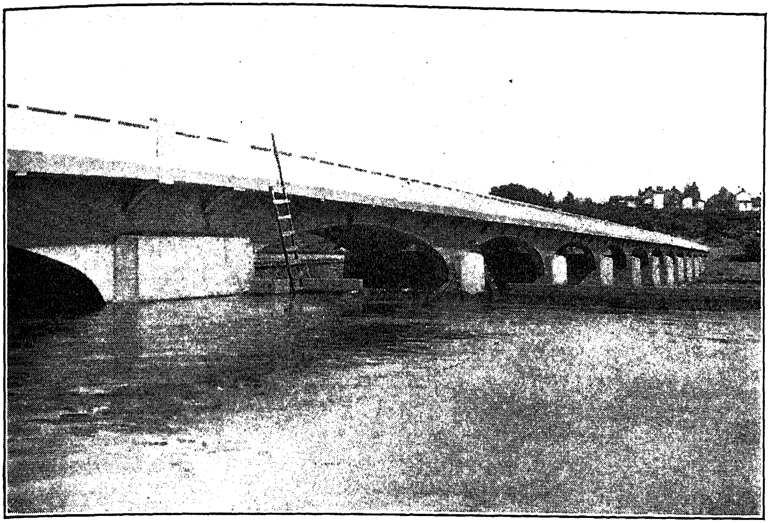
Courtesy of Mr. Morton Macartney, City Engineer, Spokane.

FIG. 232.—Latah Creek bridge, Spokane, Wash.



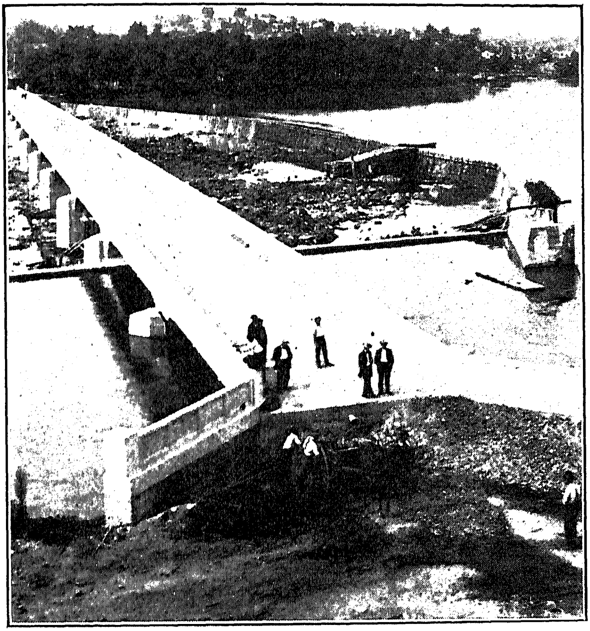
Courtesy of Mr. Morton Macartney, City Engineer, Spokane.

FIG. 233.—Construction view of Latah Creek bridge, Spokane, Wash.



Courtesy of Mr. Wilbur J. Watson, Consulting Engineer, Cleveland.

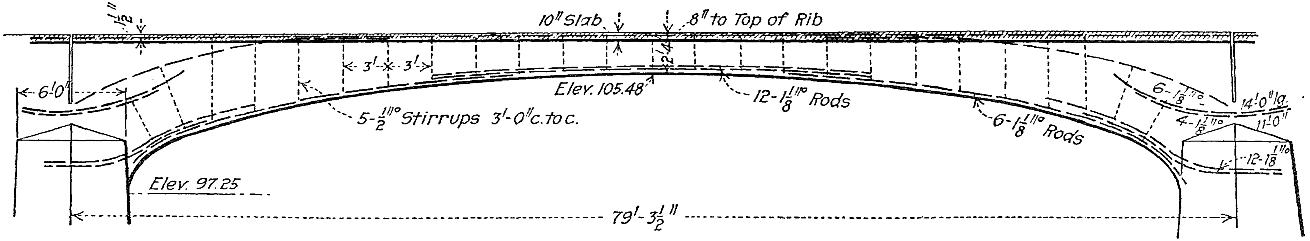
FIG. 234.—Highway bridge at Danville, Va.



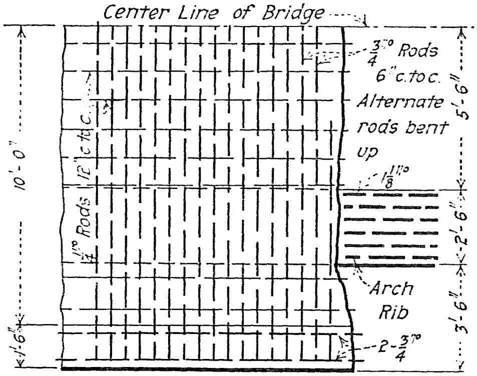
Courtesy of Mr. Wilbur J. Watson, Consulting Engineer, Cleveland.

FIG. 235.—General view of highway bridge at Danville, Va.

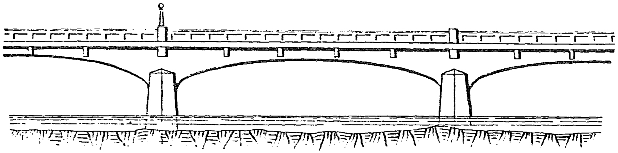
PLATE XXXVIII


$\frac{1}{2}''$
10" Slab
8" to Top of Rib
2'-4"
6'-0"
3'-3'
Elev. 105.48
12-$1\frac{1}{8}''^{\circ}$ Rods
5-$\frac{1}{2}''^{\circ}$ Stirrups 3'-0" c. to c.
6-$1\frac{1}{8}''^{\circ}$ Rods
6-$1\frac{1}{8}''^{\circ}$
14'-0" lg.
4-$1\frac{1}{8}''^{\circ}$
11'-0"
12-$1\frac{1}{8}''^{\circ}$
Elev. 97.25
79'-$3\frac{1}{2}''$

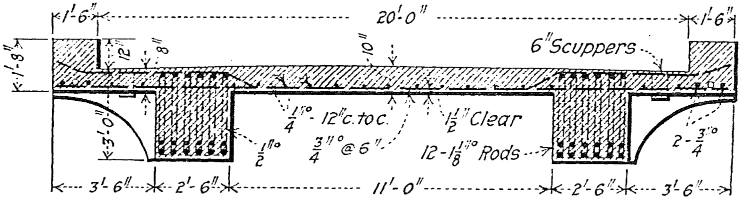
Longitudinal Section on Center Line of Bridge.



Part Plan of Floor Slab

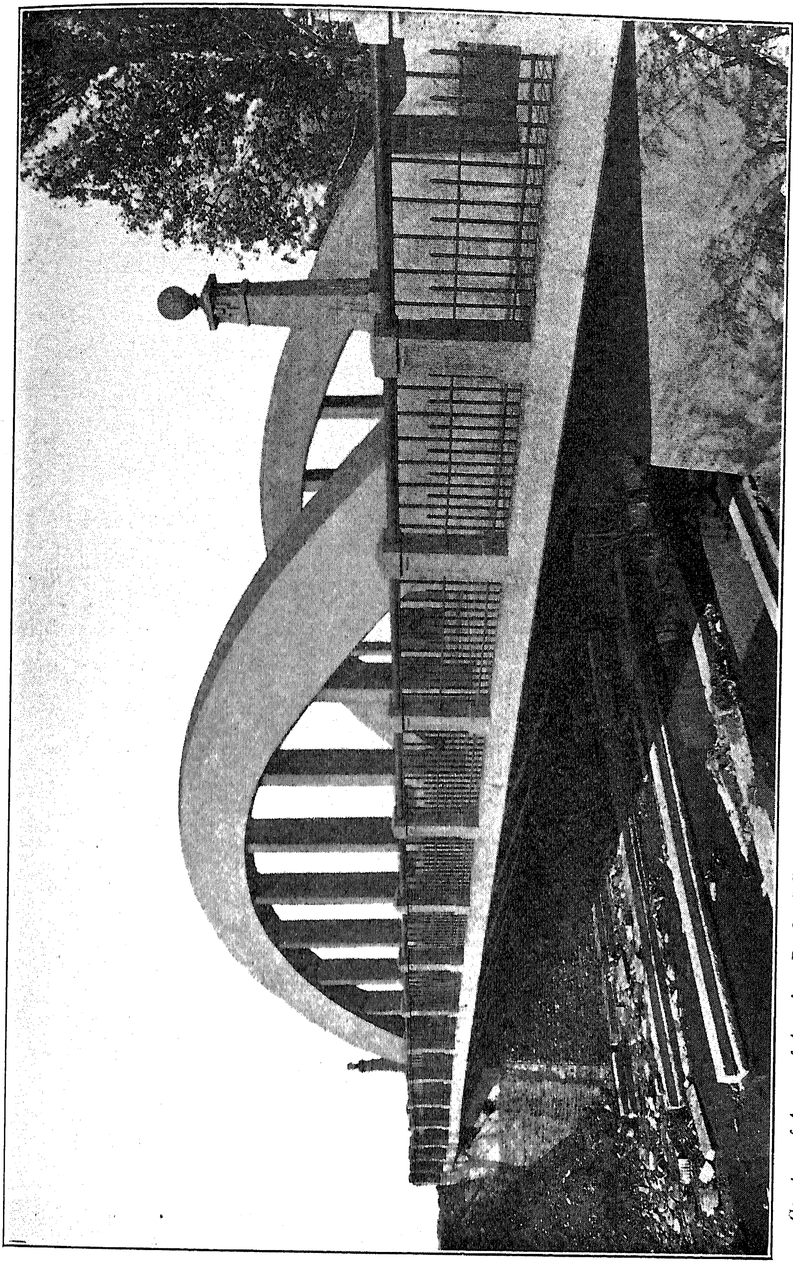


Elevation


1'-6"
20'-0"
1'-6"
1'-8"
12"
8"
10"
6" Scuppers
3'-0"
$\frac{1}{4}''^{\circ}$-12" c. to c.
$\frac{1}{2}''$ Clear
$\frac{1}{2}''^{\circ}$
$\frac{3}{4}''^{\circ}$ @ 6"
12-$1\frac{1}{8}''^{\circ}$ Rods
2-$\frac{3}{4}''^{\circ}$
3'-6"
2'-6"
11'-0"
2'-6"
3'-6"

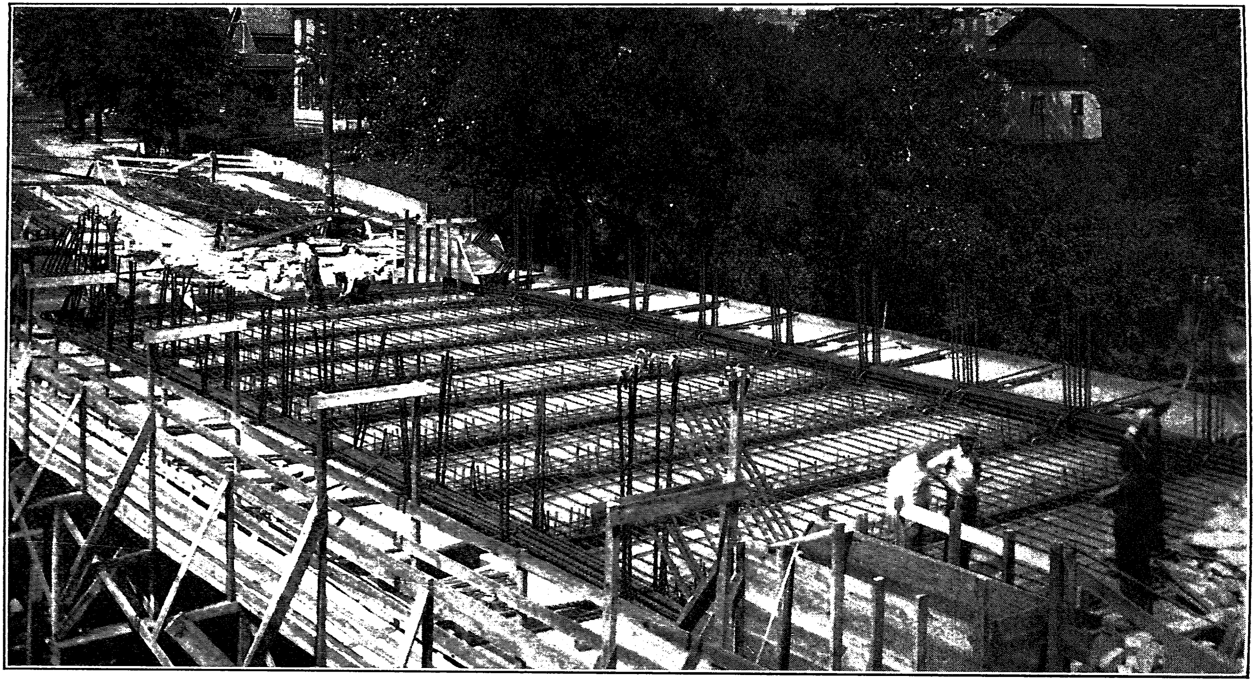
Section at Crown

Highway bridge at Danville, Va.



Courtesy of Assoc. of American Portland Cement Mfgrs.

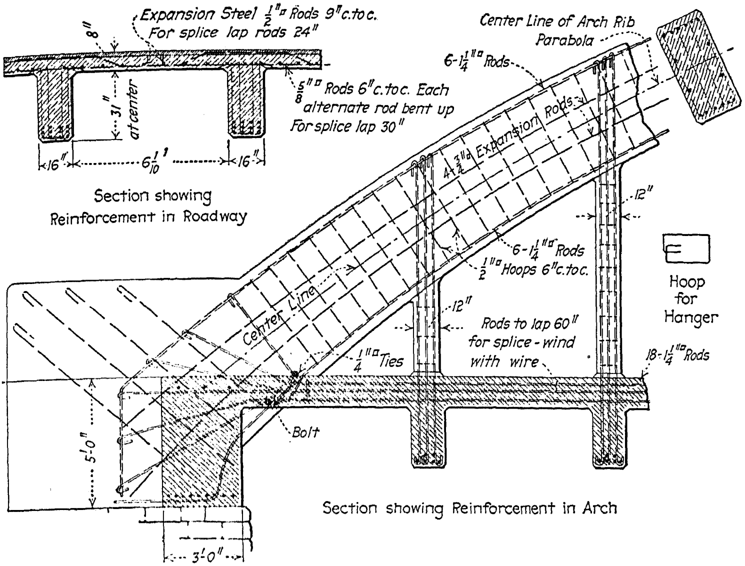
FIG. 236.—Benson Street bridge over Mill Creek at Lockland, Ohio.



Courtesy of Mr. E. A. Gast, Deputy County Surveyor, Hamilton County, Ohio.

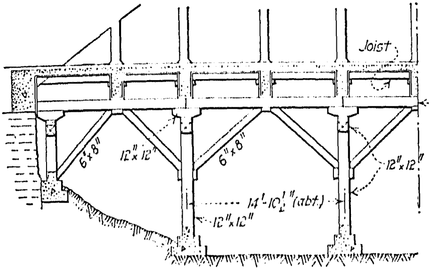
FIG. 237.—Construction view, Benson Street bridge over Mill Creek at Lockland, Ohio.

PLATE XXXIX

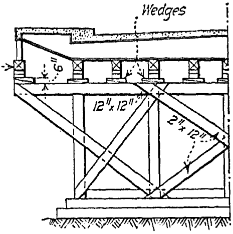


Section showing Reinforcement in Roadway

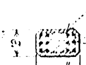
Section showing Reinforcement in Arch



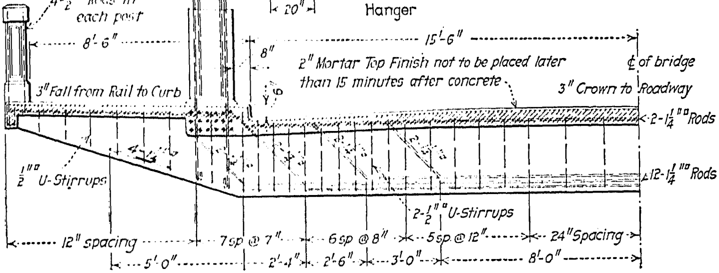
Falsework



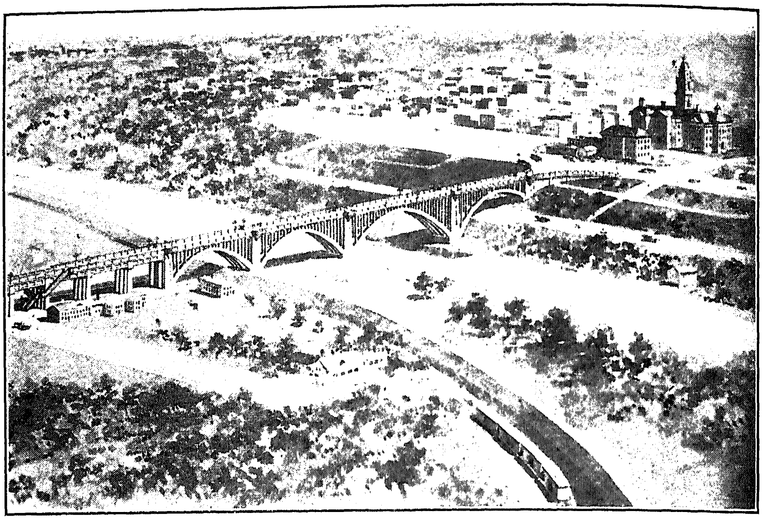
Half Bent



Section of Hanger



Benson Street bridge over Mill Creek at Lockland, Ohio.



Courtesy of Mr. S. W. Bowen of Brenneke and Fay, Consulting Engineers, St. Louis.

FIG. 238.—Main Street viaduct, Fort Worth, Texas, from an artist's perspective.

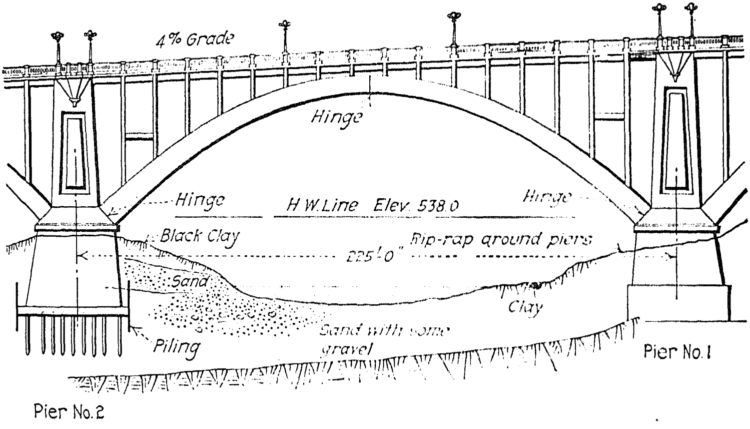
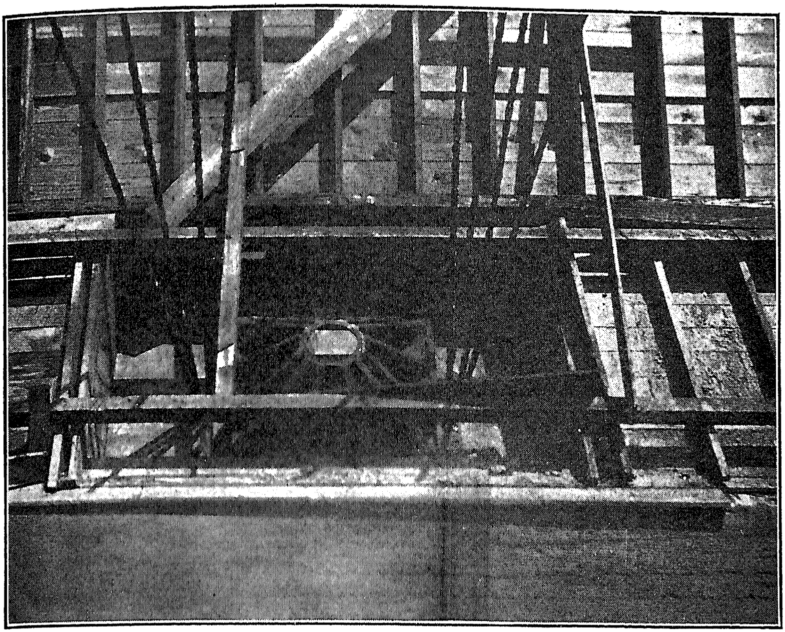
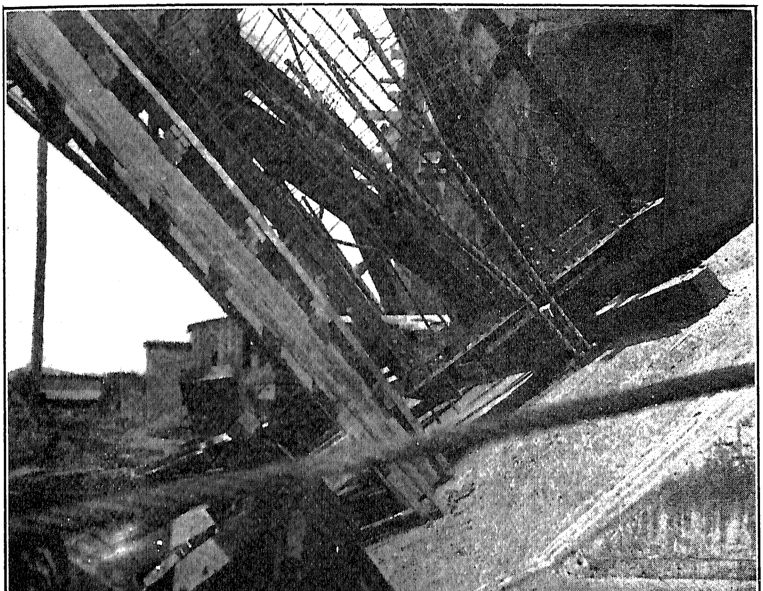


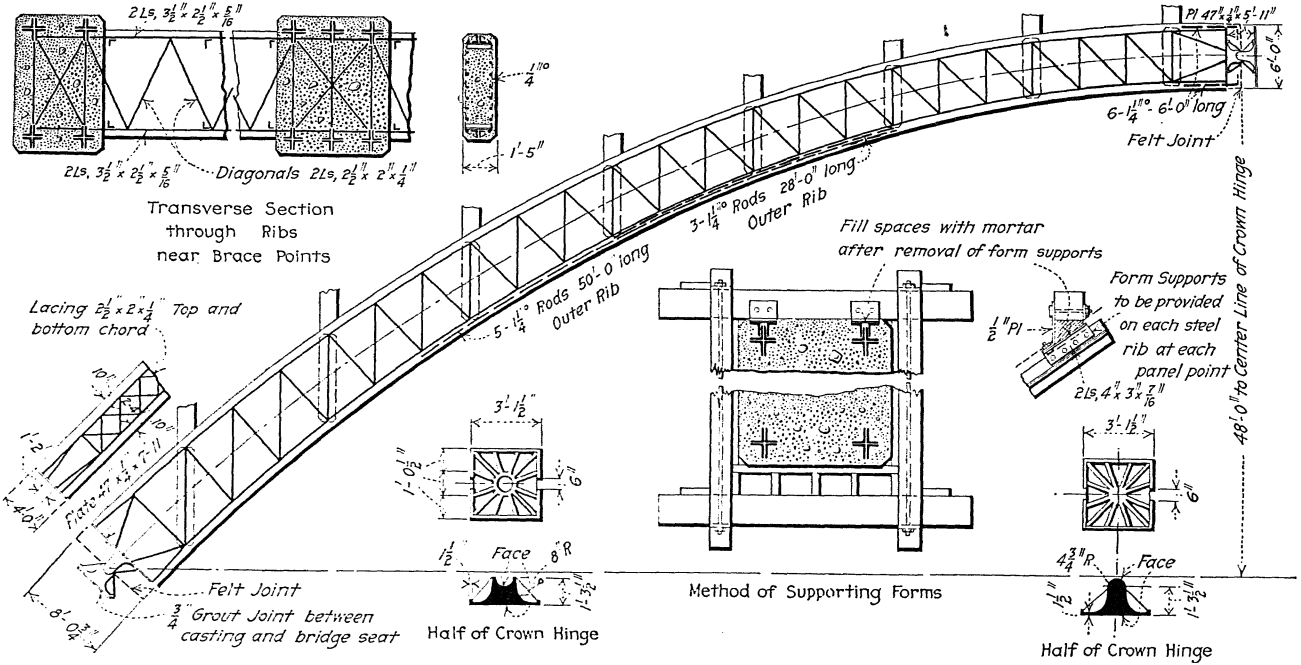
FIG. 239.—River span, Main Street viaduct, Fort Worth, Texas.



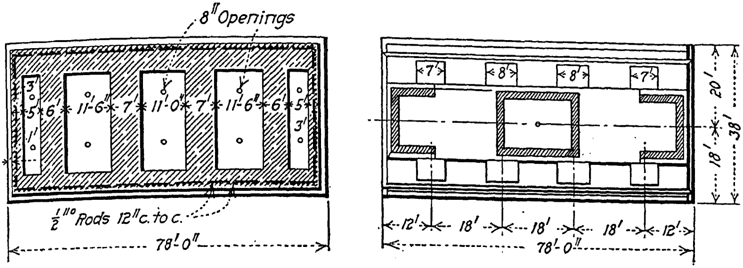
Courtesy of Mr. S. W. Bowen, St. Louis.

FIG. 240.—Pier casting of center rib, Main Street viaduct, Fort Worth, Texas. Casting ready to be concreted in.



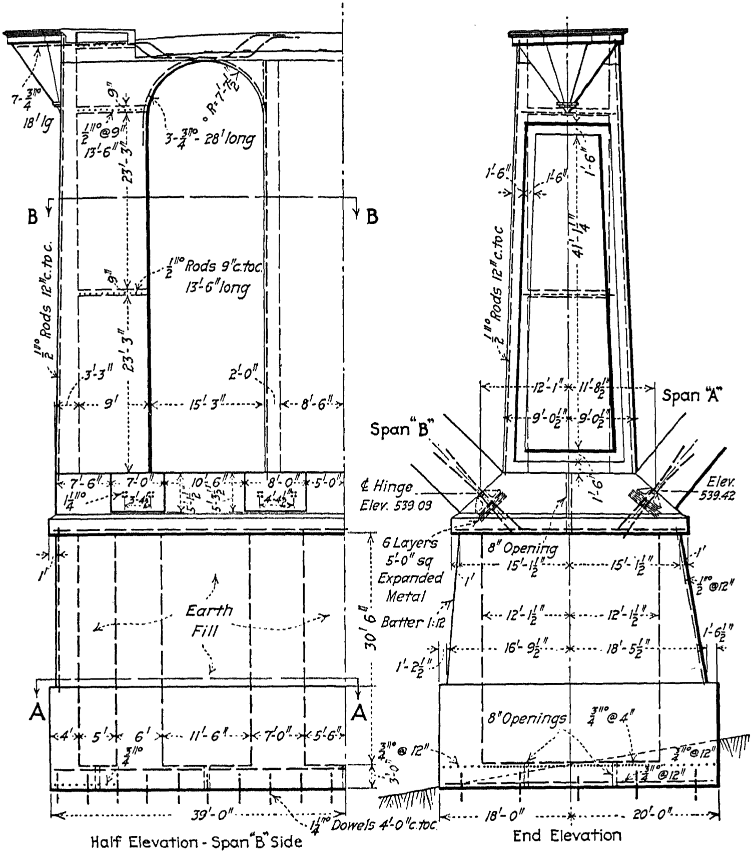

2Ls, 3½" × 2½" × 5/16"
2Ls, 3½" × 2½" × 5/16"
Diagonals 2Ls, 2½" × 2" × ¼"
Transverse Section through Ribs near Brace Points
¼"⌀
1'-5"
Lacing 2½" × 2" × ¼" Top and bottom chord
10"
10"
1'-2
4'-0"
Plate 47" × ¼" × 7'-11
8'-0¾"
Felt Joint
¾" Grout Joint between casting and bridge seat
5-1¼"⌀ Rods 50'-0 long Outer Rib
3-1¼"⌀ Rods 28'-0" long Outer Rib
Pl 47" × ¼" × 5'-11"
6'-0"
6-1¼"⌀-6'-0" long
Felt Joint
48'-0" to Center Line of Crown Hinge
Fill spaces with mortar after removal of form supports
Form Supports to be provided on each steel rib at each panel point
½" Pl
2Ls, 4" × 3" × 7/16"
3'-1½"
1'-0½"
6"
½"
Face
8"R
1'-3½"
Half of Crown Hinge
Method of Supporting Forms
3'-1½"
6"
4¾"R
Face
½"
1'-3½"
Half of Crown Hinge

Main Street viaduct, Fort Worth, Texas.



Section A-A

Section B-B



Half Elevation - Span "B" Side

End Elevation

Main Street viaduct, Fort Worth, Texas.

PART II

SLAB AND GIRDER BRIDGES

Since the methods of designing slabs, beams, and girders have been explained at length in Volumes I and II, no attempt will be made in the following chapters to treat of these methods in detail. Many things must be considered, however, in designing bridges of this class, aside from the proportioning of simple and continuous beams, and such matters will be given due consideration.

The erection of forms and other operations in slab-and-girder bridge construction are essentially the same for ordinary conditions as the corresponding operations in the construction of buildings. On this account, constructional methods will be referred to only incidentally under this heading.

The loadings to use in design are, of course, the same as for the floors in arch bridges of open-spandrel construction (see Art. 7). In fact, it should be noted that the framed structure which is supported by a ribbed arch is virtually a trestle form of girder bridge and the loadings and general design are identical.

Impact may properly be neglected in arch-ring analysis but becomes important in bridge-floor design. An increase of 25 per cent is usually made in the live load or live-load stresses for highway bridges and 50 per cent in those for railroad bridges.

From the standpoint of economy, slab bridges should in general be limited in span length to about 25 ft., and ordinary girder bridges to about 50 ft.

CHAPTER XIV

SLAB BRIDGES

65. Slabs Under Concentrated Loading.—Recent tests made on simply-supported slabs at the University of Illinois and at the United States Office of Public Roads gave results in sufficient agreement to permit of simple rules being formulated for estimating the effective width of slabs supporting concentrated loads. In one series of tests, $\frac{1}{2}$-in. plain round rods were used and the loads were applied centrally over a bearing area 6 in. in diameter. In another series somewhat similar loads were applied at the one-third points of the span. In a discussion of the tests above mentioned before the American Society for Testing Materials, June, 1913, Mr. W. A. Slater, first assistant of the Engineering Experiment Station, University of Illinois, recommended that where the total width of slab is greater than twice the span, the effective width e (Fig. 242) be assumed as

$$e = \frac{4}{3}x + d$$

where x is the distance from the concentrated load to the nearest support and d is the width at right angles to the span over which the load is applied. Mr. Slater pointed out that the tests so far made showed the effective width to be but little influenced by the depth of the slab or by the percentage of longitudinal reinforcement. He stated, however, that he would limit the latter to 1 per cent. "because of the possibility that in a beam with a large amount of longitudinal reinforcement and a relatively small depth, failure may be caused by transverse tension in the concrete and not by longitudinal steel stress."

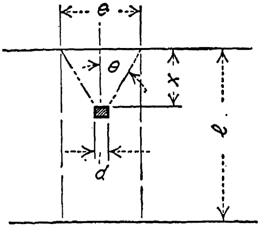


FIG. 242.

Tests were made on beams having various percentages of transverse reinforcement, but results did not show conclusively that such reinforcement is economical when used for the purpose of resisting transverse bending stress. Additional tests are needed to determine what increase may be made in the effective width for different percentages of transverse steel. At least a small amount should be used in every case for the purpose of distributing deformation due to variations in temperature.

The formula given above for finding effective width shows that this width is very nearly a constant proportion of the span; and consequently it follows that the depth of slab required to support a given concentrated load is approximately the same for all spans. Of course, the depth referred to here is only that necessary to support the live load. Considering both dead and live load, an increase in span length would obviously cause considerable increase in depth or in percentage of reinforcement.

The above discussion refers to a total width of slab greater than twice the span. For a slab whose total width is less than this, the effective width may be found from Fig. 243 which shows the ratio of the effective width of the span as determined from the measured steel stresses in the University of Illinois tests.

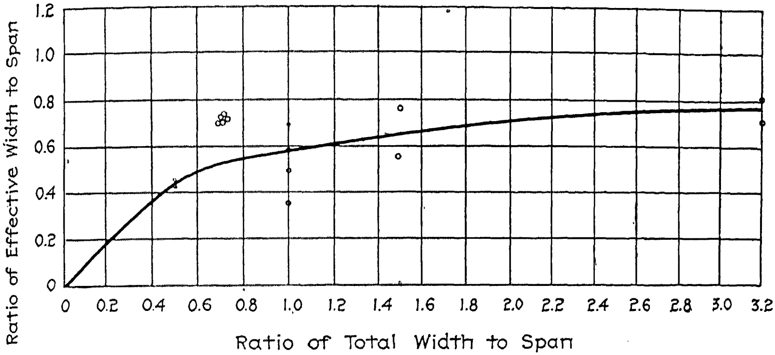


Fig. 243.

Mr. C. R. Young, Assistant Professor of Structural Engineering in the University of Toronto, in an article in *Engineering News*, issue of July 30, 1914, calls attention to the fact that the loads caused by wheels or rollers on highway bridges are applied over considerably elongated areas. In view of the tests referred to above, he presents a practical method of estimating the effective width of slabs supporting the type of concentrated loads found in practice. The following is taken from the article mentioned:

"Let a concentrated load, bearing on an area of length *c* and width *d*, be applied in any general position to a slab having a total width *b* not less than twice the span length l, as shown in Fig. 244. If any small element of this load, as mn, distant x from the nearer support, be considered, it is directly evident from the experimental investigations cited above, that such part of it as is delivered to the nearer support may be regarded as wholly transferred to that support within lines drawn through the end points of the element making angles θ with the direction of the span of the slab, the tangent of which angle is between 0.67 and 0.80. For brevity, the angle between these two bounding lines, that is 2θ, may be called the *effective angle.* The reaction at the near support may thus be regarded as uniformly distributed over a width of $r = 2x \tan \theta + d$, or if $\tan \theta$ be assumed as 0.67, a conservative value in the light of experimental results, this width becomes $1.33x + d$.

"Although there is no direct experimental evidence that the effective angle to the farther support is the same as to the nearer, careful consideration of the matter leads to the conclusion that it is. If the farther support were moved in so that the element of load, mn, became central, the effective angles to the two supports would be equal. There would then be applied to the shifted support, along a length that may be called the effective reaction-width of the reduced span, a uniformly-distributed reaction which, when the first support was replaced in its original position, would become a concentrated load on the original span. The virtual load would be delivered to the support nearest to it, but farthest from the original eccentric element of load mn within the same effective angle as would apply to the nearer support in any case. It thus appears that the effective angles from any element of loading to the two supports are equal.

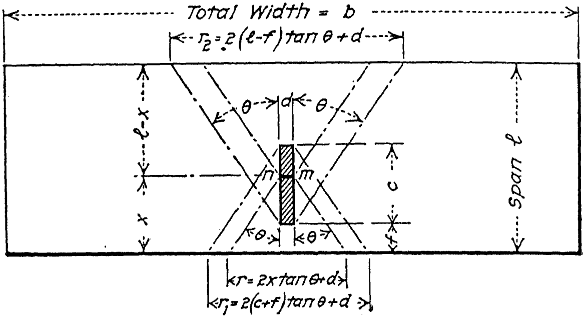


FIG. 244.

"When the selected element of loading is not at the center of the span and the reactions at the two supports are thus not distributed over the same effective widths, the question arises as to what should be the effective width of the slab for the calculation of resisting moment. It is probable that the average of the two reaction-widths would be the correct width to assume for this purpose, and in the absence of direct experimental evidence on the matter, this assumption is recommended.

"Considering as a whole the concentrated load shown in Fig. 244 it follows that the part of it delivered to either support may be regarded as wholly transferred to that support within lines drawn through the remotest corners of the loaded area and making angles with the direction of the span of the slab equal to θ, one-half of the effective angle. If f be the distance of that edge of the loaded area nearest a support, from the support, the effective reaction width, r_1, for the nearer support would be

$$r_1 = 2(c + f)\ tan\ \theta + d$$

and for the farther support it would be,

$$r_2 = 2(l - f)\ tan\ \theta + d$$

As in the case of an element of loading, the effective width of the slab for the computation of moment of resistance may be taken as the average of the reaction widths r_1 and r_2.

"Where the concentrated load is a moving one, the critical position will be at the center of the span. The effective reaction-widths are then equal to each other and the effective width of the slab for the computation of moment of resistance is equal to either.

"Since loads arising from wheels or rollers are often applied over considerably elongated areas, the orientation of the load on the span is an important consideration. For such loads, the distance d is always small—in theory only the width of a line—but the distance c may, for rollers, be as much as 48 in., and is commonly from 12 to 20 in.

"The two extreme arrangements of the load, indicated in Fig. 245, will be compared. In Fig. 245(*a*), the axis of the machine or vehicle is at right angles to the direction of the span of the slab, while in Fig. 245(*b*), it is parallel to this direction. For the usual system of floor construction in a bridge, with the slabs supported on longitudinal stringers, the axis of the machine for the case shown in Fig. 245(*a*) would be parallel to the axis of the bridge and for the case indicated in Fig. 245(*b*), transverse to it. The latter is a contingency that must be considered in wide bridges. When the load is placed as shown in Fig. 245(*a*), it follows from what has already been said that the effective width of the slab e_1 is given by the expression

$$e_1 = (l + c)\ tan\ \theta + d$$

and when the arrangement is that of Fig. 245(*b*), the effective width is

$$e_2 = (l + d)\ tan\ \theta + c$$

Since c is ordinarily very much greater than d, e_2 is greater than e

and the location and orientation of a wheel load most seriously stressing a slab is as shown in Fig. 245(*a*) and *not* as shown in Fig. 245(*b*).

"For bearing areas differing in shape from those shown in Figs. 244 and 245, equivalent rectangles may be substituted.

"When the length *b* of the slab is less than twice the span length *l*, it becomes necessary to make some reduction in the effective width. The writer would suggest that until further experimental evidence is available the values of *tan θ* in any calculation of effective widths be assumed substantially in accordance with those given by Mr. Slater's graph, or as follows:

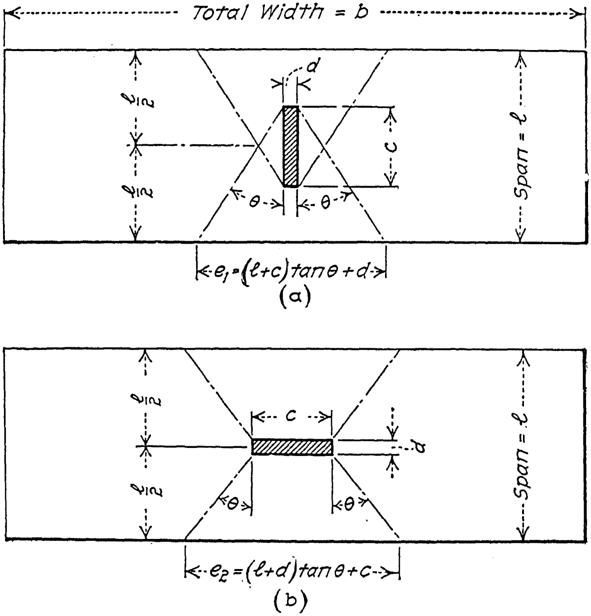


FIG. 245.

$\frac{b}{l}$	*tan θ*
0.4	0.35
0.5	0.42
0.6	0.48
0.8	0.54
1.0	0.59
1.2	0.62
1.4	0.64
1.6	0.65
1.8	0.66
2.0 and over	0.67

"Although no experimental investigation of the relative distributing effects of various types of slab reinforcement has been made, it is probable that a form involving diagonal strands or rods is most efficient. Maximum stiffness along lines radiating from the load within the effective angle and the provision of a large number of strands directly under the load would thus be secured. Upon these considerations the delivery of the load to the supports over a considerable effective width largely depends.

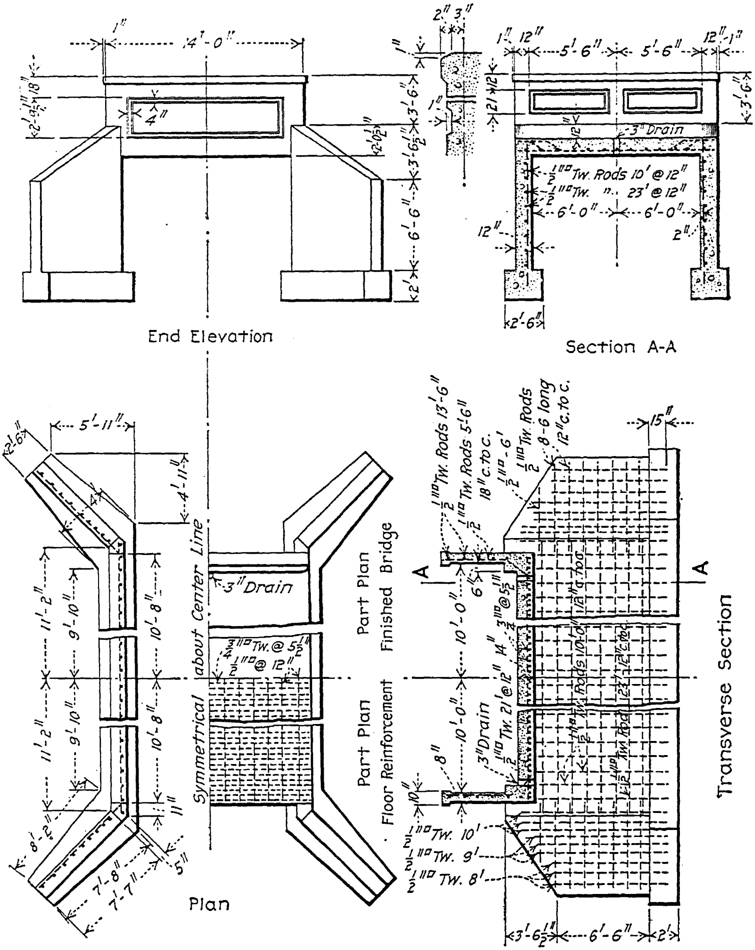
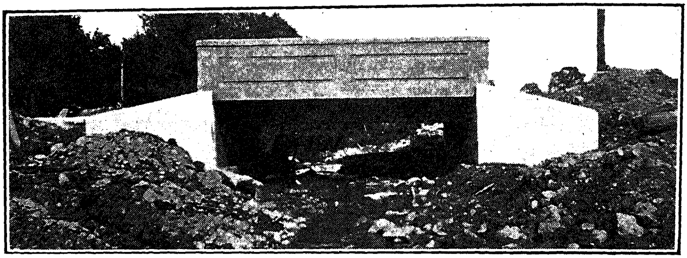


FIG. 246.—Standard design for slab bridges of 12 ft. span, Wisconsin Highway Commission.

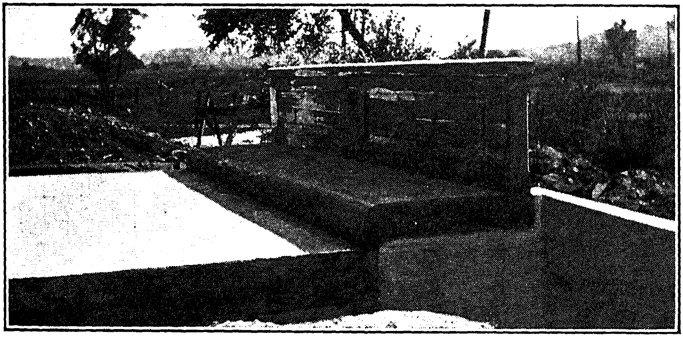
"The effect of a gravel cushion or other wearing surface in distributing concentrated loads over bridge floors has not been considered, as this is a matter not entering into the present discussion. Having found the area of the slab to which the load is applied by this surfacing, the problem is of precisely the same kind as arises when loads are applied directly to the slab. The only difference is that the bearing area of the concentrated load is increased with the use of a wearing surface and consequently the effective width of the slab also becomes greater."



Courtesy of Mr. M. W. Torkelson, Bridge Engineer, Wisconsin Highway Commission.

FIG. 247.—Christiana bridge over Brewery Creek, Town of Cross Plains, Dane County, Wis.

66. Slab Bridges of Single Span.—The floor of a slab bridge may be designed as any simply-supported slab except that shearing stresses may require very careful attention where the live load is relatively large, as in railroad structures. The whole reinforcing system may be made absolutely rigid by wiring the main reinforcing rods to the transverse reinforcement, employing extra transverse spacing rods at the ends of the bent-up



Courtesy of Mr. M. W. Torkelson, Bridge Engineer, Wisconsin Highway Commission.

FIG. 248.—Construction view of Christiana bridge, Town of Cross Plains, Dane County, Wis.

steel. Both the straight longitudinal rods and those bent up to provide for diagonal tension should be hooked at the ends.

Practice varies in regard to the use of expansion joints between the slab floor and the abutments. There are none provided in Figs. 246, 249, and 250, but such joints are placed at both abutments in Fig. 251. In the latter figure, vertical end expansion joints are provided at the angle points between abutment wings and slab, making it necessary to cantilever the outer portions of the slab width on account of insufficient abutment support.

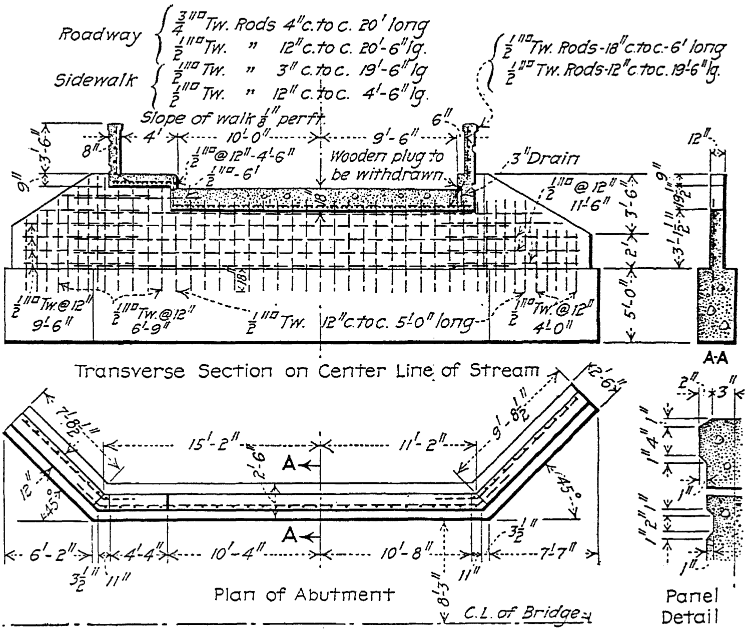


FIG. 249.—Details of Christiana bridge, Town of Cross Plains, Dane County, Wis.

In Figs. 246, 249, and 250 the main wall or vertical slab of the abutment is supported at the top by the floor and at the bottom by the footing, and may be figured for earth pressure as a simple slab with the main steel near the outer or stream face. Of course, the main wall should also be designed to act as a column to support the superstructure. Since a joint of low friction is not provided between floor and abutments, the unit tensile stress in the steel of the superstructure is usually kept low (12,000 lb. per square inch) so as to provide properly for the additional tension in the steel caused by the contraction of the bridge in cold weather. The wings may be designed as self-supporting retaining walls of the cantilever type, using the methods explained in Part I of Volume II. Theoretically the maximum efficiency of the footing for the wing walls can be obtained by placing the wing wall at about the outer middle-third point of the base, but in many cases considerable saving in excavation

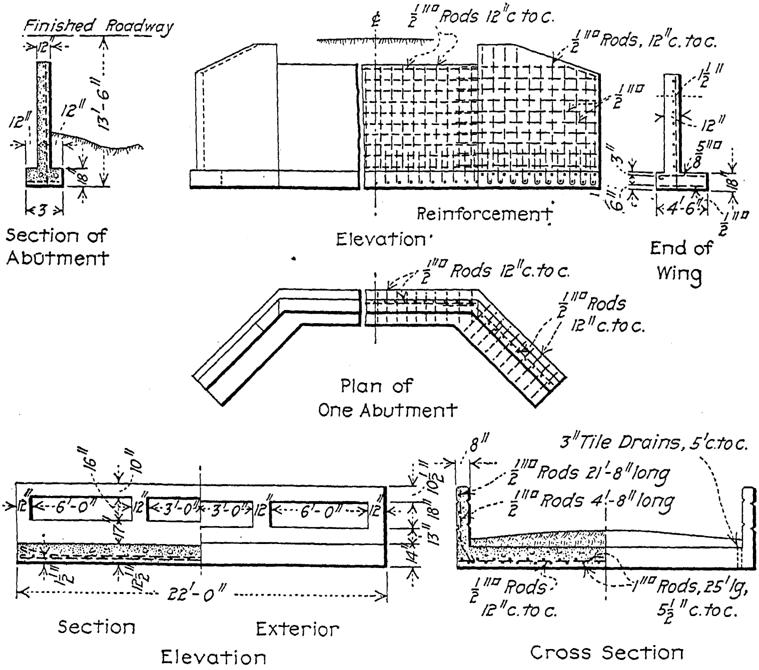
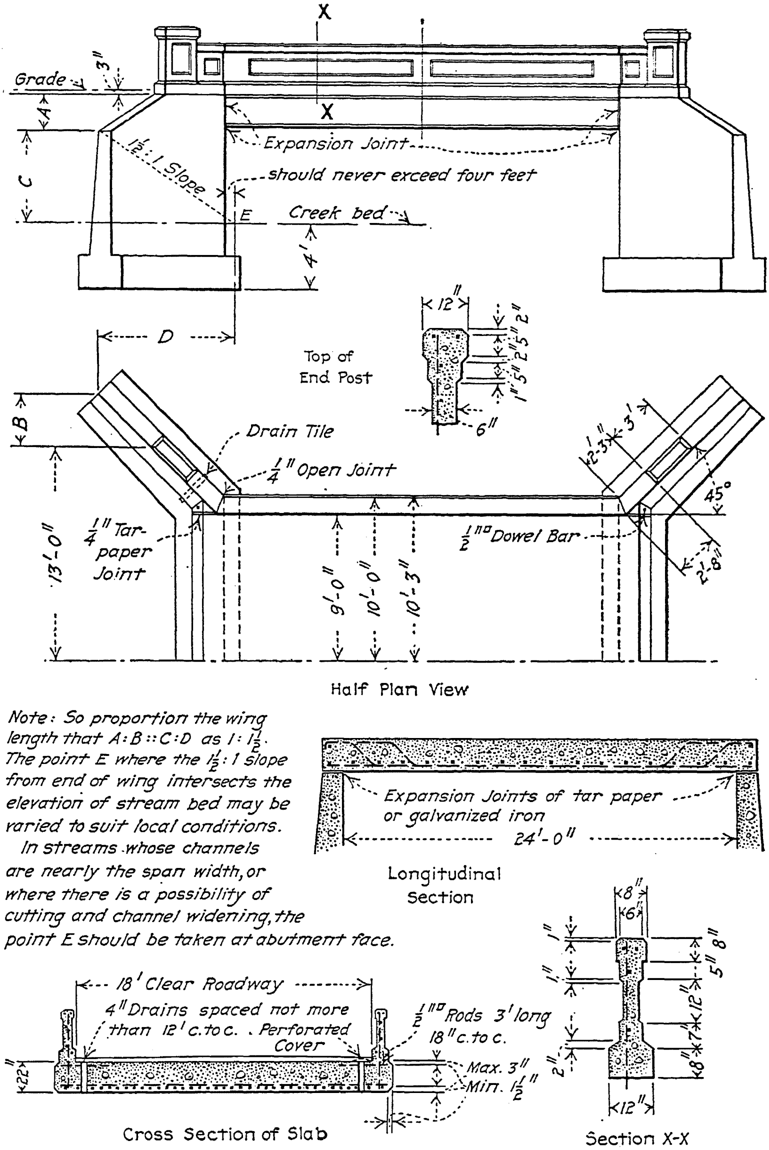


FIG. 250.—Typical slab bridge, Illinois Highway Commission. Abutment wings of the cantilever type. Main wall vertical slab supported at top by superstructure and at base by footing.

may make it more desirable to shift the footing a little toward the stream bed. Counterforted walls are advisable only for abutments over 20 ft. in height.

Figs. 251 and 252 show an unusual abutment design adopted by the engineers of the Iowa Highway Commission for both slab and girder bridges. Expansion joints being provided at each end of the superstructure, both the main portion and the wings were designed as self-supporting retaining walls. The main portion,



Note: So proportion the wing length that A:B::C:D as $1:1\frac{1}{2}$. The point E where the $1\frac{1}{2}:1$ slope from end of wing intersects the elevation of stream bed may be varied to suit local conditions.

In streams whose channels are nearly the span width, or where there is a possibility of cutting and channel widening, the point E should be taken at abutment face.

FIG. 251.—Typical slab superstructure, Iowa Highway Commission.

however, was not only analyzed in the ordinary manner for pressure on the base, but was also analyzed taking into account the stability due to the weight of the wings. The mean value of the maximum pressure at the toe of the foundation by the two methods being found safe, and an analysis for sliding and overturning being satisfactory, the dimensions shown were adopted.

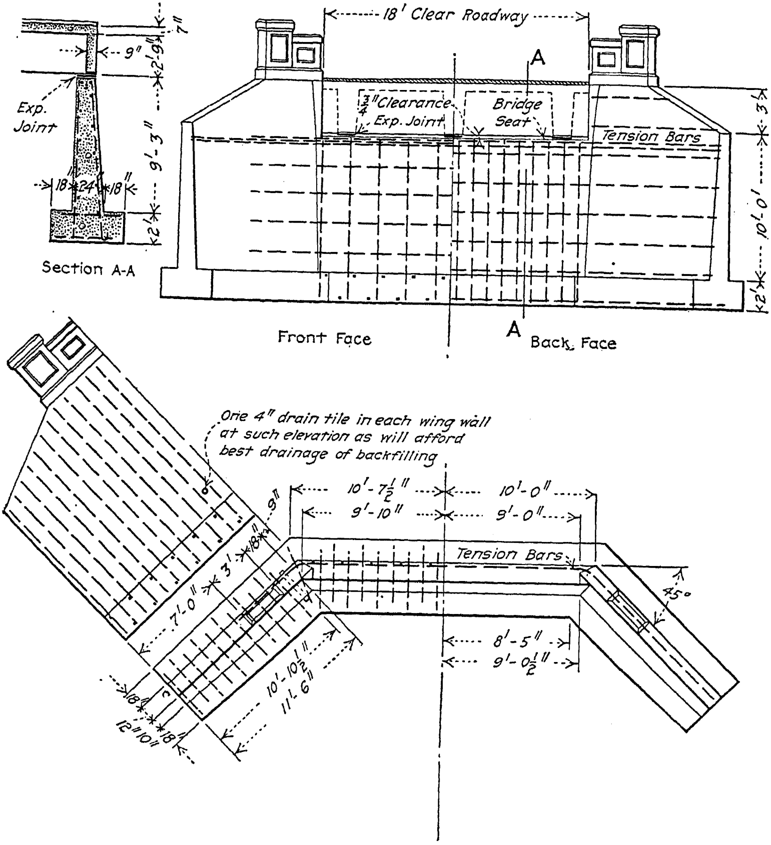


FIG. 252.—Typical substructure for slab and girder bridges, Iowa Highway Commission.

The horizontal rods designated as tension bars were inserted in order to utilize the weight above mentioned. These rods are placed in the main stem near the upper surface and extend continuously through the wings, with splices at the center of the main portion. The effect of considering the wings as a part of the abutment body is to shift the center of gravity of the entire mass farther from the stream face and thus reduce the eccentricity of pressure on the foundation. The horizontal reinforcing rods shown near the stream face of the abutment, and which are carried about 5 ft. into the wings, were employed to counteract a tendency to the formation of vertical cracks on the outside at the corner of wing and abutment. The vertical rods in the front face serve as a framework upon which to build the horizontal rods and they also prevent any tendency toward the formation of horizontal cracks in the stream face due to the clogging of an expansion joint.

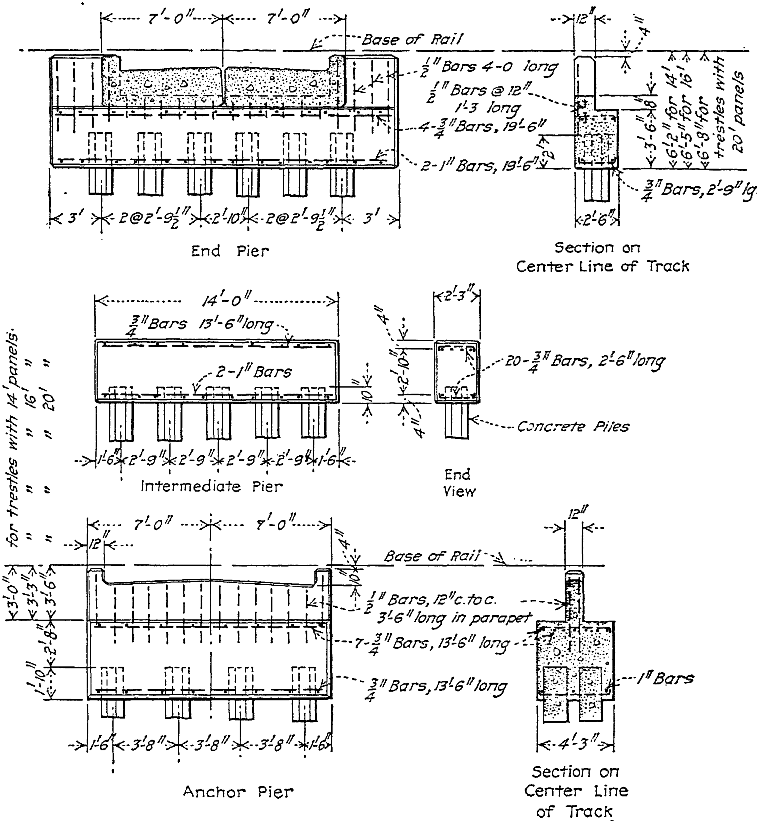


Fig. 253.—Details of substructure, standard concrete pile trestle, Illinois Central R. R.

67. Slab Bridges of Multiple Spans.—Slab bridges of multiple spans will be treated under the four following headings:

Concrete pile trestles.

Pier trestles.

Trestles with framed bents.

Cantilever flat-slab construction.

Concrete Pile Trestles.—Figs. 253 to 256 inclusive give the essential details of design of the pile trestles built by the Illinois Central Railroad. They can be considered typical of concrete pile trestles in general. These trestles replace similar wooden structures over swamps and shallow streams which may not be filled and where bridges on more permanent supports would be extremely expensive because of their great length. The con-

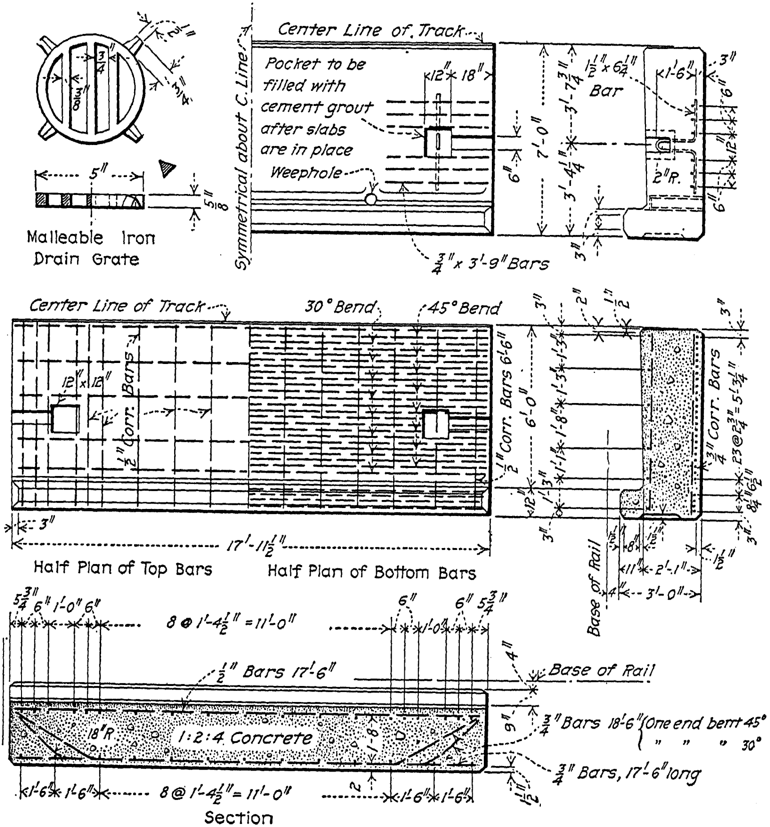
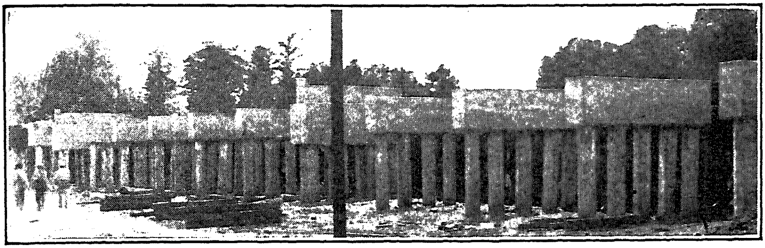


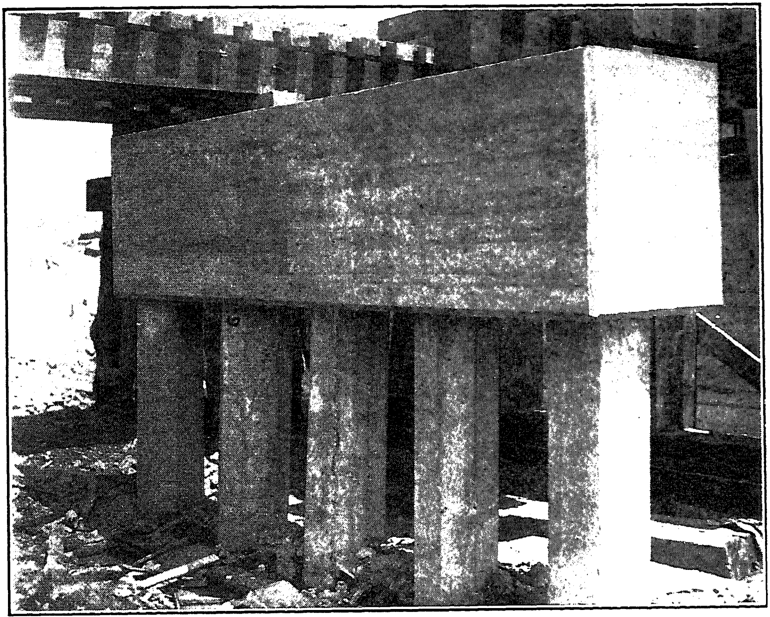
FIG. 254.—Standard slab for clear spans of 16 ft. Illinois Central R. R.

struction consists of pile bents spaced generally from 16 to 20 ft. c. to c., and with a height above ground not greater than the span. The piles are capped with reinforced-concrete girders which support the floor slabs.



Courtesy of Mr. Maro Johnson, Engineer of Bridges and Buildings, I. C. R. R

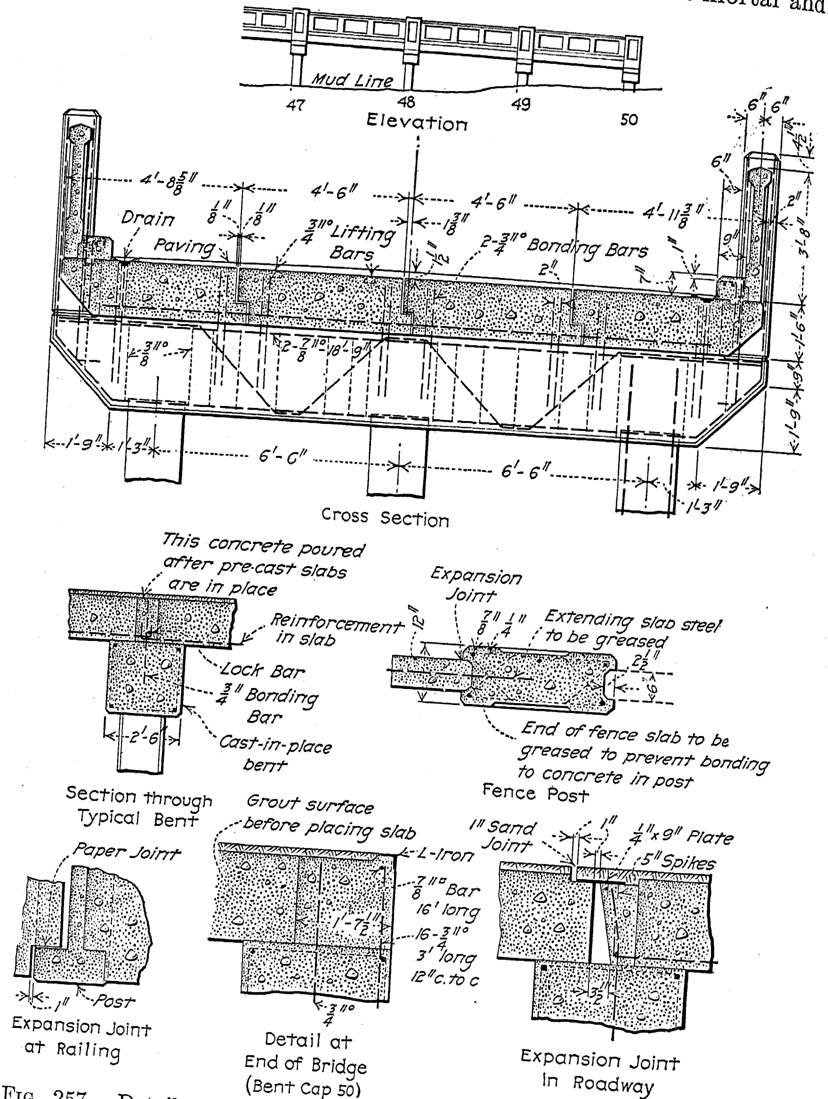
FIG. 255.—Construction view of concrete pile trestle on Illinois Central R. R. near Obion, Tenn. Concrete bents ready for slabs.



Courtesy of Mr. Maro Johnson, Engineer of Bridges and Buildings, I. C. R. R.

FIG. 256.—Typical bent of concrete pile trestle on Illinois Central R. R. near Obion, Tenn.

The piles and deck slabs are usually cast in a convenient yard, allowed to season from 60 to 90 days, and are then hauled to the bridge site. The lifting stirrups shown permit of the slabs being set in place by a wrecking crane. The ballast and track are laid directly on the slabs after the longitudinal and transverse joints (except at anchor bents) are filled with cement mortar and



Elevation

Cross Section

Section through Typical Bent

Fence Post

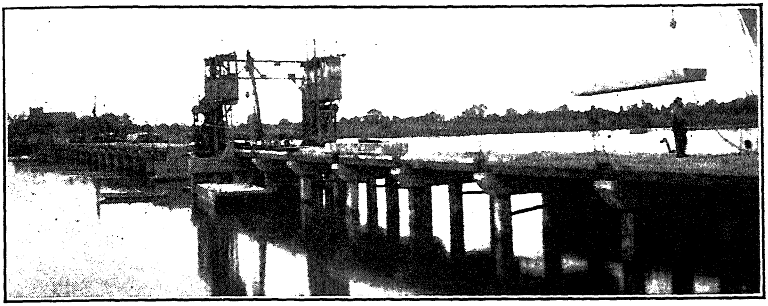
Expansion Joint at Railing

Detail at End of Bridge (Bent Cap 50)

Expansion Joint in Roadway

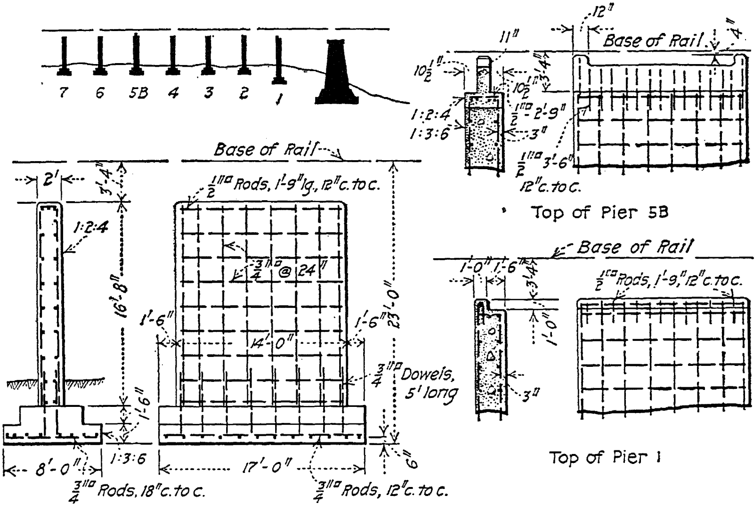
FIG. 257.—Details of pile trestle across the Miles River near Easton, Md.

after the floor surface is thoroughly waterproofed. The slabs are set on a bed of grout on the pile caps. An anchor bent is used at suitable intervals to take up longitudinal stresses due to tractive force and, by means of an expansion joint, to prevent any great accumulation of movement of the deck due to temperature changes.



Courtesy of Raymond Concrete Pile Co.

FIG. 258.—Construction view of pile trestle across the Miles River near Easton, Md.



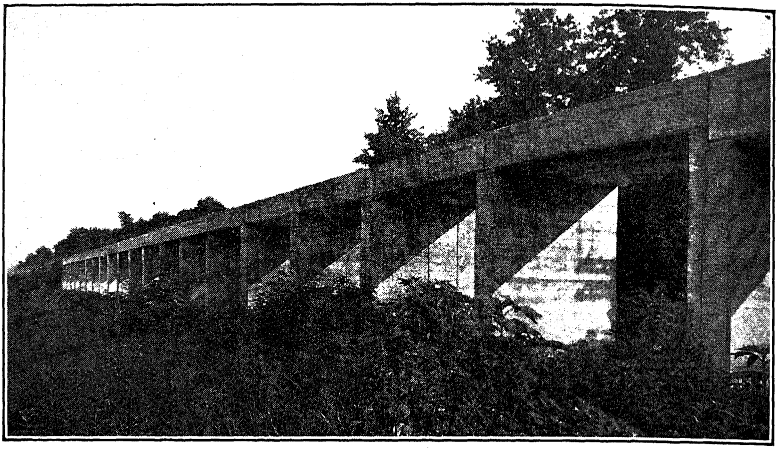
Piers 2, 3, 4, 6, & 7

FIG. 259.—Pier details, Illinois Central R. R. trestle over Kaskaskia River near New Athens, Ill.

A concrete pile trestle for carrying a highway is shown in Figs. 257 and 258.[1] It was found economical to cast the piles, deck slabs, and railing slabs at Baltimore, 60 miles away, and transport them to the site on scows. Expansion joints were located in the roadway slabs, curb, and railing slabs at every fifth bent.

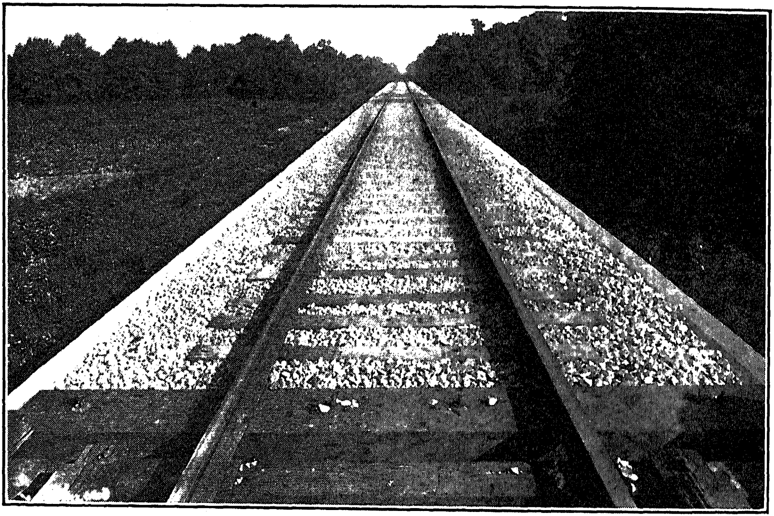
---

[1] See also *Engineering News*, issue of Feb. 5, 1914.



Courtesy of Mr. Maro Johnson, Engineer of Bridges and Buildings, I. C. R. R.

FIG. 260.—Solid benchwall type of concrete trestle on Illinois Central R. R. near New Athens, Ill.



Courtesy of Mr. Maro Johnson, Engineer of Bridges and Buildings, I. C. R. R.

FIG. 261.—Roadbed of trestle shown in Fig. 260.

Pier Trestles.—Thin concrete piers are preferable to pile bents when the height of bridge above the ground line is greater than

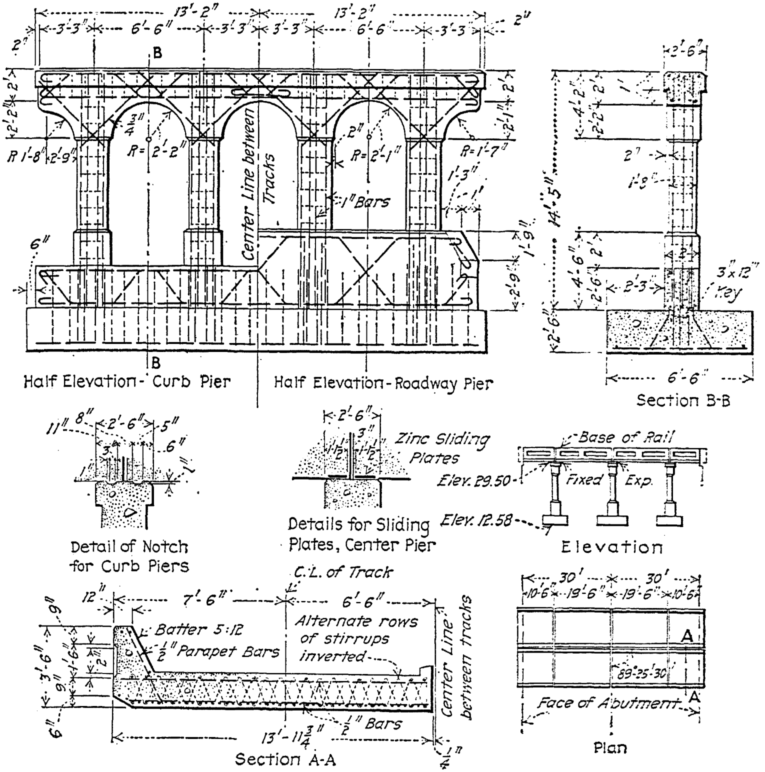
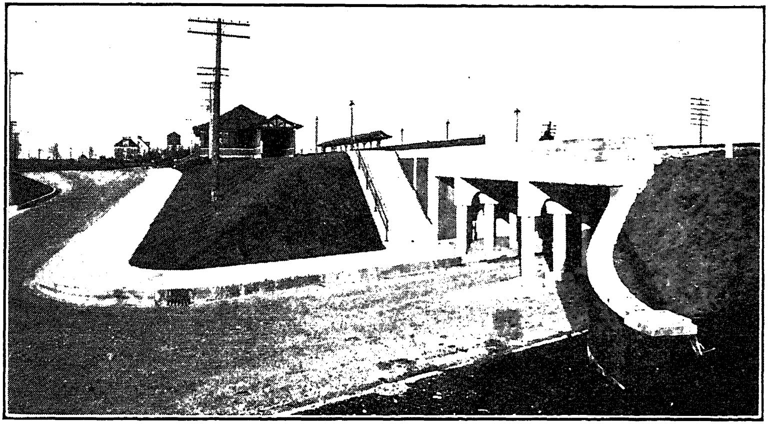
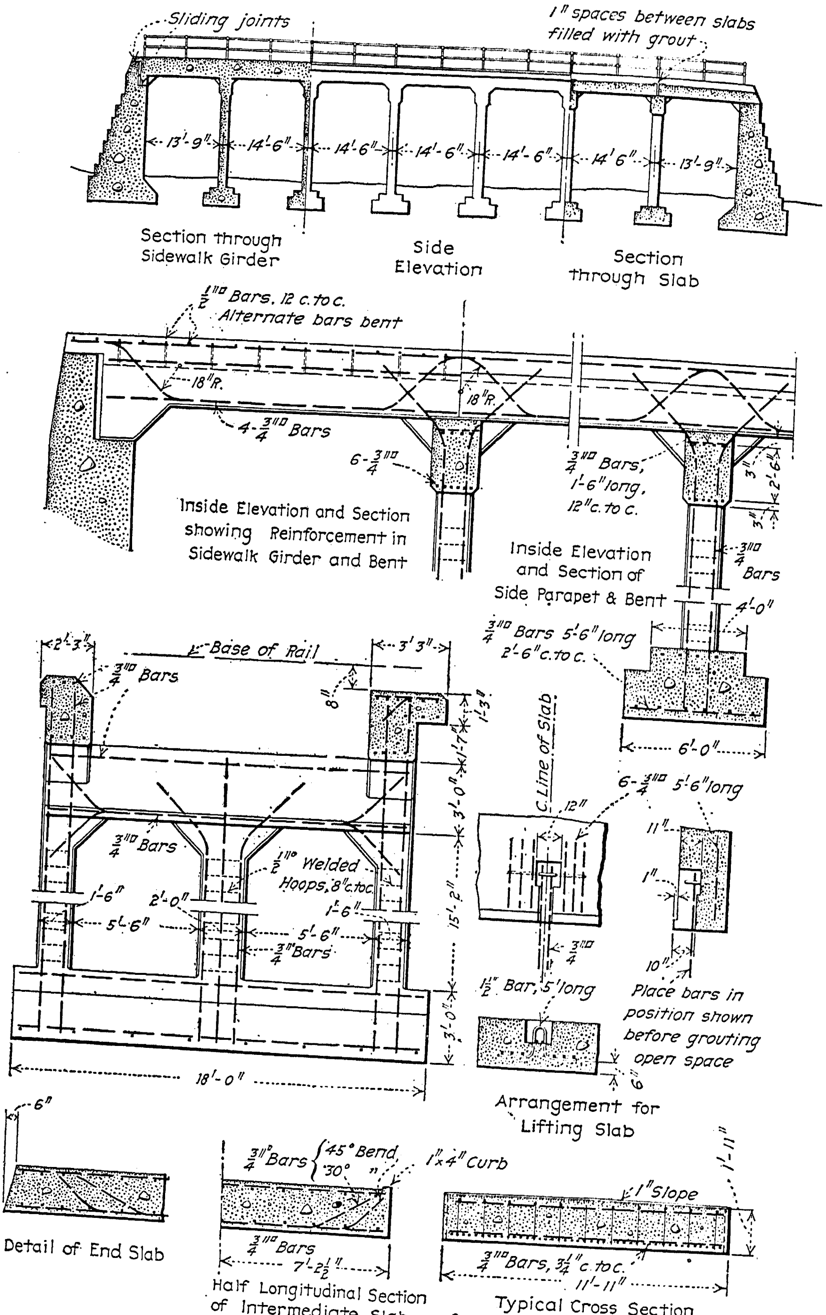


Fig. 262.—Details of Mozart Street subway, Bloomingdale Road track elevation, Chicago, Milwaukee & St. Paul Railway.



Courtesy of Mr. Maro Johnson, Engineer of Bridges and Buildings, I. C. R. R.



Section through Sidewalk Girder

Side Elevation

Section through Slab

Inside Elevation and Section showing Reinforcement in Sidewalk Girder and Bent

Inside Elevation and Section of Side Parapet & Bent

Arrangement for Lifting Slab

Detail of End Slab

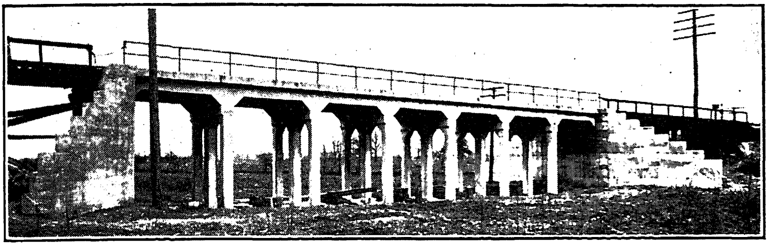
Half Longitudinal Section of Intermediate Slab

Typical Cross Section

about 16 ft. Figs. 259, 260, and 261 show a typical trestle of the solid bench-wall type built by the Illinois Central Railroad.

Trestles with Framed Bents.—Slab bridges with framed bents forming subways are used on at least fifteen railroads in this country. A design which may be considered typical is shown in Fig. 262. The deck slabs may either be cast in place or cast at some central yard and placed in a similar manner to the slabs for pile or pier trestles. In Fig. 262 the design is shown for slabs to be cast in place. A general view of a typical Illinois Central subway is shown in Fig. 263.

A framed-bent trestle with continuous side girders to resist stresses due to traction is shown in Figs. 264 and 265. The girder on one side acts simply as a tie and parapet, while the other with a cantilever projection at the side acts also as a sidewalk. The slabs were cast in a yard at some distance from the bridge site, loaded on flat cars, taken to the job, and swung into place with derricks. Expansion is allowed for at both ends by providing a sliding joint between the bridge superstructure and the abutments. The shallow 4-in. curbs at the ends of the slabs were provided to prevent seepage from getting into the 1-in. grout joint between slabs. The pier footings are reinforced longitudinally in top and bottom, and were figured as continuous T-beams uniformly loaded by the pressure on the soil. The cross girders at the top of the columns support the deck slabs previously referred to, and are made continuous.



Courtesy of Mr. Maro Johnson, Engineer of Bridges and Buildings, I. C. R. R.

FIG. 265.—Trestle approach to Cumberland River bridge, Illinois Central R. R. Side girders continuous.

Cantilever Flat-slab Construction.—Fig. 266 is a flat-slab structure of the Turner Mushroom type. The methods which may

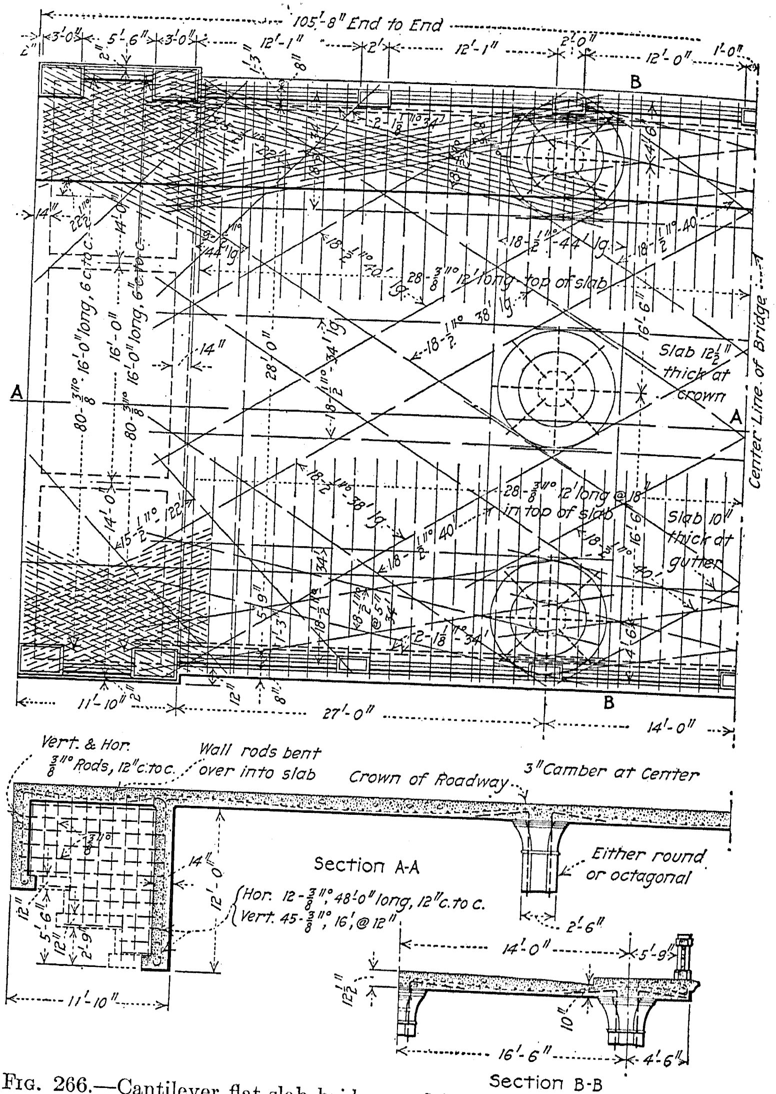
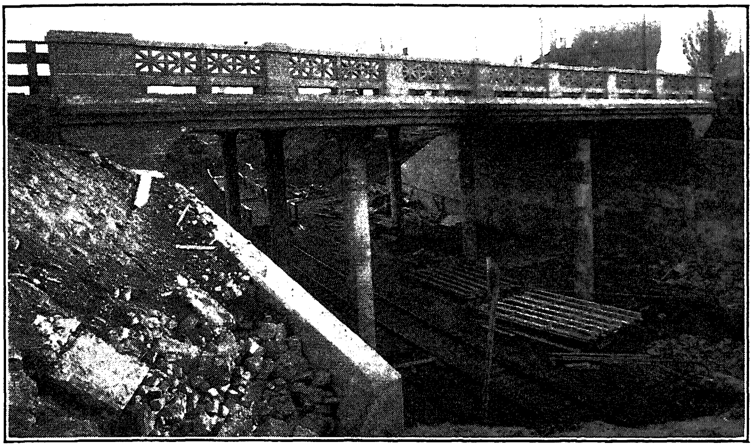


FIG. 266.—Cantilever flat-slab bridge on Mississippi River Boulevard, St. Paul, Minn.

be used in the design of the roadway slab are treated in Volume II. The hollow abutments should be noted.

A cantilever flat-slab bridge in which the abutments are of the ordinary reinforced-concrete type is shown in Fig. 267. The abutment walls are considered as held at the top by the superstructure to which they are anchored by bending the vertical rods into the slab.



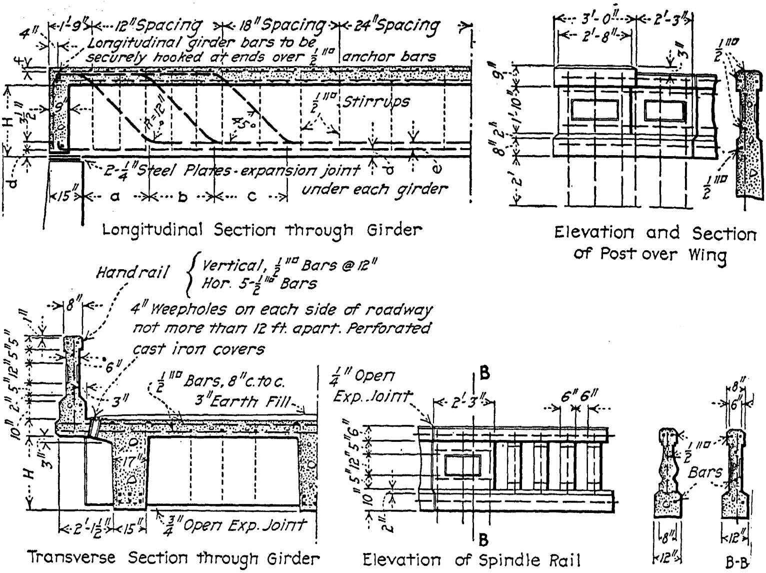
Courtesy of Mr. A. M. Wolf, Prin. Asst. Eng'r., Condron Co., Chicago.

FIG. 267.—Lafayette St. bridge, St. Paul, Minn.

CHAPTER XV

SIMPLE GIRDER BRIDGES

68. Deck Girders.—The deck-girder type of construction usually proves more economical than the through-girder type where-


4″
1′-9½
12″ Spacing
18″ Spacing
24″ Spacing
Longitudinal girder bars to be securely hooked at ends over ½″□ anchor bars
½″□ Stirrups
45°
R=12″
H
d
2-¼″ Steel Plates - expansion joint under each girder
15″
a
b
c
e
f
Longitudinal Section through Girder
3′-0″
2′-3½
2′-8″
1″□
½″□
9″
1′-10″
8″
2′
Elevation and Section of Post over Wing
Handrail
Vertical, ½″□ Bars @ 12″
Hor. 5-½″□ Bars
4″ Weepholes on each side of roadway not more than 12 ft. apart. Perforated cast iron covers
8″
6″
3″
½″□ Bars, 8″ c. to c.
3″ Earth Fill
17″
2′-1½″
15″
¾″ Open Exp. Joint
Transverse Section through Girder
¼″ Open Exp. Joint
B
2′-3″
6″ 6″
Elevation of Spindle Rail
½″□ Bars
8″
6″
12″
B-B

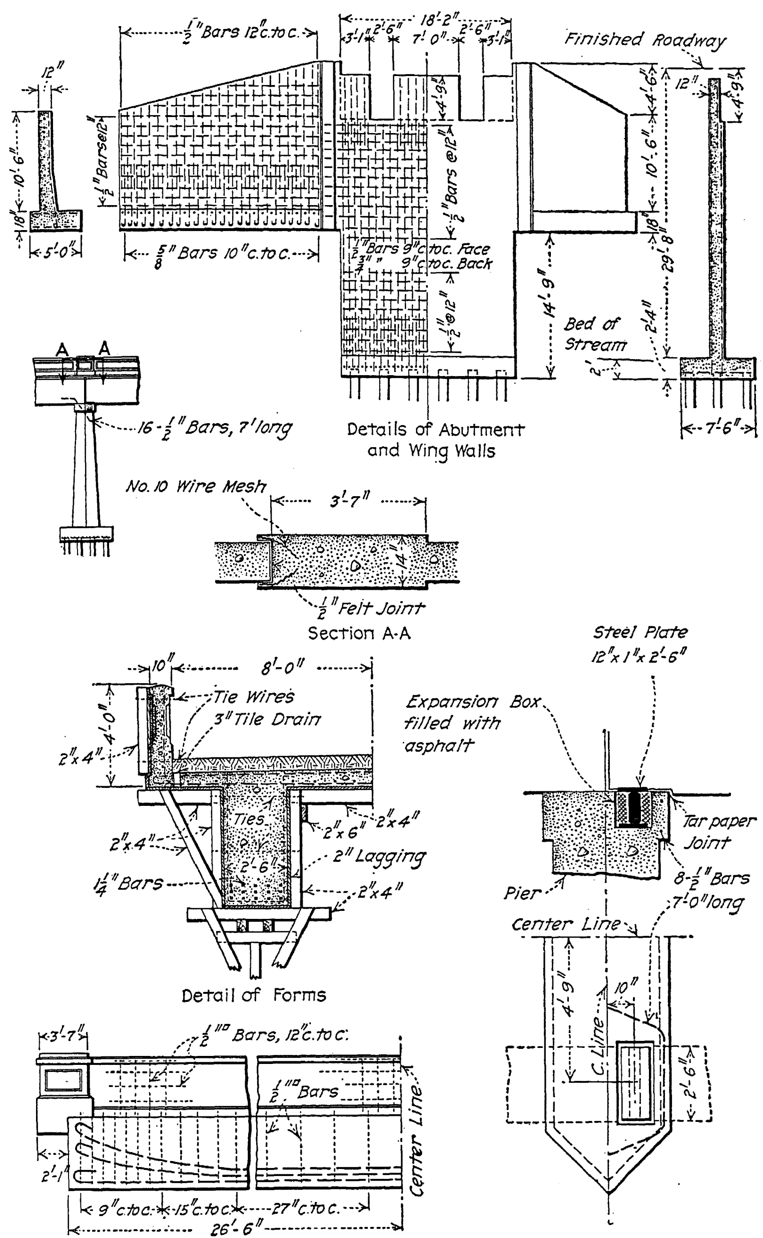
Sections

Table of Dimensions and Girder Reinforcement

Span	H	d	e	f	Stirrup Spaces			a	b	c	Girder Bars	
					12″	18″	24″				Top Row	Bottom Row
24′	2′-2″	2½″	3″	2½″	10	6	2	2′-3″	2′-3″	2′-0″	3 - 1″□	3′- 1″□
26′	2′-4″	2½″	3″	2½″	12	6	2	2′-6″	2′-1″	2′-5″	3 - 1″□	3′- 1″□
28′	2′-6″	2½″	3″	2½″	12	6	3	2′-5″	2′-2″	2′-11″	3 - 1″□	1′- 1″□ 2′- 1⅛″□
30′	2′-9″	3″	3½″	2½″	10	6	5	2′-8″	2′-6″	2′-10″	3 - 1″□	3 - 1⅛″□
32′	2′-10″	3″	3½″	3″	10	6	6	2′-7½″	2′-6½″	3′-1″	3 - 1⅛″□	3 - 1⅛″□
34′	3′-1″	3″	4″	3″	10	10	4	2′-11″	2′-10″	3′-3″	2 - 1″□ 1 - 1¼″□	3 - 1¼″□
36′	3′-3″	3″	4″	3″	10	10	5	3′-0″	2′-11″	3′-10″	3 - 1⅛″□	3 - 1¼″□
38′	3′-5″	3″	4″	3″	10	10	6	3′-4″	2′-10″	3′-4″	2 - 1⅛″□ 1 - 1¼″□	3 - 1¼″□
40′	3′-8″	3⅜″	4″	3⅛″	12	6	9	3′-6″	3′-2″	3′-10″	3 - 1¼″□	3 - 1¼″□

FIG. 268.—Standard details of concrete deck girder bridges, Iowa Highway Commission.

ever sufficient head-room is available. The girders, of course, should be relatively thin and deep for the greatest economy, and



Details of Abutment and Wing Walls

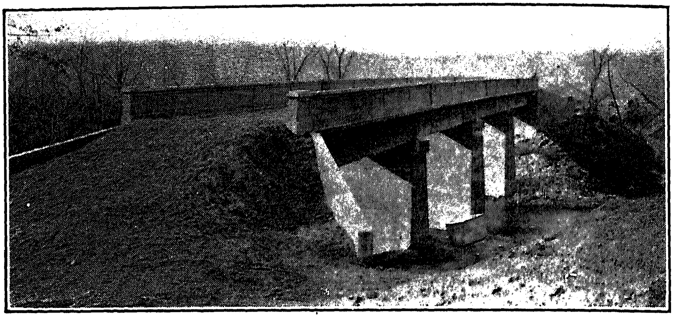
Section A-A

Detail of Forms

FIG. 269.—Details of Embarrass River bridge, Cumberland Co., Ill.

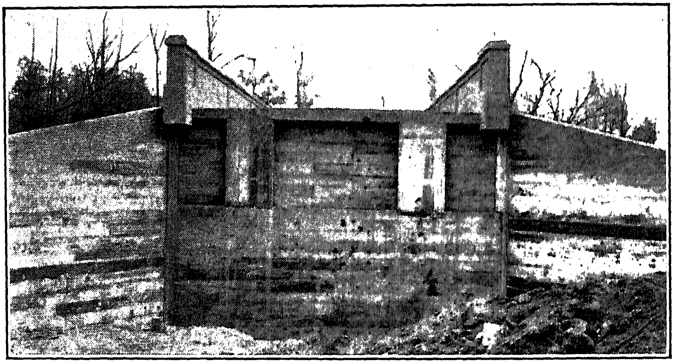
a curtain wall should be provided between the girders at each end of span to retain the earth fill, thereby avoiding complicated parapet walls on the abutments.

Standard details of deck-girder bridges designed by the engineers of the Iowa Highway Commission are shown in Fig. 268. The floor slab was analyzed both as fixed and as continuous, and was designed to resist maximum stresses caused by either method of analysis. The method of fastening the girder steel to anchor rods should be noted. Expansion joints are provided under the girder stems by means of sliding steel plates anchored into the body of both superstructure and substructure.



Courtesy of Engineering and Contracting.

Fig. 270.—Embarrass River bridge, Cumberland Co., Ill.



Courtesy of Engineering and Contracting.

Fig. 271.—Rear view of abutment and wing walls, Embarrass River bridge, Cumberland Co., Ill.

Figs. 269, 270, and 271 illustrate the type of deck-girder bridge adopted as standard by the Illinois Highway Commission. The following description of the methods employed in providing for expansion in girder bridges is given in the fourth report of the Commission:

"Two methods of providing for expansion in girder bridges have been used and both have proved satisfactory. In one method, the wing walls of one abutment are entirely separated from the abutment wall proper, the latter being free to move at the top with the expansion or contraction of the superstructure. The wing walls are designed to be self-supporting. As girder spans designed by the commission have so far been limited to 60 ft., the amount of movement either way from the normal is small and is taken up by deflection of the main wall or a slight rocking of the wall on the footing. Earth pressure against the wall is of little importance in this connection as it but tends to reduce the tension in the girder steel during expansion and to cause the abutment wall to follow the superstructure during contraction. It does not increase the stress in the compression area of the girder as the load is applied at the bottom of the girder, tending by this eccentricity of application to reverse the dead- and live-load stresses in the girder.

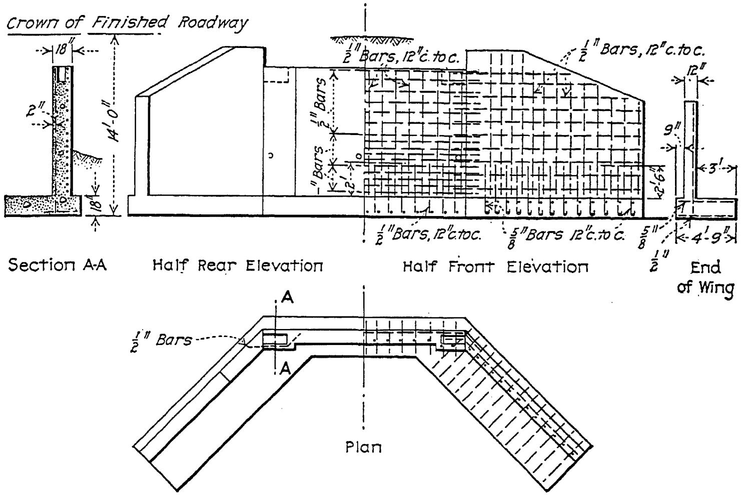


FIG. 272.—Type of abutment used for girder bridges by the Illinois Highway Commission. Wings of cantilever type. Main wall supported by wings as counterforts.

"This method has been found to be entirely successful, but is somewhat objectionable as a slight movement of the wings due to earth pressure and unequal settlement sometimes causes the wing walls to move forward slightly at the top, making a somewhat unsightly offset between the wing and abutment walls. This has never been more than

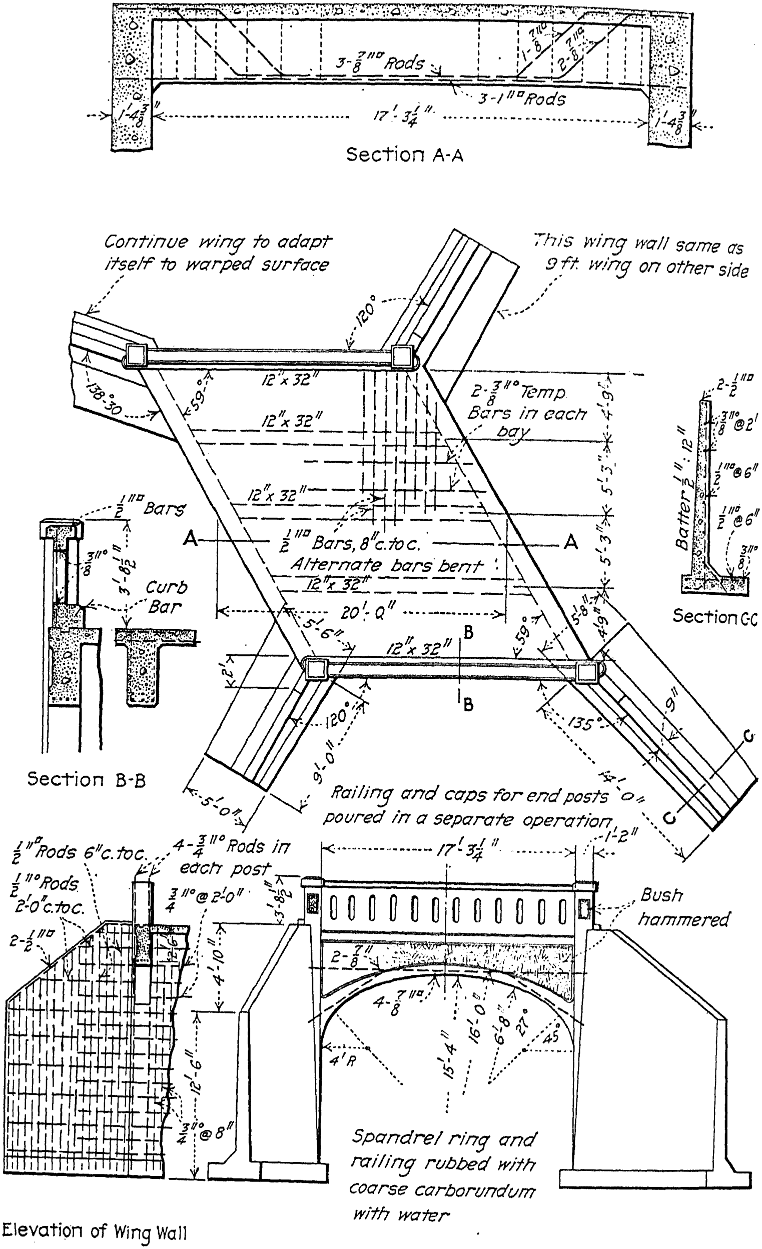
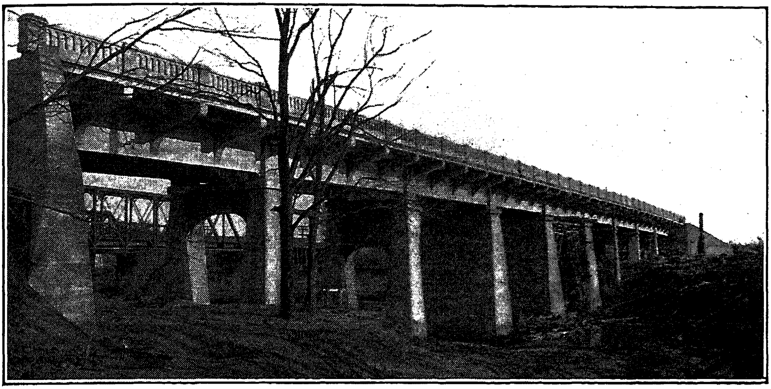


FIG. 273.—Lindsey Creek crossing near Wyeth, Oregon.

2 or 3 in. for the highest walls, but as it is not understood by the ordinary observer, an impression of weakness is sometimes caused.

"The present method of providing for expansion is to design the abutments and wings in the ordinary way, separating the superstructure completely from one of the abutments by a thick paper joint and supporting each girder at the free end on a single cast-iron rocker of large diameter. The reaction is transmitted to the girder and abutment from the rocker through planed structural-steel plates stiffened with I-beams when necessary. The rocker surfaces in contact with the bearing plates are turned to insure perfect bearing on the plates. The diameter of the rocker is made proportional to the load imposed per linear inch, in the same manner as is commonly used in proportioning roller nests for steel bridges. The upper and lower plates are bedded in the concrete of the superstructure and abutment. The rocker is located in a pocket built in the abutment. This pocket is filled with a soft asphalt to prevent the entrance of water or dirt and to protect the metal from corrosion.



Courtesy of Brenneke & Fay, Consulting Engineers, St. Louis, Mo.

FIG. 274.—North Samuels Avenue viaduct, Fort Worth, Texas.

"The rocker method of providing for expansion has proved very satisfactory and is but little more expensive than the other method, especially when it is considered that the wings may be tied to the main wall when rockers are used and advantage taken of the mutual support thus obtained."

Fig. 272 shows a type of abutment adopted by the Illinois Highway Commission in cases where the girders are supported on cast-iron rockers and the wings are nearly parallel to the roadway or make an angle of more than 45° with the face of the abutment. The wing walls are considered to act as counterforts and the reinforcing steel in the main wall is horizontal and placed near the stream face of the wall.

Fig. 273 gives the details of a girder bridge, the main portion of the abutments and the wings of which were designed and

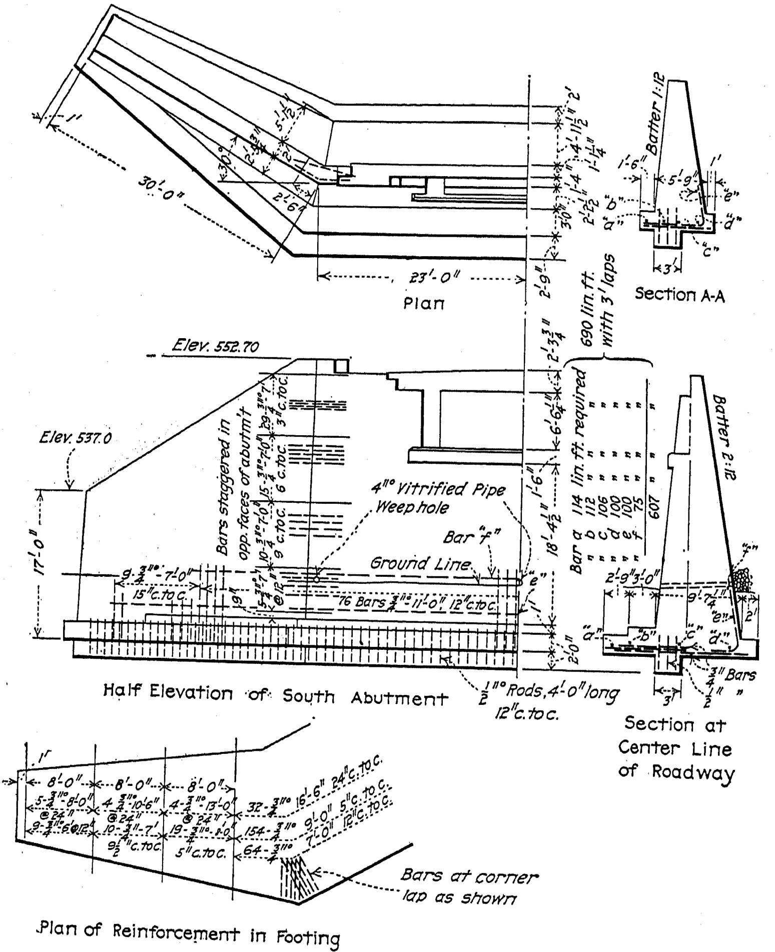

Plan
30'-0"
23'-0"
Section A-A
Batter 1:12
Elev. 552.70
Elev. 537.0
17'-0"
Bars staggered in opp. faces of abutm't
4" Vitrified Pipe Weep hole
Bar "f"
Ground Line
76 Bars 3/4" - 11'-0", 12" c. to c.
1/2" Rods, 4'-0" long 12" c. to c.
Half Elevation of South Abutment
690 lin. ft. with 3' laps
Bar a 114 lin. ft. required
" b 112 "
" c 106 "
" d 100 "
" e 100 "
" f 75 "
607 "
Batter 2:12
3/4" Bars
1/2" "
Section at Center Line of Roadway
Bars at corner lap as shown
Plan of Reinforcement in Footing

FIG. 275.—Details of south abutment, North Samuels Avenue viaduct, Fort Worth, Texas.

figured in the same manner as the slab bridges of Figs. 246, 249, and 250.

In the structure shown in Figs. 274 to 278 inclusive, cross girders and stringers were provided in addition to the longitudinal

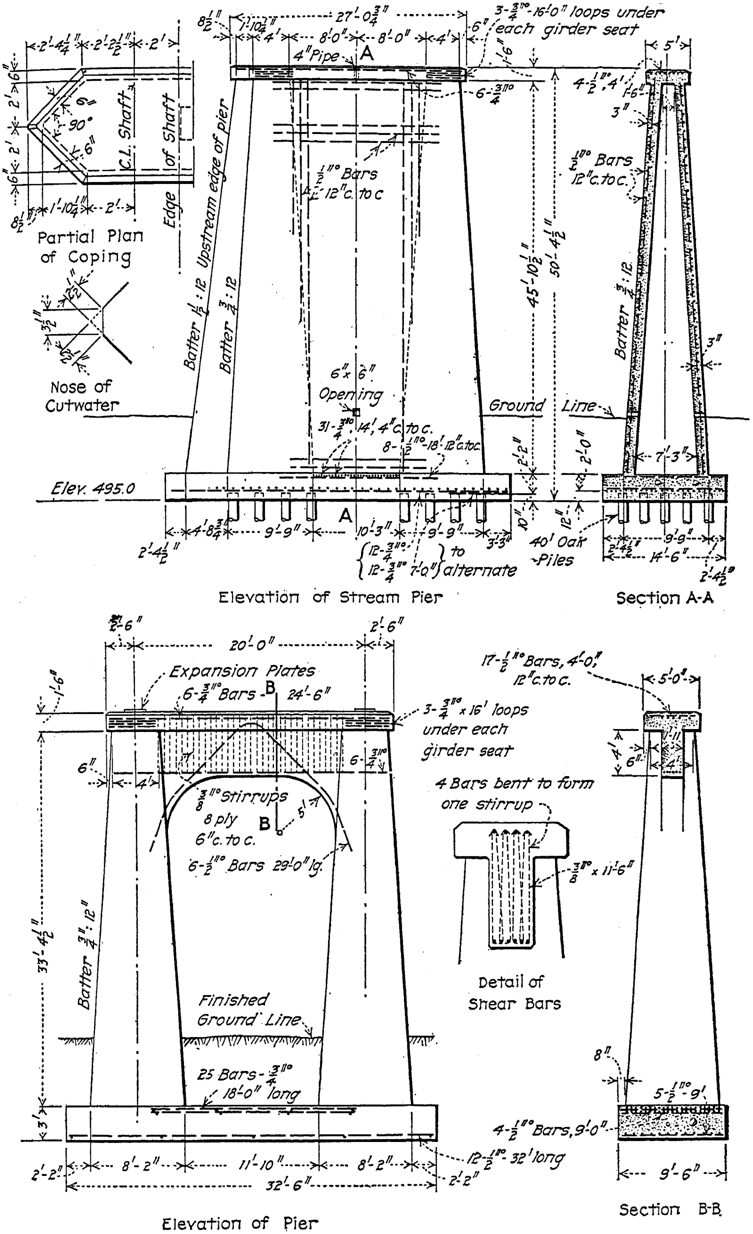


FIG. 276.—Details of typical piers, North Samuels Avenue viaduct, Fort Worth, Texas.

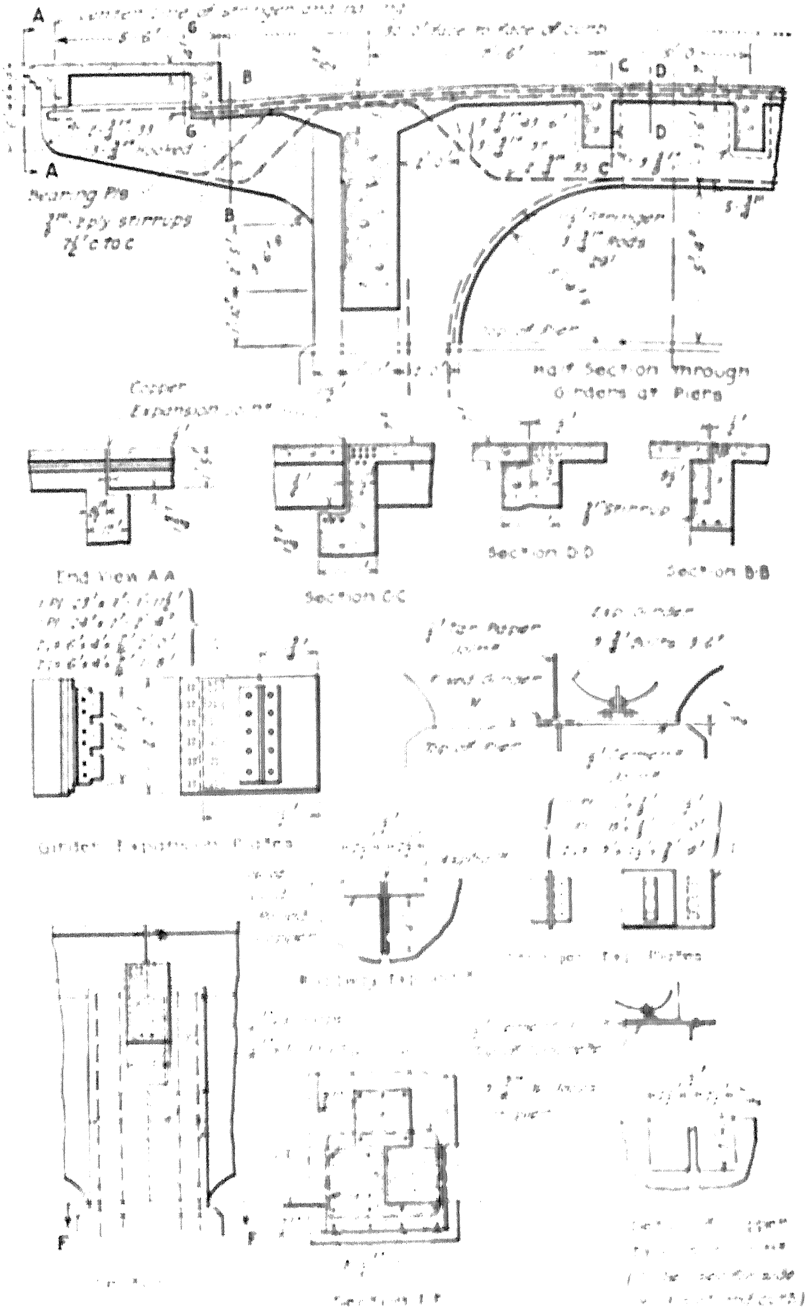


Fig. [illegible].—Details of floor system, [illegible] Viaduct.

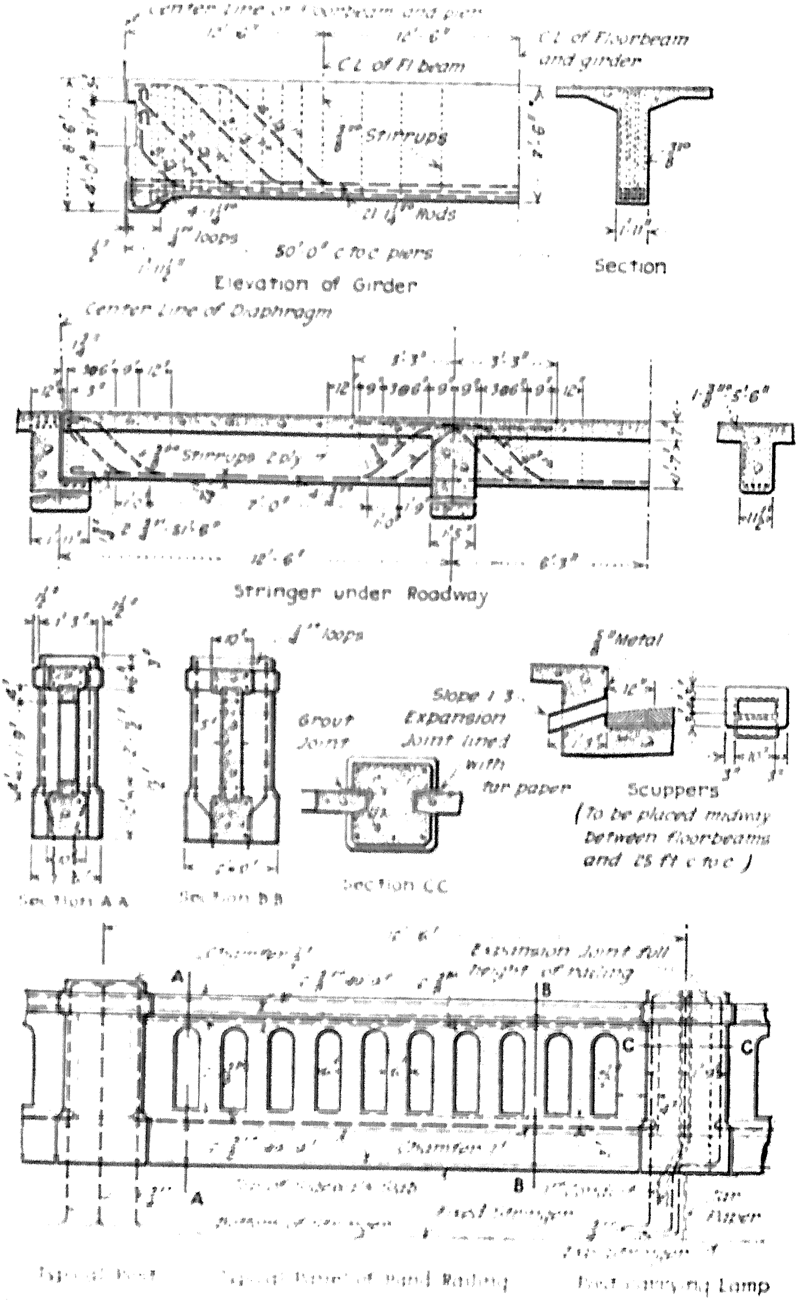


FIG. 278.—Details of floor system and railing, North Samuels Avenue via-

to the pier and the other end is allowed to expand and contract in a joint packed all around with ½-in. of tar paper and bearing on a pair of milled steel plates. Expansion joints were also made in the roadway slab and railing. Cast iron scuppers were placed in each curb on 25-ft. centers. For maximum stresses in the cross girders, the sidewalks were assumed to be unloaded.

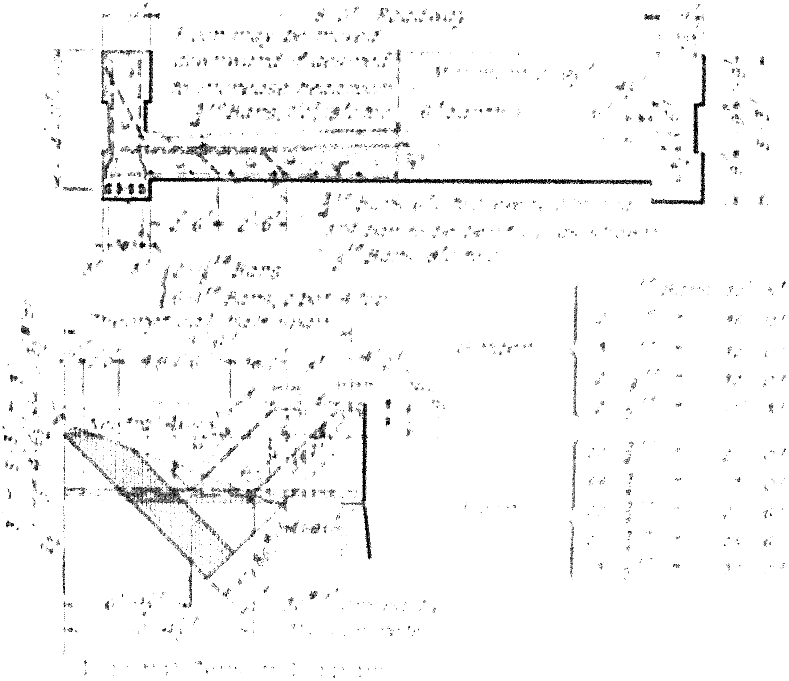


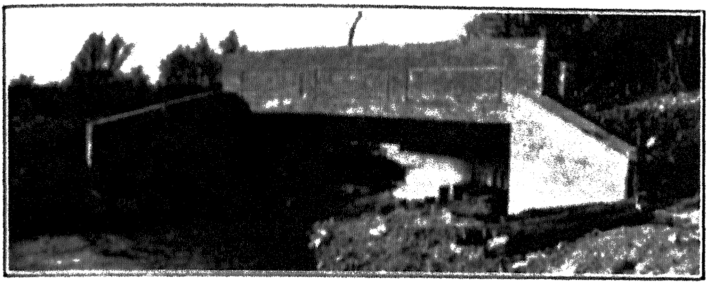
FIG. 279.—Standard sections for through girder bridges [illegible] span. Iowa Highway Commission.

Reinforced-concrete deck girder bridges for railroad traffic are rather unusual, but a few such structures have been built. The superstructure in such bridges is usually supported by plain concrete piers and abutments.

69. Through Girders. From the standpoint of economy, the through girder bridge should not be built except where insufficient head room or other local conditions prevent the use of the deck girder.

The standard type of through girder bridge adopted by the

Iowa Highway Commission for locations where the deck type is impracticable, is shown in Fig. 279. The girders themselves were designed as simple beams and the floor as a slab partially fixed, the point of contraflexure being arbitrarily assumed at about 1 ft. from the edge of the girder. This is approximately in accordance with tests made by the Illinois Highway Commission in 1907.



Courtesy of Mr. M. W. Torkelson, Bridge Engineer, Wisconsin Highway Commission.

Fig. 280.—Funkelien bridge, Town of Christiana, Dane Co., Wis.

In the tests referred to, stress measurements were made on a through-girder span with an 18-ft. roadway loaded with crushed stone and pig iron. The full applied load was 418 tons, which gave a distributed load per square foot of floor of 1450 lb. Deformation measurements were made in the reinforcing steel of the sus-

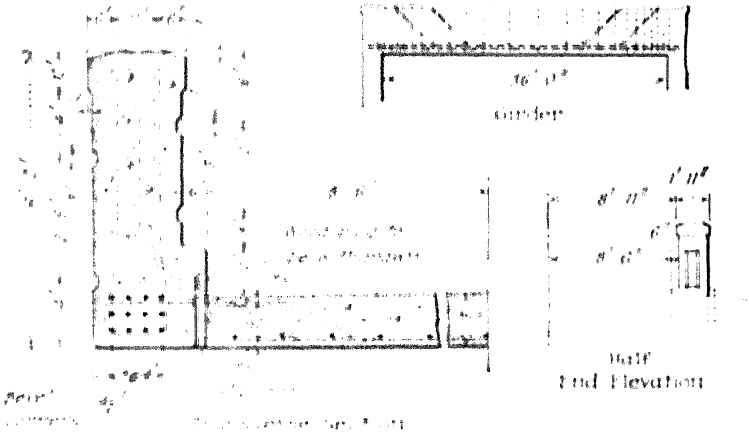


Fig. 281.—Details of superstructure of Funkelien bridge, Town of Christiana, Dane Co., Wis.

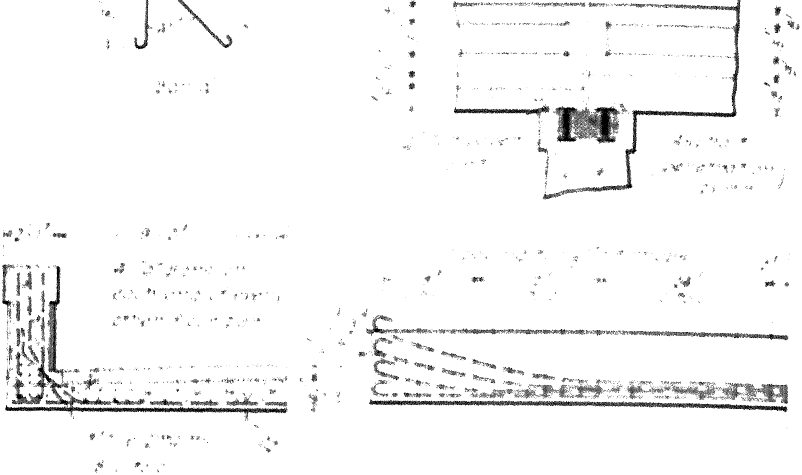


Fig. 282.—Details of through girder bridge of 15 ft. span, Illinois Highway Commission.

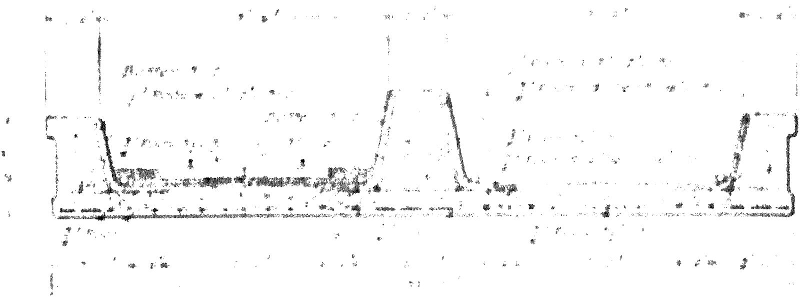


Fig. 283.—Cross-section of through girder bridge, C. B. & Q. R. R.

[illegible]

Fig. 284.—[illegible]

pended floor which indicated stress equivalent to that theoretically resulting from a simple span of 15.5 ft. In other words, since the roadway was 18 ft. between girders, the point of contraflexure was apparently about 15 in. from each girder face.

Figs. 280, 281, 282, and 283 illustrate other designs of through-girder superstructures.

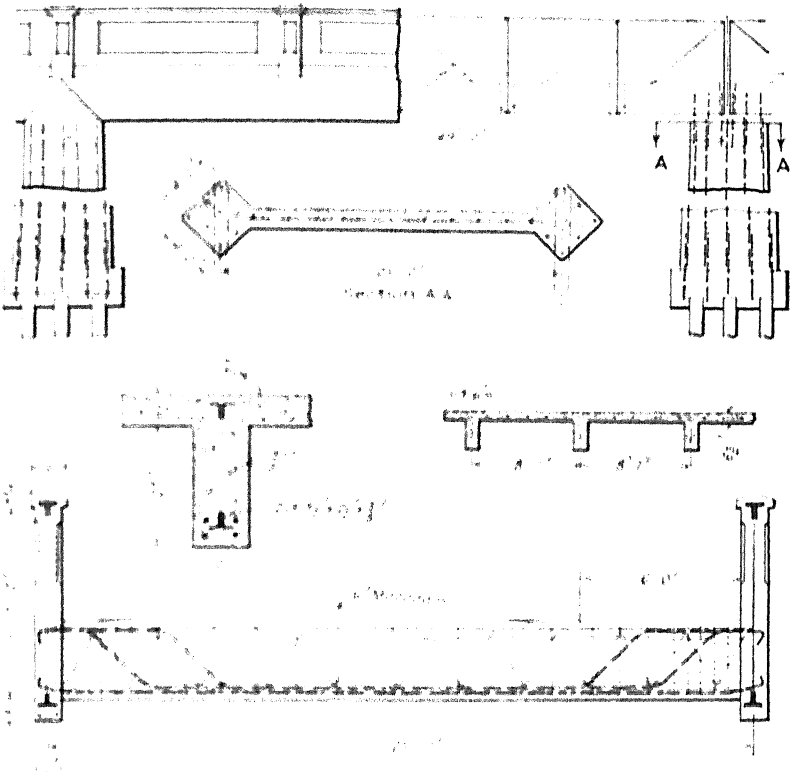


Fig. 283. Detail of Bridge over Muddy Creek, Hamilton County, Ohio.

Figs. 284 and 285 show a rather unusual type of through bridge on account of the fact that the girder reinforcement is in the form of a truss of sufficient strength to carry the dead and construction loads. The piers are simply columns braced between by either a vertical slab or by struts. As no falsework is necessary, this type of construction is especially adapted for highway bridges over railroad and electric lines, or at locations where the soil is very soft.

CHAPTER XVI

CONTINUOUS GIRDER BRIDGES

70. Durability of Monolithic Construction. The designs of most of the continuous girder bridges built monolithic with supports are conservative and safe, but still there are some bridges of this type, especially those with exceedingly high piers or bents, which are classed by many engineers as dangerously extreme considering our present knowledge of reinforced concrete. The fact that light sections in concrete construction have quite often developed cracks to a more or less extent has made these engineers hold to the conservative idea that the difficulty in providing intelligently for movement due to temperature, change in the moisture content of the concrete, etc., is so great as to make light construction something to be avoided wherever possible. It will suffice to say here that bridges of the monolithic type when carefully designed and constructed are giving entire satisfaction, and only time will prove whether or not such light structures are durable.

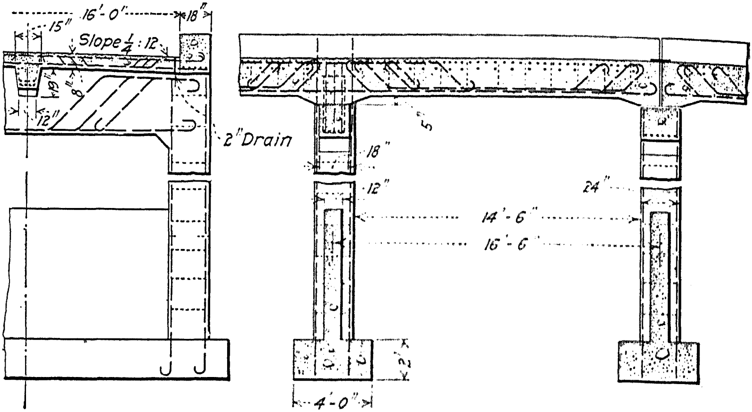
71. Details of Design. Since the superstructure and the piers are usually made monolithic, the latter may be made of comparatively small cross-section if properly reinforced for bending. The piers may become in reality nothing more than two or more columns connected by a suitable portal and the stress due to bending in the columns, caused by unsymmetrical eccentricity of the floor loading, may be computed in the same manner as for columns in buildings. (See Chapter XIV of Volume II.) In order to prevent an accumulation of movement due to contraction and expansion, expansion joints should be provided at least every 100 ft. in length of the structure. If this is not done, severe stresses are likely to occur in the end columns. For high trestles, the columns should be braced longitudinally as well as laterally in the same manner as for high trestles of structural steel.

In most railroad trestles or viaducts the steel in the bottom of the longitudinal girder at the center of span is proportioned for the maximum bending moment produced by the given loading for a simply-supported beam, and six-tenths of the reinforcement thus found necessary is placed in the top of the girders over supports to provide for the continuous action developed. In view of what has been said in Art. 69 of Volume II this method requires 25 per cent. more steel than if the girders were designed throughout for $\frac{1}{12}wl^2$. The above practice, however, is quite prevalent among designers of railroad structures in cases where safety is paramount, since if the piers should settle, or cracks should develop over the piers for other reasons, there would be enough reinforcement at the center of span so that the girders would carry the load as simply-supported beams.

In long bridges an expansion joint is usually provided between the superstructure and the abutments for the reason that, if an abutment with a heavy pressure of earth against it is rigidly connected with a number of continuous spans, the expansion and contraction tend to act in one direction only — that is, away from the abutment — the earth pressure back of the abutment not allowing movement in the opposite direction. Such a condition would lead to difficulties at the center of the bridge, or over the expansion piers next to the abutments, and the abutments and piers would also be severely over-stressed due to the continuous movement in one direction. Each time an abutment would move slightly due to contraction, the earth against it by reason of the heavy moving loads would fill in the small space left by the contractive movement, and when expansion again took place, the abutment would be restrained by the earth so that enormous stresses might be developed.

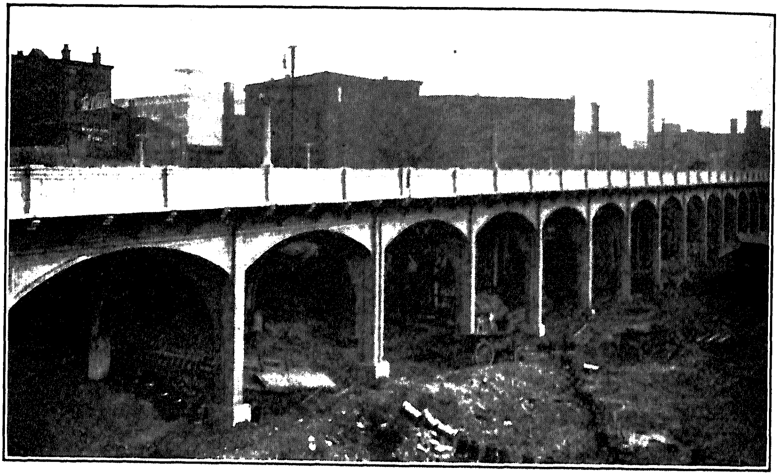
72. Examples of Typical Bridges of the Continuous Girder Type. A rather simple highway trestle, applicable to comparatively low crossings, is shown in Fig. 286. The longitudinal beams are continuous over three spans, an expansion joint occurring over every third pier. Intermediate piers are made monolithic with the floor by means of rods from the columns and stirrups from the cross beams. Because of the indeterminate degree of fixity, the lightness of the structure, and unknown construction factors, all members affected were designed both as fixed beams and as beams freely supported. The slabs were designed as continuous, with equal positive and negative steel throughout. The center longitudinal beam was designed as a T-beam with a 36-in. flange. The pier web, or wall between

A low trestle or viaduct type of construction is shown in Figs. 287 and 288. The slab-beam-and-girder spans were selected since arches, it was thought, would not appear to advantage



Half Transverse Section Longitudinal Section on Center Line

FIG. 286.—Details of standard trestle spans, Engineering Department, State of Arizona.



Courtesy of Mr. Frank L. Raschig, Division Engineer, Division of Structures, Cincinnati, O.

FIG. 287.—Gilbert Avenue viaduct, Cincinnati, Ohio. Note expansion joint near center of illustration.

for such low construction. The cost of the girder type was also found to be much less than for a series of arches, due principally to decrease in the dead weight of the structure and

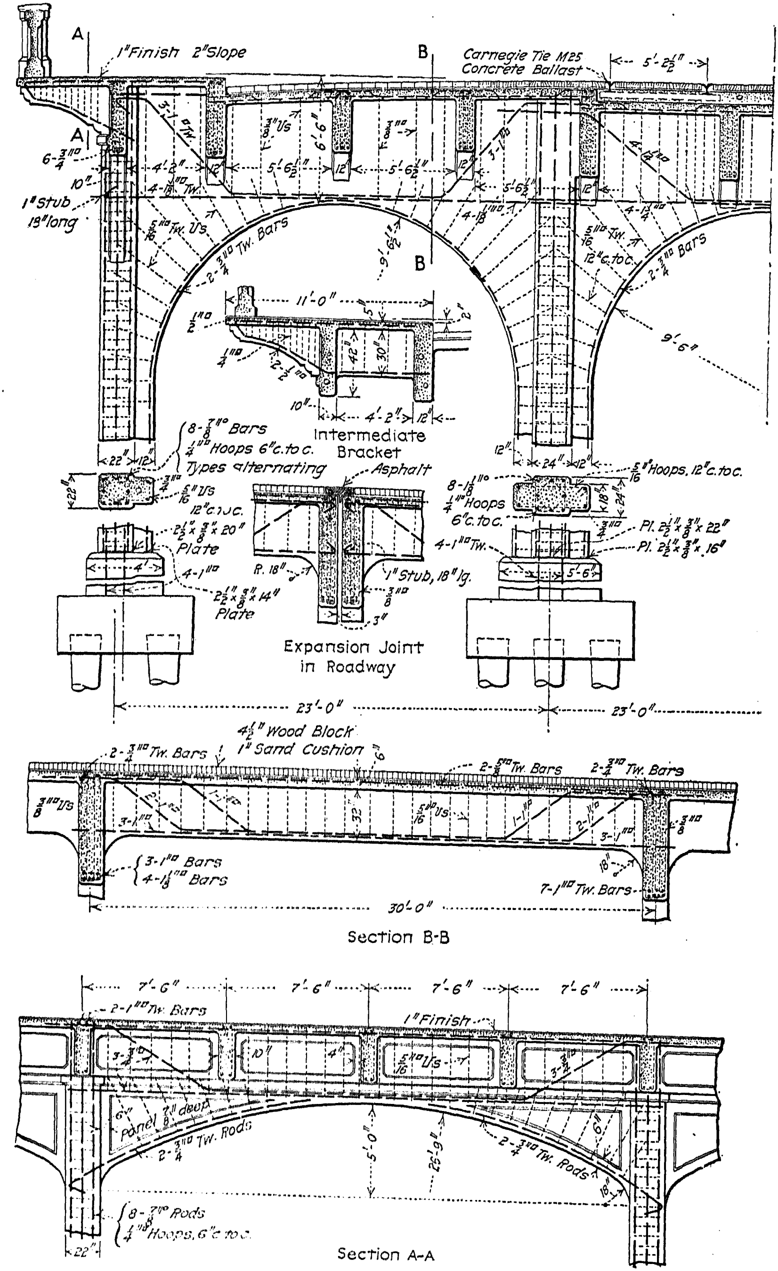


Fig. 288. —Details of Gilbert Avenue viaduct, Cincinnati, Ohio.

to simplicity in form work. Expansion joints occur about every 200 ft. and make the viaduct virtually a number of independent structures, a double row of columns being provided at each joint. The arched girders capping the column bents were designed as straight rectangular beams and no account was taken of the possible arch action. The entire top surface of the roadway slab was waterproofed with a layer of burlap and two layers of felt laid in hot asphalt.

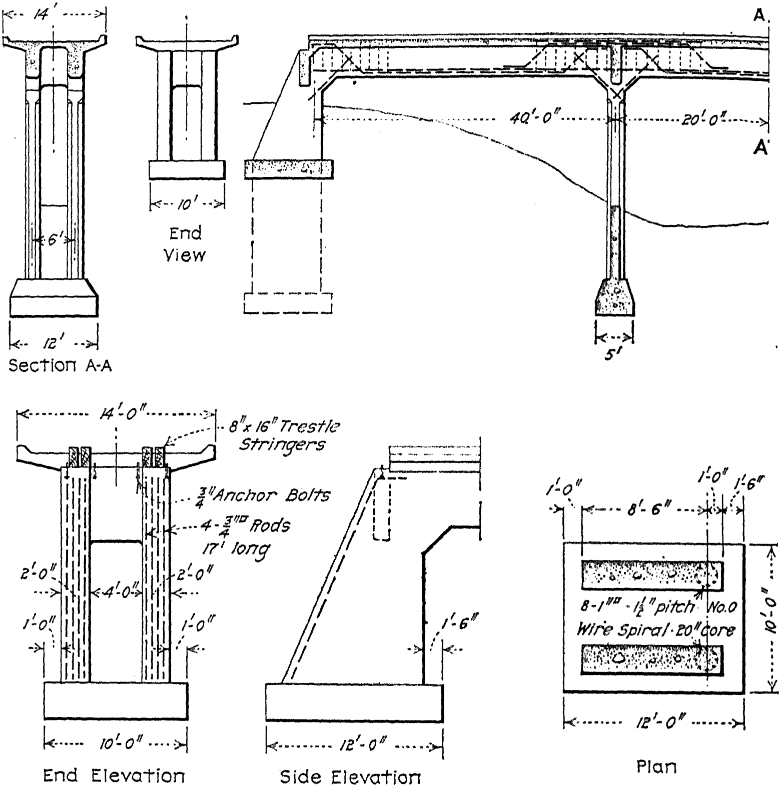
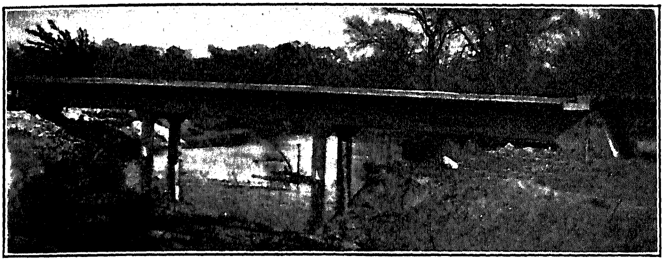


FIG. 289.—Details of railway bridge over White Rock Creek near Dallas, Texas.

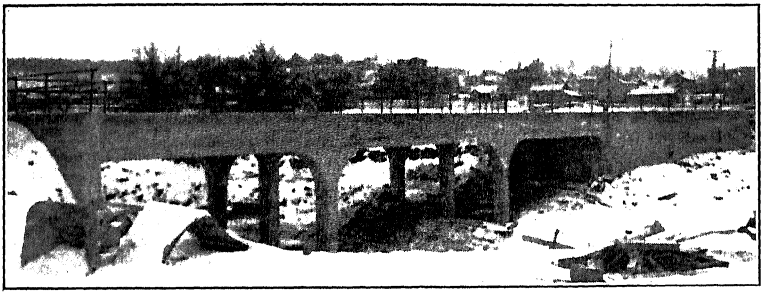
Fig. 289 gives the details of a continuous girder bridge to span a stream which is generally dry, but which at flood times reaches over a wide area of bottom lands. This railway bridge is also within the backwater area of the White Rock reservoir of the Dallas water supply so that ample provision had to be made for high-water conditions. The floor consists of a double T-beam which is monolithic with the bents and the abutments. There is no expansion joint in the structure since it was considered of sufficient strength to take all movement from end to end. Both ends of the structure are open between girder supports (Fig. 290), with short trestles connecting with the bridge which at some future time may be filled in if high water gives no trouble at this point. The girders were designed for Cooper's E-30 loading with an impact allowance of 100 per cent of the live load. The ribbed abutments should be noted.



Courtesy of Mr. Wm. E. Beilharz, Architect and Engineer, Dallas, Texas.

Fig. 290.—Railway bridge over White Rock Creek near Dallas, Texas.

A continuous girder bridge or trestle with no expansion joints and with abutments of the ordinary type is shown in Figs. 291 and 292. The structural steel core in the column bents is unusual but undoubtedly adds greatly to the rigidity.

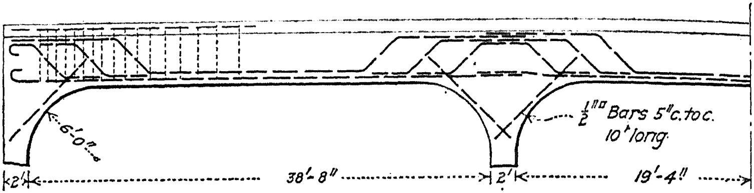


Courtesy of Mr. M. W. Torkelson, Bridge Engineer, Wisconsin Highway Commission.

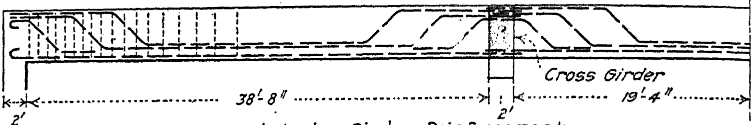
Fig. 291.—Mill Creek bridge, Village of Cazenovia, Richland Co., Wis.

Fig. 293 is an example of high trestle construction with the long columns braced both laterally and longitudinally.

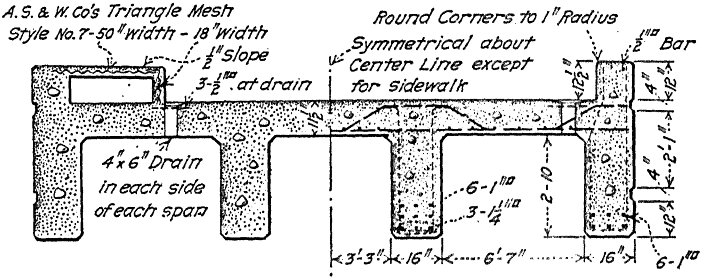
Fig. 294 gives the details of one of the typical bridges in



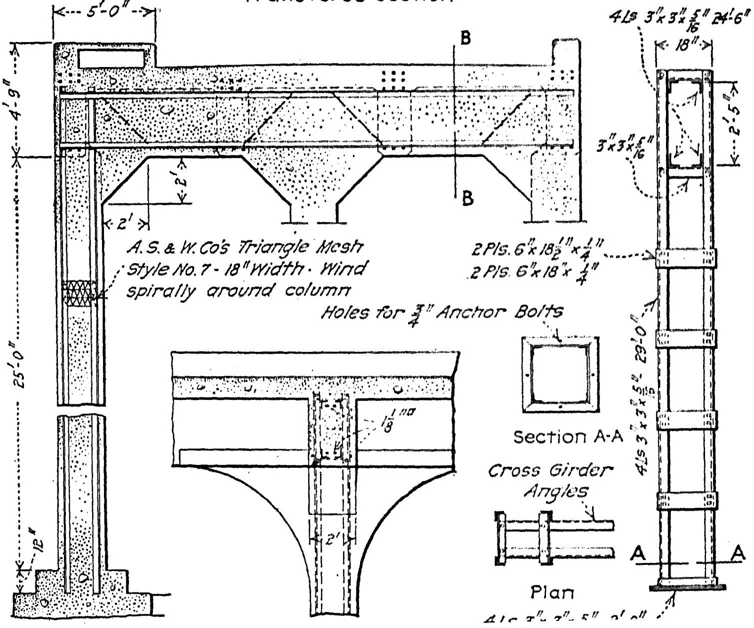
Side Girder Reinforcement



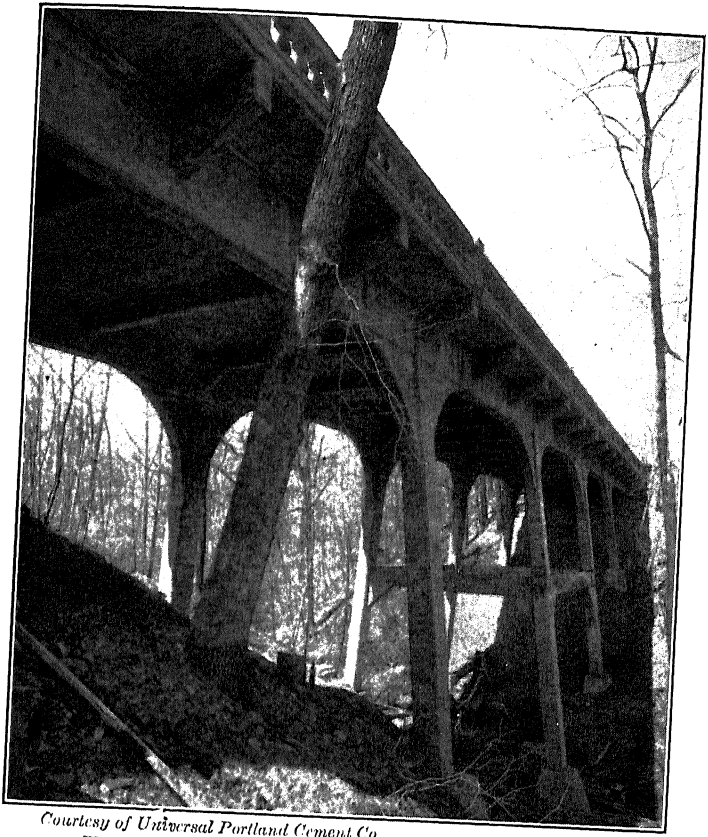
Interior Girder Reinforcement


A.S. & W. Co's Triangle Mesh Style No. 7 - 50" width - 18" width
1/2" Slope
3-1/2" at drain
Round Corners to 1" Radius
Symmetrical about Center Line except for sidewalk
1/2" Bar
4" x 6" Drain in each side of each span
1 1/2"
1 1/2"
4"
12"
2'-1"
4"
12"
2-10
6-1"
3-1 1/4"
6-1"
3'-3"
16"
6'-7"
16"

Transverse Section

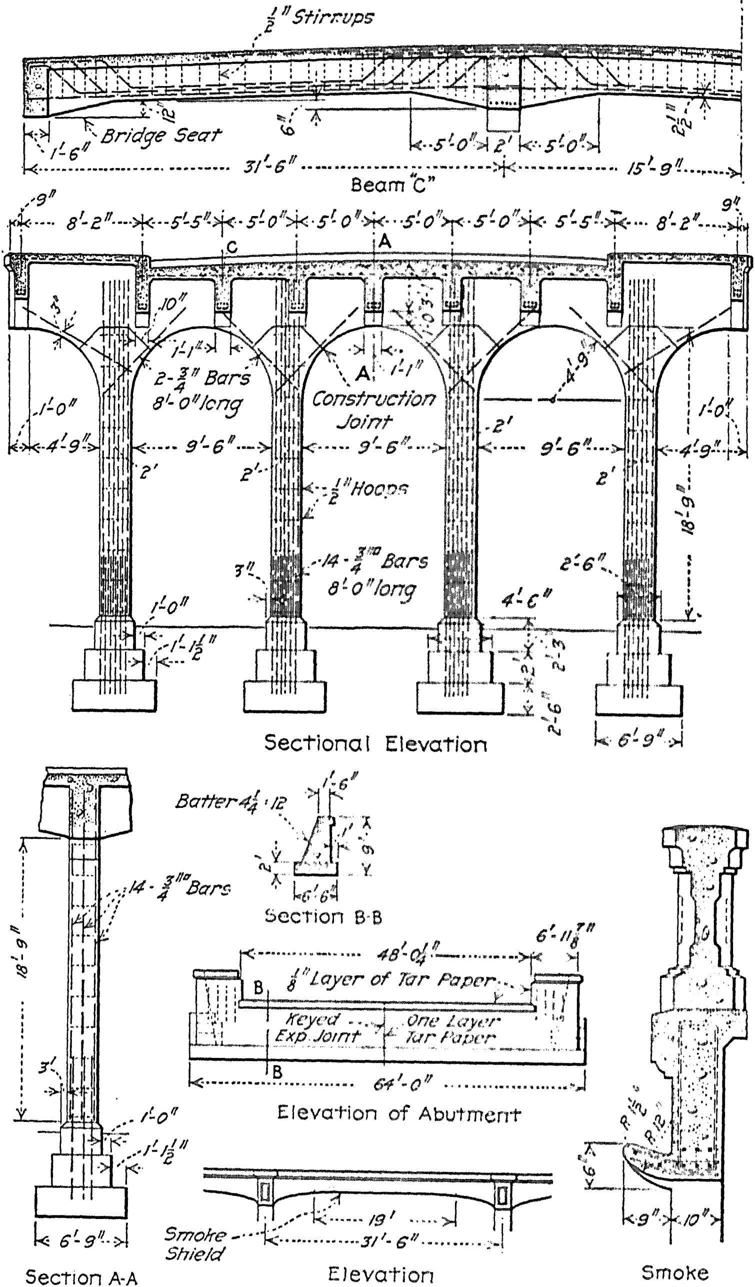

5'-0"
4'-9"
25'-0"
12"
B
B
2'
2'
A.S. & W. Co's Triangle Mesh Style No. 7 - 18" Width - Wind spirally around column
Holes for 3/4" Anchor Bolts
1/8"
2'
Section A-A
Cross Girder Angles
Plan
A
A
4 Ls 3" x 3" x 5/16" 24'-6"
18"
2'-5"
3" x 3" x 5/16"
2 Pls. 6" x 18 1/2" x 1/4"
2 Pls. 6" x 18" x 1/4"
29'-0"
4 Ls 3" x 3" x 5/16"

the Chicago, Milwaukee & St. Paul track depression work at Minneapolis. The girders are continuous from end to end with expansion joints at the abutments—that is, the girders were considered continuous over the two interior supports and simply supported at the ends. The moments and shears were calculated by influence lines in accordance with the theory of continuous structures (see Chapter XIII, Volume II) assuming constant I and unyielding supports. Corrections were made for $\frac{1}{8}$-in. settlement of supports for variable I. An unusual feature of the bridges is a curved shelf on the outer face of the outside girder which is intended to act as a smoke shield by diverting the smoke from the parapet as engines pass under the bridge.



Courtesy of Universal Portland Cement Co.

FIG. 293.—Forest Park bridge, Highland Park, Ill.


Beam "C"

Sectional Elevation

Section B-B

Elevation of Abutment

Section A-A

Elevation

Smoke Shield

FIG. 294.—Details of Bryant Avenue bridge over tracks of the

CHAPTER XVII

CANTILEVER BRIDGES

A type of bridge which in appearance is a concrete arch, but which in reality is composed of balanced cantilevers, is shown in Figs. 295 to 301 inclusive. A structure of this type can be made with longer spans than the ordinary girder and is suited to locations where the real arch would be exceedingly costly on account of unsatisfactory foundation conditions.

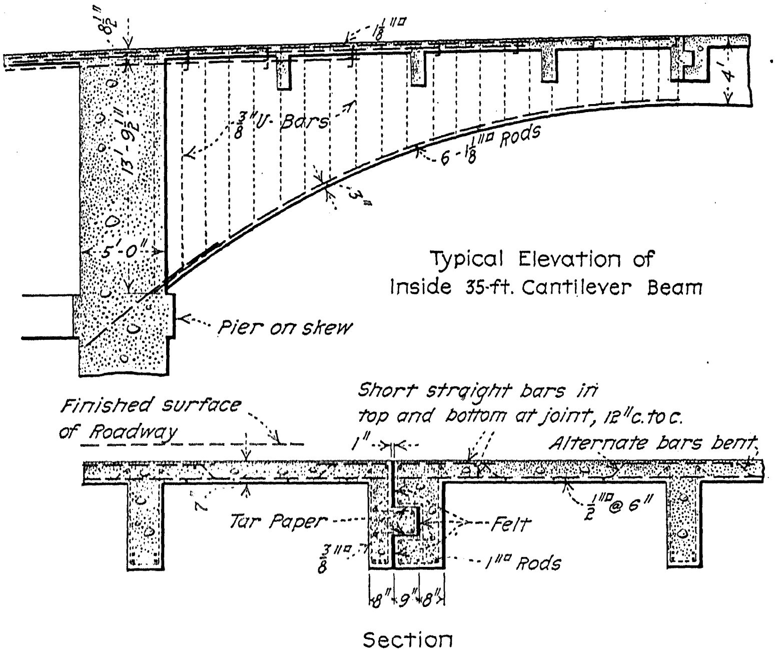
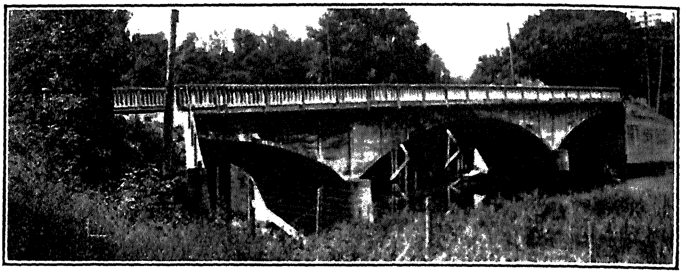


FIG. 295.—Details of Hopple Street viaduct, Cincinnati, Ohio.

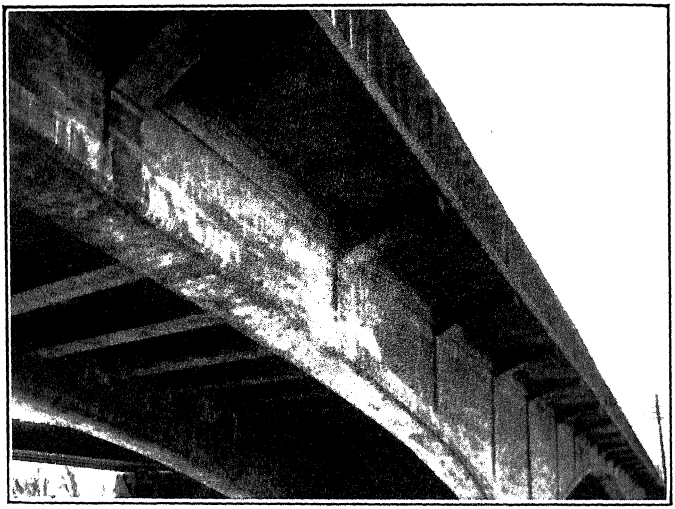
73. Theory of Design.—A pier and the cantilever arms on each side compose a unit, the arms being balanced for dead load and for full live load. The piers are designed for bending due to the maximum eccentric load that can be applied, and considering the load on only one of the cantilever arms at a time. The pier footings are designed so that the pressure on the base of a pier due to this same eccentric loading will not cause an intensity greater than the unit bearing value of the soil.

74. Examples of Cantilever Bridges.—The viaduct shown in Fig. 295 consists of twenty-five skewed spans, each span comprising two curved cantilever arms supported on reinforced-concrete piers. A single cantilever arm occurs at each end of the viaduct. Each cantilever arm comprises four curved ribs which were designed as cantilevers from the skewed piers. The joint at the center of each span is shown in detail. In designing, this joint was con-



Courtesy of Universal Portland Cement Co.

FIG. 296.—Cantilever bridge over Rouge River, Wayne County, Mich.



Courtesy of Universal Portland Cement Co.

FIG. 297.—Close view of cantilever bridge, Wayne County, Mich.

sidered as transmitting only shear from one cantilever arm to another and not any bending or arch action. Since each pier and its cantilever arms are symmetrical about the center line of the pier, no bending exists in the pier due to dead load or to full live load on both cantilevers. Piers, footings, and piling were designed to withstand the overturning effect produced by the loading of a single cantilever with the full live load.

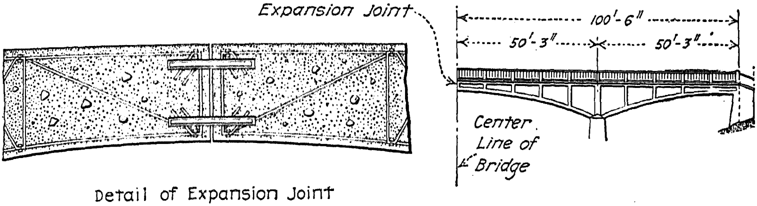
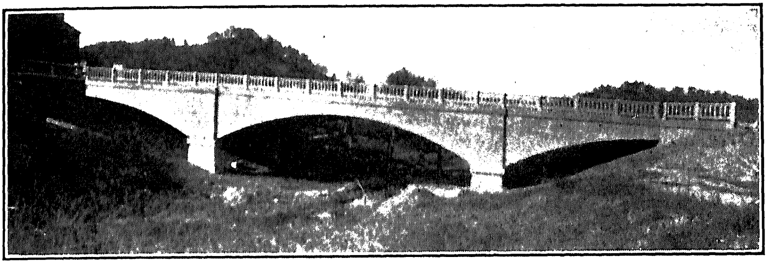


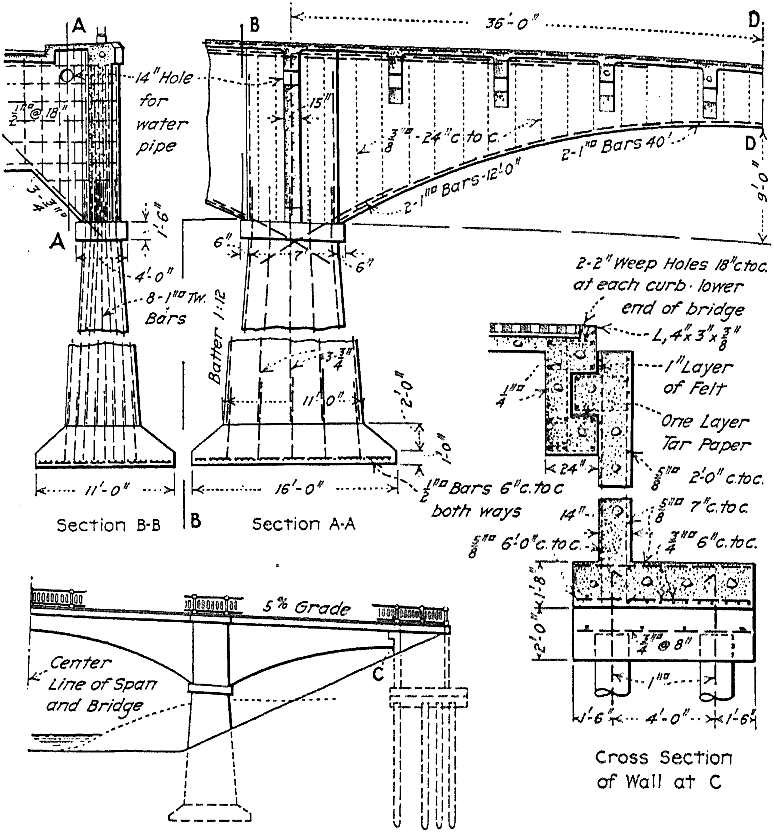
Fig. 298.—Half-elevation and detail of expansion joint of arch-shaped cantilever bridge over Rouge River, Wayne County, Mich.

The expansion joint at the center of the structure, shown in Figs. 296 to 298 inclusive, is entirely different from that employed in the bridge just described. It should be noted, however, that the same result is accomplished—that is, that the two abutting arms are permitted to move longitudinally, but not laterally or vertically. The joint consists of two 3-ft. pieces of $2\frac{1}{2}$-in. steel shafting each inserted in two $1\frac{1}{2}$-ft. pieces of 3-in. gas pipe embedded in the concrete. End joints were made by placing sheets of three-ply tar paper on top of the abutments before the end cantilevers were poured, thus permitting a slight movement of the ends of the structure under changes of load and temperature. There was no apparent deflection at any of the expansion joints due to live load.



Courtesy of Mr. Frank L. Raschig, Division Engineer, Division of Structures, Cincinnati, O.

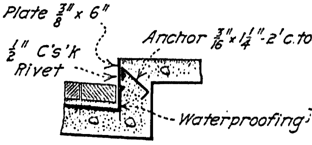
Fig. 299.—Cantilever bridge on Runnymede Avenue, Cincinnati, Ohio.



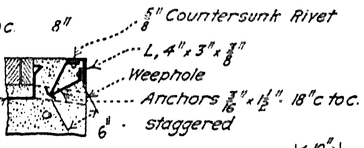
Section B-B

Section A-A

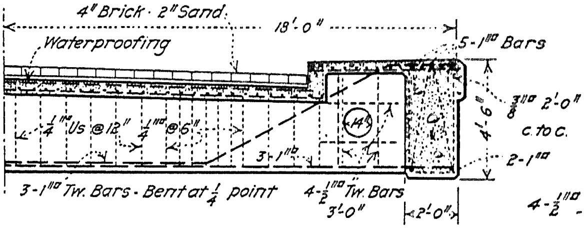
Cross Section of Wall at C



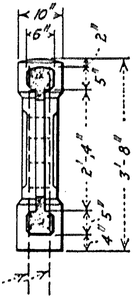
Curb Protection



Protection Angles at each end of Bridge

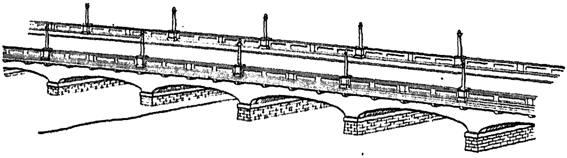


Cross Section D-D

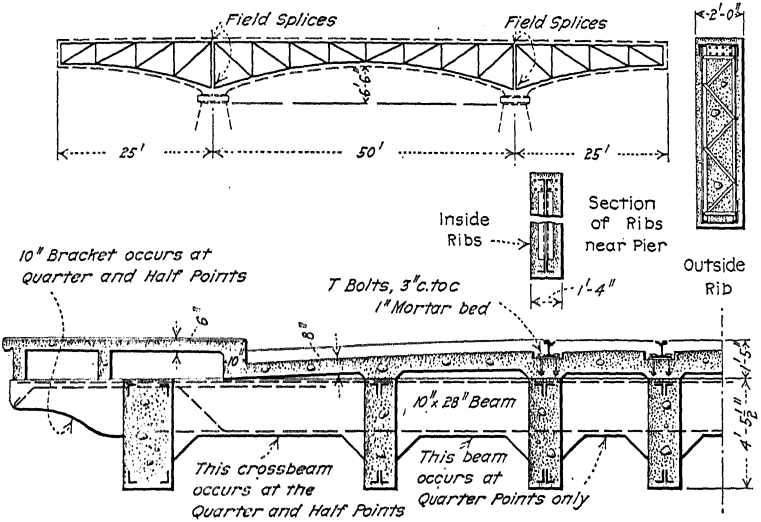


Post

The joints at the ends of the bridge shown in Figs. 299 and 300 are of the same type as employed in the viaduct spans of Fig. 295. The absence of such a joint at the center of the middle span is, however, the principal feature. In spite of this continuity between piers, no account was taken of continuous action upon supports and the bridge was designed in the same manner as the cantilever viaduct previously referred to. In fact, it has been found that a joint at the center of span is an unnecessary refinement in cantilever bridge design, and might be omitted. The bridge was designed and constructed so as not to rest on the abutments at all, the abutments being used merely to hold back the earthfill at each end and to serve as anchorage for the end cantilevers. A structure of this type with end openings closed



Perspective View


Field Splices
Field Splices
6'-9"
25'
50'
25'
2'-0"
Inside Ribs
Section of Ribs near Pier
Outside Rib
1'-4"
10" Bracket occurs at Quarter and Half Points
T Bolts, 3"c.to c
1" Mortar bed
6"
8"
10"
1'-5"
4'-5½"
10"x 28" Beam
This crossbeam occurs at the Quarter and Half Points
This beam occurs at Quarter Points only

Half Section at Quarter and Half Points of Span

FIG. 301.—Details of Washington Street bridge, Norwalk, Conn.

by earthfill has all the appearances of a real arch. In such a case abutments are not needed.

If a number of bridges similar to the bridge of Figs. 299 and 300 were placed end to end, the result would be essentially a structure such as shown in Fig. 301. It should be noted that the Norwalk bridge is continuous in sections of maximum length of 100 ft. The structural steel work was designed to be self-supporting during erection and to carry the erection stresses of the forms and the fluid concrete in the ribs, cross-girders, and sidewalk brackets. Although a deflection of $\frac{1}{2}$ in. at the free ends of the cantilevers was anticipated, a deflection of only $\frac{1}{16}$ in. to $\frac{1}{8}$ in. actually resulted due principally to the rigidity of the forms and to the fact that the concrete was continuously setting during the process of placing. The combined steel and concrete in the ribs was proportioned to carry the roadway slab, paving, and all live loads. The trusses were proportioned to carry all shear not safely taken by the concrete but were not proportioned to carry all the tension developed by the bending moment since extra horizontal rods were embedded in the concrete adjacent to the top chords of the trusses. The following information is given in a letter to the writer from Mr. W. Walters Pagon, Resident Engineer on the South Norwalk bridge:

"No plans were made by the contractors for the forms around the ribs, but they were of standard construction made of $\frac{7}{8}$-in. dressed lumber secured to 4 × 4-in. studs, spaced about 2 ft. centers, and braced through the ribs by means of heavy twisted wire which passed around the waling pieces. These waling pieces were laid longitudinally and at vertical distances apart of approximately 5 ft. and consisted of 4 × 4-in. and 4 × 6-in. timbers. The bottom forms were secured to the structural-steel frames which comprise the reinforcing for the ribs, by means of pairs of $\frac{3}{4}$-in. round bolts spaced every $12\frac{1}{2}$ ft. At these points 6 × 8-in. timbers were placed, supported by the bolts just mentioned, and running transversely from outside rib to outside rib, their purpose being not only to support the bottom forms but also to brace the steel ribs together until the concrete cross beams were cast. The shape of the intrados of the ribs was segmental, relieved by little three-centered fillets at the ends. The forms for these fillets were made up in advance and were supported by the bolts at the ends of the segmental curve.

"We employed two types of steel ribs both built up of structural angles, but those for the concrete ribs, 24 in. in width, were made of box section and those for the intermediate concrete ribs were of I-section. My experience was that it was possible to get much better results with the wider beams because, owing to the fact that there was a 12-in. space between the angles of each flange, it was possible to pour the concrete into the middle of the rib; whereas in the narrow ribs the concrete while being poured was divided by the top chord into two parts which followed down the two sides of the rib, with the result that there was considerable difficulty encountered in preventing separation of the materials. I also found that it was inadvisable to use lattice bars on compression members, for in places where latticing was not used, but battens instead, the placing of the concrete was very much facilitated."

CHAPTER XVIII

REINFORCED CONCRETE IN STEEL BRIDGE CONSTRUCTION

It is not the intention to go into great detail in this chapter in treating of reinforced concrete as applied to the supports and floors of steel bridges. The idea in including any discussion on this subject is simply to call attention to the fact that reinforced concrete is not limited strictly to bridges built entirely of concrete.

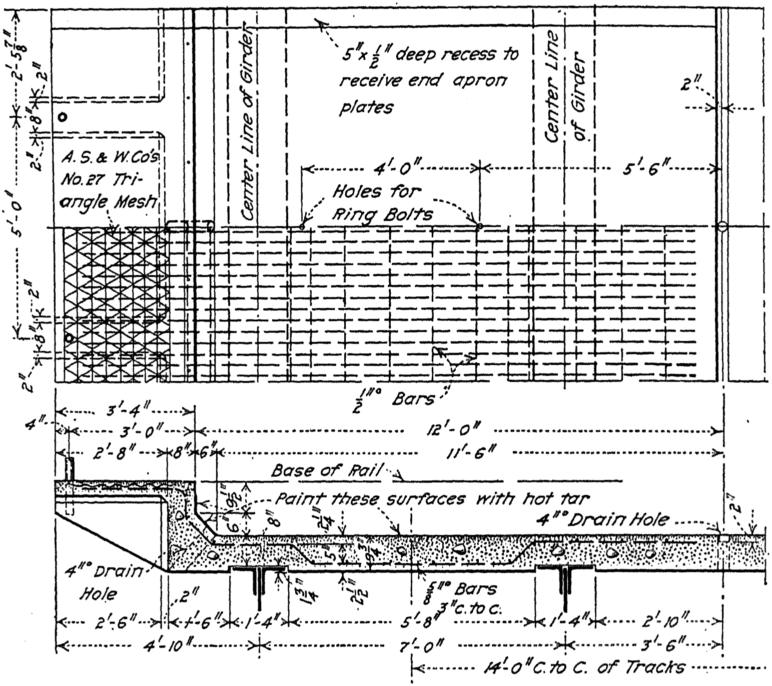


FIG. 302.—Reinforced-concrete deck for steel girder spans on D. M. & N. Ry.

75. Concrete Floors on Steel Bridges.—Within recent years it has become the general opinion among railroad engineers that a ballasted solid floor is the most satisfactory form of floor for steel bridges. Perhaps the best type of such a floor is the reinforced-concrete slab resting directly on the steel-floor members.

Figs. 302 and 303 show details of reinforced-concrete deck slabs for plate-girder spans. The slab floors are seen to rest directly

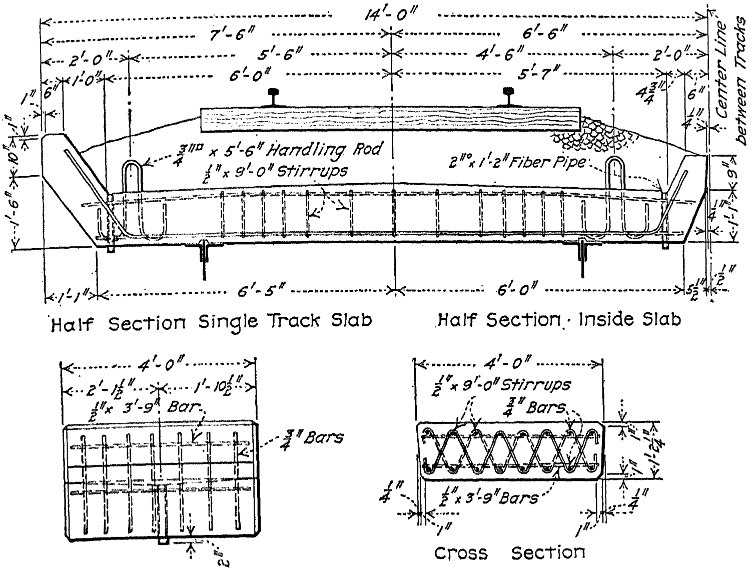


Fig. 303.—Standard reinforced-concrete slab for deck girders of C. M. & St. P. Ry.

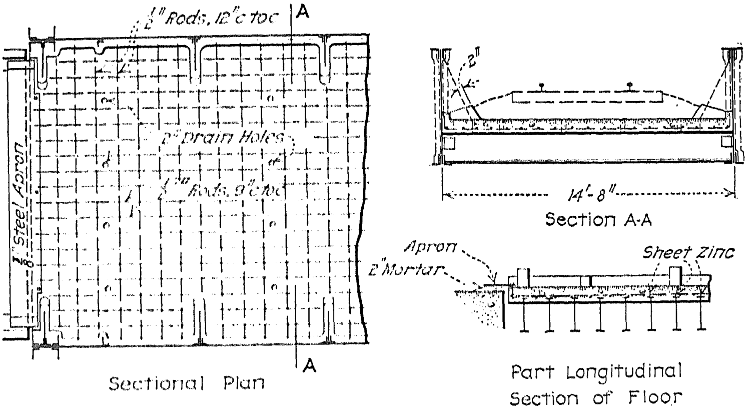

$\frac{1}{2}$" Rods, 12" c to c
A
2" Drain Holes
$\frac{1}{2}$" Rods, 9" c to c
$\frac{1}{8}$" Steel Apron
A
Sectional Plan
2"
14'-8"
Section A-A
Apron
2" Mortar
Sheet Zinc
Part Longitudinal Section of Floor

Fig. 304.—Reinforced-concrete floor for through plate girder bridge, C. B. & Q. R. R.

upon the top flange of the steel girders. Comparison of the two designs is of value since they show a wide difference in the of the reinforcement. The slabs are usually made at some convenient location and hoisted into place when sufficiently cured. Before adding the ballast, the upper surface of the slabs is thoroughly waterproofed

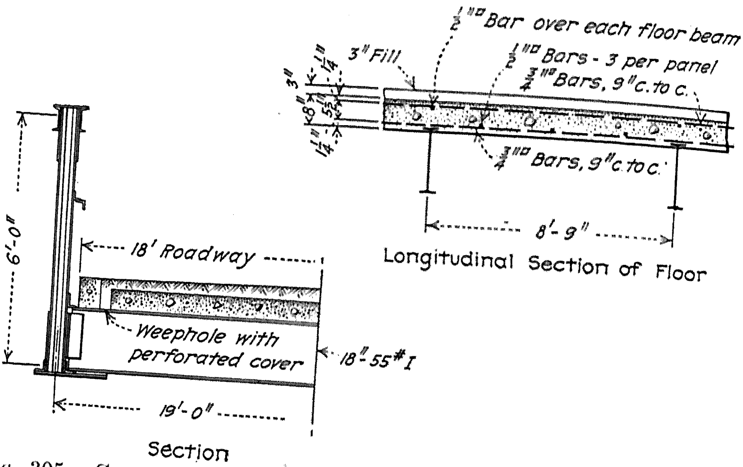


Fig. 305.—Concrete slab floor for highway steel truss span, Iowa Highway Commission.

by painting with tar paint. Drain holes are placed in such a position as to keep the drip clear of the steel members.

A reinforced-concrete floor for a through plate-girder bridge is shown in Fig. 304. The concrete of the floor slab is seen to

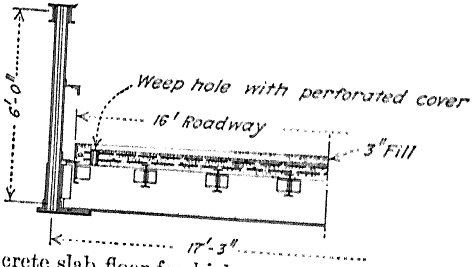


Fig. 306.—Concrete slab floor for highway steel truss span, Iowa Highway Commission.

extend up on the sides to form curbs, and these curbs extend entirely around the gusset plates.

Steel I-beams encased in concrete and supporting a reinforced-concrete floor slab is the most common type of highway bridge with steel floor members. On account of the ease with which forms may be constructed to hold the concrete, this bridge for short spans is sometimes used in preference to slab bridges of all concrete. The only disadvantage of this bridge is in point of economy.

Timber floors for highway bridges are not in great favor at the present time. Since the expense of maintaining wood floors is considerable, the engineers of a number of highway commissions design practically all steel bridges with concrete floors covered by a wearing surface of gravel or macadam. For exceedingly light traffic on country bridges, driving is sometimes allowed directly on top of the floor slab, making an allowance of at least 1 in. in the thickness of the slab for wear and cutting transverse grooves to prevent slipping. Figs. 305 and 306 show typical designs of reinforced-concrete floors for steel-truss spans.

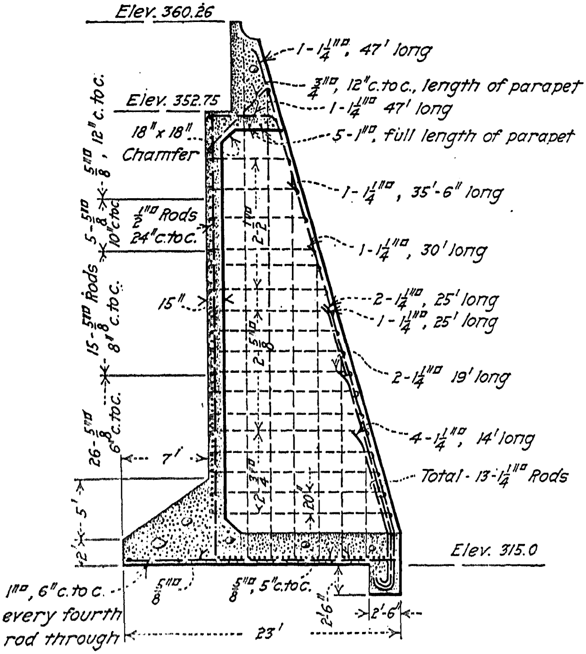
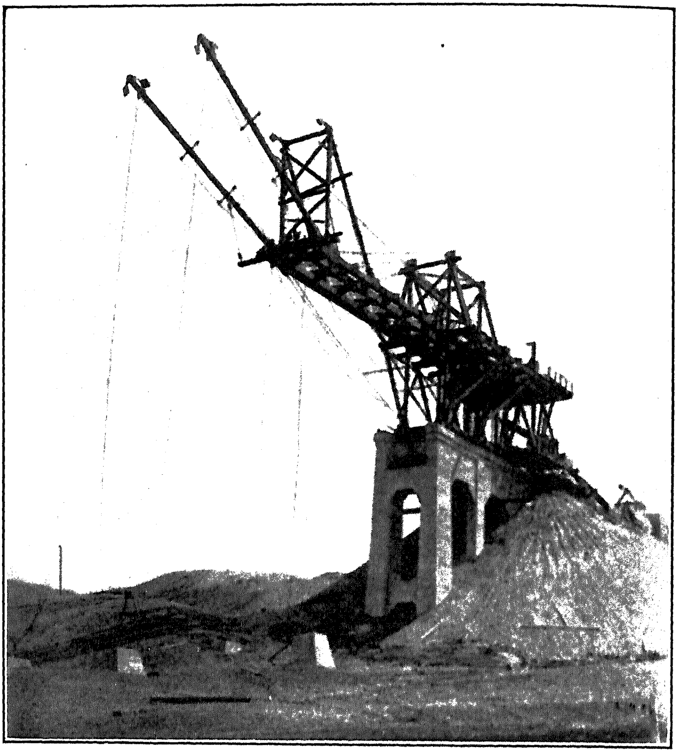


FIG. 307.—Section through bridge seat of typical reniforced-concrete abutment for steel bridges.

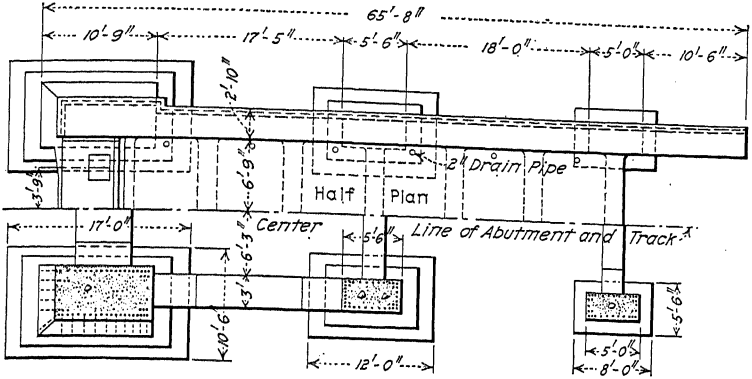
76. Reinforced-concrete Abutments for Steel Bridges.—For high abutments to steel bridges, the use of reinforced concrete leads to great economy. Abutments of this material for moderate heights are usually of the counterforted type (see Art. 12, Volume II) with the bridge seat formed by a heavy reinforced-concrete slab extending over the top of the supporting counterforts. A cross-section through a typical bridge seat is shown in Fig. 307.



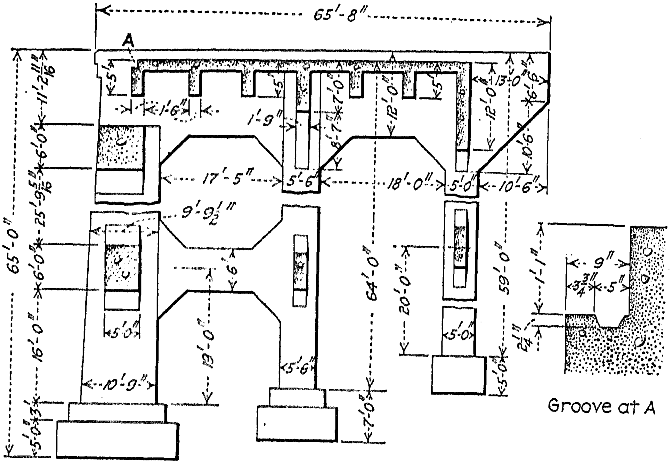
Courtesy of Mr. H. C. Lothholz, Engineer of Design, C. M. & St. P. Ry.

FIG. 308.—East abutment, Judith River viaduct, Chicago, Milwaukee & St. Paul Ry.

A skeleton type of reinforced-concrete abutment has been adopted by the Chicago, Milwaukee & St. Paul Railway for steel viaducts of considerable height. Abutments of this type carry the track over the end slopes of high embankments in such a manner as to avoid the heavy horizontal earth pressures which would be developed if ordinary abutments were constructed



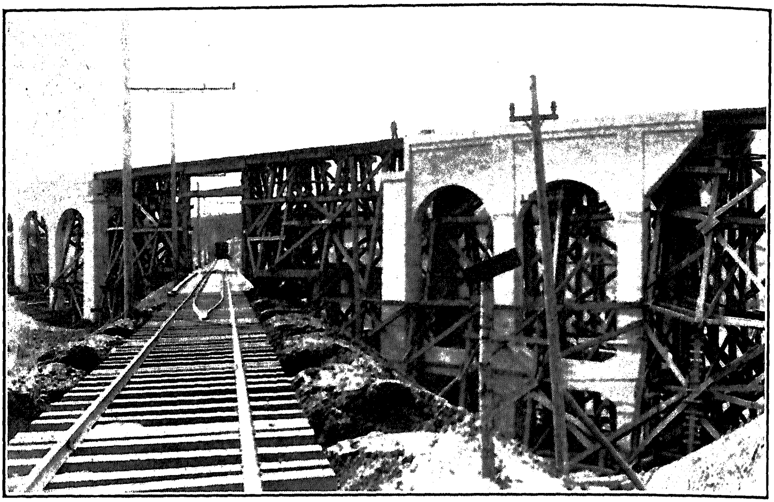
Half Section



Section at Center Line of Track

FIG. 309.—Details of abutments of the Judith River viaduct in Montana, C. M. & St. P. Ry.

under the same conditions. Views of two typical skeleton abutments designed by this company are presented in Figs. 308 and 310. The structure of Fig. 308 is shown in some detail in Fig. 309. It should be noted that in these structures a high embankment need not be brought to an abrupt termination as the separated concrete shafts and cantilevered girders allow the fill to assume freely its natural slope. A valuable feature of this type of abutment lies in the fact that it can be constructed in the midst of a timber trestle with very little interference with the timber work.



Courtesy of Mr. H. C. Lothholz, Engineer of Design, C. M. & St. P. Ry.

FIG. 310.—Skeleton abutments for steel truss span on Chicago, Milwaukee & St. Paul Ry. Timber trestle within abutments being removed.

PART III

CULVERTS

The term culvert is usually applied to structures built to carry surface water or small streams through highway or railroad embankments. When the area of waterway required is comparatively small, a pipe culvert is usually the most economical. For the larger openings either the box or arch form should be employed depending upon the available head-room, the depth of fill, the condition of the foundation, and whether or not an artistic arch design is especially desirable.

The ordinary type of arch culvert and the box culvert without a load-supporting floor (called *open-box* culvert) are in reality small bridges, and it is sometimes a question of how large such structures may become before they should be considered strictly in the bridge class. No arbitrary division will be adhered to in the following chapters except that a culvert will be kept in mind as a structure which can be completely and economically standardized, based on a given area of waterway and height of embankment.

CHAPTER XIX

FACTORS IN CULVERT DESIGN

77. Culvert Efficiency.—A culvert to be efficient in the amount of water it can discharge should have its head-walls or wings arranged so as to facilitate the flow, and its bed should be considerably inclined for those cases where the channel below the culvert will permit the water to flow away freely. If any well-defined stream bed exists, the bed of the culvert should have the same inclination as that of the stream, as otherwise either the outlet or inlet end will clog depending upon whether the slope of the culvert is greater or less respectively than the slope of the stream bed.

Any projections in the culvert bed should be avoided as they will retard the water and diminish efficiency. It is also important that culverts be placed across roadways in the direction of the stream flow since, if this is not done, clogging and subsequently washouts will be likely to occur.

A culvert will discharge twice as much under a head of 4 ft. as under a head of 1 ft., but water should not be allowed to dam up in this manner unless the culvert is well constructed through a water-tight embankment.

78. Waterway Required.—Assuming an efficient culvert design, the area of waterway required depends principally upon the maximum rate of rainfall, the area and shape of the watershed, the kind and condition of the soil throughout this watershed area, and the character and inclination of both drainage surface and stream bed. A number of empirical formulas have been proposed by which to calculate the required culvert opening, but obviously a problem of this kind does not admit of an exact mathematical solution and the desired size of culvert should be determined by direct observation whenever that is possible.

In a new country an empirical formula is often the only method by which the required area of waterway can be determined. Talbot's formula is the one most generally employed and is as follows:

$$A = C\sqrt[4]{a^3}$$

where A = area of waterway in square feet, a = drainage area in acres, and C is a coefficient which varies from 1 to $\frac{1}{6}$ in the following manner: "For steep and rocky ground, C varies from $\frac{2}{3}$ to 1. For rolling agricultural country subject to floods at times of melting of snow, and with the length of valley three or four times its width, C is about $\frac{1}{3}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $\frac{1}{5}$, $\frac{1}{6}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert."[1]

The proper area of waterway can best be determined by noting the dimensions of existing openings on the same stream and by careful observation of the stream and the amount of water which it carries at flood times. This amount of water can be determined by measuring a cross-section of the stream at some narrow place near the culvert site.

---

[1] Selected Papers of the Civil Engineers' Club of the University of Illinois, No. 2, page 14.

79. Length of Culverts.—The length of a culvert should depend upon the width of roadway and the depth of fill on top of the culvert. The slope of an earth fill can generally be taken as $1\frac{1}{2}$ to 1—that is, for every 1 ft. in height, the horizontal distance is $1\frac{1}{2}$ ft.

In highway construction the roadway should never be narrowed at a culvert since such a practice is dangerous and the construction unsightly.

80. Design of Ends.—The arrangement of the head-walls or wings may be substantially the same for all arch and box culverts. The arrangement should be such that the embankment is protected and the flow of water facilitated. Wing walls may be built parallel with or at right angles to the axis of the culvert, or they may be so placed as to make an angle (usually 30 or 45°) with this axis.

For the shorter spans (including spans for pipe culverts), wings parallel with the roadway are generally used for low fills and, in highway construction at least, these head-walls are carried up above the grade line to provide a low guard rail. This type of end, however, is not economical for the larger spans since the straight wings under such conditions need to be made of considerable length and height to retain the fill efficiently. A low guard rail may be formed with flared wings by raising and coping both head- and wing-walls. The top of the wings, of course, should have a slope consistent with the slope of the earth fill.

Flared wings, especially at the upstream end, are the best for hydraulic reasons and, when used, the culvert is less likely to become choked than when either of the other two forms of wing walls are employed. Straight wings—namely, wings parallel to the axis of the culvert are of advantage in railroad construction when an extension of the culvert is likely to be made in the near future to accommodate another track.

It is common practice to design wing and head-walls of sufficient length to keep the culvert opening clear when the earth is assumed to fall around the ends on a $1\frac{1}{2}$ to 1 slope. In some cases a steeper slope could be assumed, but some soils take even flatter slopes than the standard.

Box and arch culverts are built both with and without a floor, but in almost every case the smooth waterway that can be obtained by using a concrete floor will greatly increase the capacity vent any danger from erosion of the stream bed and undermining of the foundation. The floor at the ends of the culvert should be provided with an apron or baffle wall at its outer edge, and this wall should in all cases be carried as low as the bottom of the footings. If especially desirable, the floor should extend out to the end of the wing walls.

Mr. C. B. McCullough, Designing Engineer of the Iowa State Highway Commission, writes as follows in a letter to the writer under date of June 5, 1915:

> "I had the opportunity last summer to make an inspection of some seventy or eighty concrete bridges and culverts, a large number of them being culverts. These were structures constructed before the going into effect of the present state highway law and, of course, in many cases were constructed without any engineering supervision of any kind. The foundations almost universally were carried to a very slight depth below the stream bed and in practically every case there was an utter lack of proper study of stream conditions. The most serious defects noticed in the inspection as far as the small culverts were concerned could be traced to erosion of the creek bed.
>
> "A tabulation of some of the results disclosed by this inspection trip was published in our Service Bulletin for December, 1914. These and other observations have led me to believe that the fundamental principle controlling the design of culverts is that erosion and the destructive effects incident thereto is greater in small spans than for large ones, and that extraordinary care in placing these foundations well below the line of ultimate scour is of the greatest importance."

CHAPTER XX

PIPE CULVERTS

One or more lines of pipe with suitable head-walls to protect the embankment is the simplest form of culvert. The pipe may be of burned clay, cast iron, plain concrete, or reinforced concrete; but, on account of frequent breakages, there seems to be a tendency at the present time to discontinue the use of vitrified and cast-iron pipe (including pipes of plain concrete), except possibly in those localities where materials for making concrete are scarce. All kinds of pipe culverts have the same type of concrete head-walls, consequently this chapter will treat only of pipe culverts of reinforced concrete.

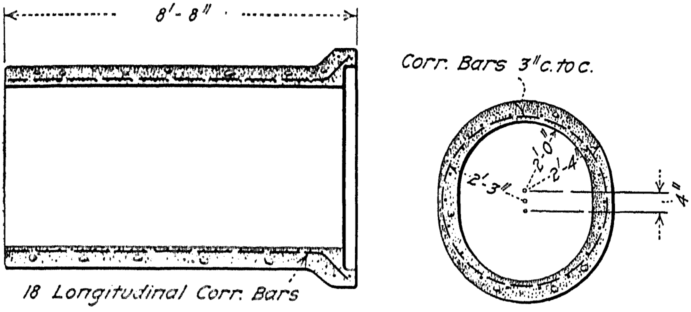
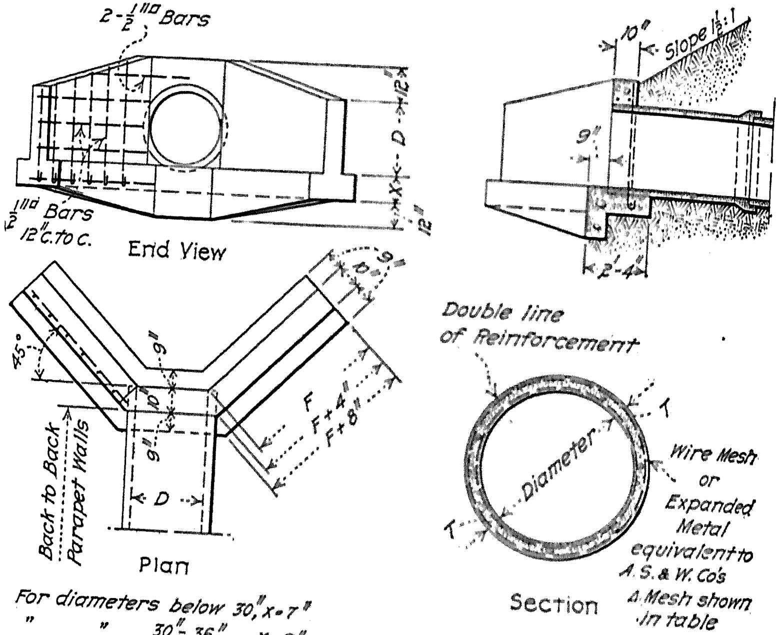


FIG. 311.—Reinforced-concrete pipe.

Since it is desirable to make as few openings as possible through an embankment, water is usually conducted along the side of the roadway until at least a 15-in. diameter of pipe is required. In the following discussion a pipe of at least this size will be assumed.

Reinforced culvert pipes are usually made in from 4 to 8-ft. lengths, and with bell and spigot joints. The largest diameter of pipe yet made is 72 in. The pipes usually have a hoop reinforcement which is near the interior surface at the top and bottom of the pipe, and near the exterior surface at the sides (Fig. 311). Pipe with a double line of reinforcing is also used, as shown in Fig. 312. Longitudinal reinforcement is provided in pipes of the longer lengths due to the likelihood of beam action if settlement takes place.

81. Pressure in Trenches.—The most elaborate and thorough investigation of the subject of pressure on pipes in ditches was


2-½″□ Bars
½″□ Bars
12″ c. to c.
12″
D
X
12″
End View
10″
Slope 1½:1
9″
2′-4″
45°
9″
9″
10″
9″
10″
D
F
F+4″
F+8″
Back to Back Parapet Walls
Plan
Double line of Reinforcement
Diameter
T
T
Wire Mesh or Expanded Metal equivalent to A. S. & W. Co's. A Mesh shown in table
Section

For diameters below 30″, X = 7″
" " 30″-36″, X = 9″
" " 37″-42″, X = 12″
" " 18″, F = 4′-0″
" " 24″, F = 4′-6″
" " 36″, F = 5′-0″
" " 42″, F = 5′-6″

Table of Dimensions for Circular Pipe

Diameter	Thickness of Shell = T	Required Steel Area for each line A S & W Co. sq. in.
15″	2.25″	Style 6 - .058 per 12″
18″	2.50″	" 5 - .077 " "
24″	3.00″	" 4 - .102 " "
30″	3.50″	" 42 - .151 " "
36″	4.00″	" 23 - .170 " "
42″	4.50″	" 32 - .225 " "

Fig. 312.—Standard dimensions for concrete pipe culverts with concrete head-walls, Iowa Highway Commission.

made a few years ago at the Iowa State College of Agriculture and Mechanic Arts, the results of which were published in Bulletin No. 31[1] of the Engineering Experiment Station. The investigation was made with special reference to drain tile and sewer pipe, but the results apply equally well to culverts laid in trenches. The following summary of the general principles of the theory of loads on pipes in ditches (a theory which was developed by reason of this investigation) is taken from the bulletin above mentioned. The correctness and reliability of this theory were demonstrated with remarkable closeness by actual weighings of loads on pipes, the pipes ranging from 12 in. to 36 in. in internal diameter placed in ditches from 0 to 19 ft. in depth.

---

[1] Written by Prof. Anson Marston and Mr. A. O. Anderson.

TABLE NO 1.—APPROXIMATE SAFE WORKING VALUES OF "*C*," THE COEFFICIENT OF LOADS ON PIPES IN DITCHES

Ratio $\frac{H}{B}$	For damp top soil and dry and wet sand	For saturated top soil	For damp yellow clay	For saturated yellow clay
0.5	0.46	0.47	0.47	0.48
1.0	0.85	0.86	0.88	0.90
1.5	1.18	1.21	1.25	1.27
2.0	1.47	1.51	1.56	1.62
2.5	1.70	1.77	1.83	1.91
3.0	1.90	1.99	2.08	2.19
3.5	2.08	2.18	2.28	2.43
4.0	2.22	2.35	2.47	2.65
4.5	2.34	2.49	2.63	2.85
5.0	2.45	2.61	2.78	3.02
5.5	2.54	2.72	2.90	3.18
6.0	2.61	2.81	3.01	3.32
6.5	2.68	2.89	3.11	3.44
7.0	2.73	2.95	3.19	3.55
7.5	2.78	3.01	3.27	3.65
8.0	2.82	3.06	3.33	3.74
8.5	2.85	3.10	3.39	3.82
9.0	2.88	3.14	3.44	3.89
9.5	2.90	3.18	3.48	3.96
10.0	2.92	3.20	3.52	4.01
11.0	2.95	3.25	3.58	4.11
12.0	2.97	3.28	3.63	4.19
13.0	2.99	3.31	3.67	4.25
14.0	3.00	3.33	3.70	4.30
15.0	3.01	3.34	3.72	4.34
Infinity	3.03	3.38	3.79	4.50

For values of $\frac{H}{B}$ not given in table, sufficiently accurate values of "*C*" can be obtained by interpolation. H = height of fill, above top of pipe, in feet.

The weight of the filling in a drainage or sewerage ditch at the time of maximum load on the pipe is carried partly by the pipe and partly by friction against the sides of the ditch. Cohesion greatly reduces the loads carried by the pipe at ordinary times after the ditch is refilled and partly consolidated, except in the case of clean sand or gravel filling, but does not appreciably affect the maximum loads.

2. "The maximum loads on pipes in ditches, due to the weight of ditch-filling materials, will usually occur at the time of the first very thorough surface flooding of the ditch filling after construction, when there is a large settlement of the refill, but there is possibility of their occurring later, at the time of extreme saturation of the ditch filling, by surface flooding of the ditch and by overcharging of the drain or sewer. The maximum loads may even be postponed for many years in some cases, as is frequently shown by settlement of the filling in old ditches during paving construction.

3. "Safe values of the ordinary maximum loads on pipes in ditches, due to the weight of ditch-filling materials, can be computed by the following formula:

$$W = CwB^2$$

where W = load on pipe in ditches, in pounds per linear foot.
C = coefficient of loads on pipes in ditches.
w = weight of ditch-filling material, in pounds per cubic foot.
B = breadth of ditch at top of pipe, in feet.

using the values of "C" given in Table No. 1 or, more conveniently, estimating directly from Table No. 2.

4. "In calculating the maximum loads on pipes in ditches, due to the weight of ditch filling, by the above formula and Tables Nos. 1 and 2,

TABLE NO. 2.—APPROXIMATE MAXIMUM LOADS, IN POUNDS PER LINEAR FOOT, ON PIPES IN DITCHES FROM COMMON DITCH-FILLING MATERIALS

H = height of fill above top of pipe	B = breadth of ditch a little below top of pipe									
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.
	Partly compacted damp top soil 90 lb. per cu. ft.					Saturated top soil 110 lb. per cu. ft.				
2 ft.	130	310	490	670	830	170	380	600	820	1,020
4 ft.	200	530	880	1,230	1,580	260	670	1,090	1,510	1,950
6 ft.	230	690	1,190	1,700	2,230	310	870	1,500	2,140	2,780
8 ft.	250	800	1,430	2,120	2,790	340	1,030	1,830	2,660	3,510
10 ft.	260	880	1,640	2,450	3,290	350	1,150	2,100	3,120	4,150

TABLE NO. 2.—APPROXIMATE MAXIMUM LOADS IN POUNDS PER LINEAR FOOT, ON PIPES IN DITCHES FROM COMMON DITCH-FILLING MATERIALS (*Continued.*)

H = height of fill above top of pipe	B = breadth of ditch a little below top of pipe									
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.
	Dry sand 100 lb. per cu. ft.					Saturated sand 120 lb. per cu. ft.				
2 ft.	150	340	550	740	930	180	410	650	890	1,110
4 ft.	220	590	970	1,360	1,750	270	710	1,170	1,640	2,100
6 ft.	260	760	1,320	1,890	2,480	310	910	1,590	2,270	2,970
8 ft.	280	890	1,590	2,350	3,100	340	1,070	1,910	2,820	3,720
10 ft.	290	980	1,820	2,720	3,650	350	1,180	2,180	3,260	4,380
12 ft.	300	1,040	2,000	3,050	4,150	360	1,250	2,400	3,650	4,980
14 ft.	300	1,090	2,140	3,320	4,580	360	1,310	2,570	3,990	5,490
16 ft.	300	1,130	2,260	3,550	4,950	360	1,350	2,710	4,260	5,940
18 ft.	300	1,150	2,350	3,740	5,280	360	1,380	2,820	4,490	6,330
20 ft.	300	1,170	2,420	3,920	5,550	360	1,400	2,910	4,700	6,660
22 ft.	300	1,180	2,480	4,060	5,800	360	1,420	2,980	4,880	6,960
24 ft.	300	1,190	2,540	4,180	6,030	360	1,430	3,050	5,010	7,230
26 ft.	300	1,200	2,570	4,290	6,210	360	1,440	3,090	5,150	7,460
28 ft.	300	1,200	2,600	4,370	6,390	360	1,440	3,120	5,240	7,670
30 ft.	300	1,200	2,630	4,450	6,530	360	1,440	3,150	5,340	7,830
Infinity	300	1,210	2,730	4,850	7,580	360	1,450	3,270	5,820	9,090
	Partly compacted damp yellow clay 100 lb. per cu. ft.					Saturated yellow clay 130 lb. per cu. ft.				
2 ft.	160	350	550	750	930	210	470	730	1,000	1,240
4 ft.	250	620	1,010	1,400	1,800	340	840	1,330	1,870	2,370
6 ft.	300	830	1,400	1,990	2,580	430	1,140	1,900	2,630	3,410
8 ft.	330	990	1,720	2,500	3,250	490	1,380	2,360	3,360	4,400
10 ft.	350	1,110	2,000	2,920	3,880	520	1,570	2,760	3,980	5,270
12 ft.	360	1,200	2,220	3,320	4,450	540	1,730	3,100	4,560	6,050
14 ft.	370	1,280	2,410	3,650	4,950	560	1,850	3,410	5,050	6,760
16 ft.	370	1,330	2,570	3,950	5,400	570	1,940	3,660	5,510	7,440
18 ft.	380	1,390	2,710	4,210	5,810	570	2,020	3,880	5,930	8,060
20 ft.	380	1,410	2,830	4,450	6,180	580	2,090	4,070	6,280	8,610
22 ft.	380	1,430	2,920	4,640	6,500	580	2,140	4,240	6,610	9,130
24 ft.	380	1,450	3,000	4,820	6,800	580	2,180	4,380	6,910	9,590
26 ft.	380	1,470	3,060	4,980	7,080	580	2,210	4,500	7,160	10,010
28 ft.	380	1,480	3,120	5,100	7,310	580	2,240	4,610	7,380	10,430
30 ft.	380	1,490	3,170	5,230	7,530	580	2,260	4,700	7,590	10,780
Infinity	380	1,520	3,410	6,060	9,480	580	2,340	5,270	9,360	14,620

the value to use for H is the height of the filling above the top of the pipe, and the value for B is the breadth of the ditch a little below the top of the pipe. The width of the ditch above the pipe makes practically no difference in the load on the pipe, which is just as great for a vertical ditch as for one several times as wide at the top but of the same width a little below the top of the pipe.

5. "In ditches of proportions customary in actual work, the diameter of the pipe used in any particular ditch of a fixed given width makes practically no difference in the load on the pipe. A 12-in. pipe will have to carry the same load as an 18-in. pipe, if both are placed in ditches 2 ft. wide under other similar conditions.

6. "The width of the ditch a little below the top of the pipe makes a great difference in the load on the pipe, which is very much heavier for wide than for narrow ditches (see Table No. 2).

7. "In case a wide ditch is necessary for constructive reasons, the load on the pipe can be diminished greatly, in firm soil, by stopping the wide ditch a few inches above the top of the pipe and digging in the bottom the narrowest ditch practicable to receive the pipe, making bell holes at the side for the sewer pipe, if necessary.

8. "The loads on pipes in ditches, due to the weight of ditch filling, increase for greater depths of fill, but the proportion of the total weight of filling carried by the pipe decreases as the depth increases, and after the depth of fill becomes equal to ten times the breadth of the ditch at the top of the pipe there is practically no further increase in the load on the pipe for greater depths.

9. "The loads on pipes in ditches, due to the weight of ditch filling, are directly proportional to the weights per cubic foot of the ditch-filling materials. Of the common ditch-filling materials, clay is the heaviest and black top soil the lightest, sand being intermediate. For safe weights per cubic foot, see Table No. 3.

TABLE NO. 3. PROPERTIES OF DITCH-FILLING MATERIALS

Ditch Filling	Unit weight of filling, lb. per cu. ft.	Ratio of lateral to vertical earth pressures	Coefficient of friction against sides of trench
Partly compacted top soil (damp)	90	0.33	0.50
Saturated top soil	110	0.37	0.40
Partly compacted damp yellow clay	100	0.33	0.40
Saturated yellow clay	130	0.37	0.30
Dry sand	100	0.33	0.50
Wet sand	120	0.33	0.50

10. "Grades or fills built over the surfaces of completed ditches, and piles of sand, gravel, and other materials having internal friction, operate to increase the loads on pipes in ditches to the same extent as an equal added height of ditch filling, for a breadth of ditch equal to that at a little below the top of the pipe.

11. "A super load is any load applied to the upper surface of the ditch filling, except loads from fills or heaps of granular materials. A long super load is one extending a considerable length along a ditch, as compared with its depth and breadth, and may be caused by piles of paving brick, lumber, etc., over the ditch. Long super loads on completed ditches cause increases in the loads on pipes in ditches by percentages of the super load which decrease as depth increases, and safe values for which can be computed by Table No. 4. Formula (4) has been closely checked by actual weighings of the increase in loads on pipes in ditches due to super loads.

TABLE NO. 4.—APPROXIMATE SAFE VALUES OF C_1 TO USE IN FORMULA $L_1p = C_1L_1$

L_1p = loads per unit of length, on pipes in ditches, due to L_1.
L_1 = long super loads on ditches, per unit of length.

$\frac{H}{B}$	Sand and damp top soil	Saturated top soil	Damp yellow clay	Saturated yellow clay
0.0	1.00	1.00	1.00	1.00
0.5	0.85	0.86	0.88	0.89
1.0	0.72	0.75	0.77	0.80
1.5	0.61	0.64	0.67	0.72
2.0	0.52	0.55	0.59	0.64
2.5	0.44	0.48	0.52	0.57
3.0	0.37	0.41	0.45	0.51
4.0	0.27	0.31	0.35	0.41
5.0	0.19	0.23	0.27	0.33
6.0	0.14	0.17	0.20	0.26
8.0	0.07	0.09	0.12	0.17
10.0	0.04	0.05	0.07	0.11

12. "A short super load is one extending a short distance along a ditch as compared with the breadth and depth, and may come from the wheels of wagons, traction engines, steam road rollers, etc. Short super loads, on completed ditches, cause increases in the loads on pipes in ditches by percentages of the super load which decrease as the depth increases, and safe values which can be estimated, but not very reliably, by Table No. 5. Table No. 5 has not been checked by actual weighings of increase of loads on pipes in ditches.

TABLE NO. 3.—APPROXIMATE SAFE VALUES FOR C_s TO USE IN FORMULA $L_{sp} = C_s L_s$

L_{sp} = loads per unit of length, on pipes in ditches directly under L_s, due to L_s. L_s = short super loads on ditches, per unit of length, of length A along ditch.

$\frac{H}{B}$	Sand and damp top soil				Saturated top soil				Damp yellow clay				Saturated yellow clay			
	$K_a = \frac{1}{2}K$		$K_a = K$		$K_a = \frac{1}{2}K$		$K_a = K$		$K_a = \frac{1}{2}K$		$K_a = K$		$K_a = \frac{1}{2}K$		$K_a = K$	
	$A =$		$A =$		$A =$		$A =$		$A =$		$A =$		$A =$		$A =$	
	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$
0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	0.77	0.32	0.70	0.12	0.78	0.33	0.71	0.13	0.79	0.34	0.72	0.13	0.81	0.34	0.74	0.13
1.0	0.59	0.11	0.49	0.02	0.61	0.11	0.51	0.02	0.63	0.11	0.52	0.02	0.66	0.12	0.55	0.02
1.5	0.46	0.03	0.34	....	0.48	0.04	0.36	....	0.51	0.04	0.38	....	0.54	0.04	0.40	....
2.0	0.35	0.01	0.24	....	0.38	0.01	0.26	....	0.40	0.01	0.27	....	0.44	0.01	0.30	....
2.5	0.27	....	0.17	....	0.29	....	0.18	....	0.32	....	0.20	....	0.35	....	0.22	....
3.0	0.21	....	0.12	....	0.23	....	0.13	....	0.25	....	0.14	....	0.29	....	0.16	....
4.0	0.12	....	0.06	....	0.14	....	0.07	....	0.16	....	0.08	....	0.19	....	0.09	....
5.0	0.07	....	0.03	....	0.09	....	0.03	....	0.10	....	0.04	....	0.13	....	0.05	....
6.0	0.04	....	0.01	....	0.05	....	0.02	....	0.06	....	0.02	....	0.08	....	0.03	....
8.0	0.02	....	....	....	0.02	....	....	....	0.03	....	0.01	....	0.04	....	0.01	....
10.0	0.01	....	....	....	0.01	....	....	....	0.01	....	....	....	0.02	....	....	....

H = height of fill in ditch, above top of pipe.
B = breadth of ditch, a little below top of pipe.
K = ratio of lateral pressure to vertical in the ditch filling.
K_a = ratio of longitudinal pressure to vertical in the ditch filling.
Values of C_s for $K_a = 0$ are given in Table No. 4.
The formula $L_{sp} = C_s L_s$ holds true only directly under L_s. Beyond L_s, in either direction, the intensity of load on the pipe diminishes rapidly.

13. "Cracking of pipes in ditches is sometimes caused by heavy tamping of the filling material over it, or too thin a cover layer. The pressures transmitted to the pipe by tamping with rammers of different weights on cover layers of different thicknesses may be estimated, but only approximately, by the aid of Table No. 5 and the following formula:

$$T_e = \frac{2TF}{f}$$

where T_e = the maximum pressure on the earth filling resulting from the shock of a blow of the rammer.
T = weight of rammer used in tamping.
F = height of fall of rammer.
f = compression of filling material under one blow of rammer at the end of the tamping.

14. "Ordinary ditch sheeting may cause some increase in the loads on pipes in ditches from fresh filling, but does not increase the probable maximum loads unless left in permanently.

15. "Freezing, and consequent horizontal expansion of the sides of ditches against the sides of the pipe, sometimes causes cracking of drain tile and sewer pipe, where they are not covered sufficiently deep.

16. "The general effect of the lapse of time after the completion of the refilling is to decrease rather than increase the loads on pipes in ditches, though the maximum loads, as indicated in principle 2 above, generally do not occur until some time after the refilling is finished, and under certain conditions may not occur for many years."

The following examples are given in the bulletin from which the above principles are taken:

"*Example* 1. What load should be provided for as imposed by a pile of paving brick 6 ft. high on a 24-in. pipe sewer, whose top is 6 ft. below the street surface, the ditch being 3 ft. wide at the top of the pipe and the filling yellow clay?

"*Solution.* The weight of the paving brick as piled would probably be about 125 lb. per cubic foot. Hence L_1 would be $125 \times 6 \times 3 = 2250$ lb. per linear foot. If the ditch filling has been deposited recently and is not in danger of saturation, C_1 would be taken from Table No. 4, for damp clay, and for a value of $\frac{H}{B} = \frac{6}{3} = 2$. Hence $L_{1p} = 0.59 \times 2250 = 1300$ lb. per linear foot.

"If the clay filling has been thoroughly consolidated for a sufficient time to develop cohesion, L_{1p} would be much smaller, unless there is danger of saturation, as by heavy rains, which might destroy the cohesion. If the soil were sand instead of clay, however, cohesion would probably not greatly affect the result, and L_{1p} would be $0.52 \times 2250 = 1200$ lb. per linear foot.

"*Example* 2. The wheel of a steam road roller is 22 in. wide and carries a load of 8000 lb. When rolling transverse to the street, what load will it impose on an 18-in. pipe sewer in a recently settled ditch, $2\frac{1}{2}$ ft. wide at the level of the pipe, with $7\frac{1}{2}$ ft. height of yellow-clay filling?

"*Solution.* The length A of load in this case is $0.73B$. The value of $\frac{H}{B} = \frac{7.5}{2.5} = 3.0$. Assuming that the longitudinal pressure in the ditch filling is one-half the lateral, and interpolating in Table No. 5 between the values of 0.25 for $A = B$ and 0 for $A = \frac{B}{10}$, we find that $C_s =$ approximately 0.18. Hence, approximately, $L_{sp} = 0.18 \times \frac{8000}{1.83} = 800$ lb. per linear foot.

"The possible effect of cohesion may be taken into account in the case of an old ditch, with clay filling, as already noted in Example 1.

"As illustrating the possible degree of uncertainty in the above computed result, due to the fact that we are uncertain as to the proper value of K_a, the ratio of longitudinal to vertical pressure in the ditch filling, we may note that in Example 2:

For $K_a = 0$, $C_s = 0.45$, approximately (see Table No. 4).
For $K_a = \frac{1}{2}K$, $C_s = 0.18$, approximately (see Table No. 5).
For $K_a = K$, $C_s = 0.10$, approximately (see Table No. 5).

"Evidently calculations made from Table No. 5 are not very reliable, and there is great need of a series of tests of the actual loads on pipes caused by short super loads, but such tests would be very difficult to make and test results are not available.

"In the meantime, Table No. 5 will be of some value to engineers of good judgment in assisting them to make reasonable safe allowances for the probable effect on the loads on pipes in ditches from heavy concentrated loads on wagon wheels, traction engines, and road rollers.

"*Example* 3. What loads were probably imposed on a sewer pipe where a 40-lb. rammer on 6-in. cover apparently caused some cracking, and was superseded later with success by a 30-lb. rammer on 12-in. cover, the rammer being 8 in. square, and the filling material clay?

"*Solution.* Since the ramming was carefully inspected and was required to be very thorough, it seems reasonable to assume that the height of fall was at least 0.5 ft. and that the ramming was continued until a compression of about ⅛ in. (= 0.01 ft.) was produced by one blow. Hence $T_e = 2 \times 40 \times \frac{0.50}{0.01} = 4000$ lb. for the 40-lb. rammer. In Table No. 5 we find for $\frac{H}{B} = \frac{0.5}{0.67} = 0.75$, $K_a = K$, $A = B$, under damp clay, that about 62 per cent. of T_e, or 2500 lb., would be transmitted to an area of the pipe 8 in. square, directly under the rammer, with a total shock load on the pipe somewhere between 2500 and 4000 lb.

"For the 30-lb. rammer with 12-in. cover, and perhaps 0.015 ft. compression under the final blow, $T_e = 2000$ lb., with 38 per cent, or 800 lb., transmitted to an area of the pipe 8 in. square, directly under the rammer, and some further pressure outside this area.

"From these results it is apparent: First, that the 40-lb. rammer on the 6-in. cover may readily have caused some cracking of the pipe; second, that the 30-lb. rammer on the 12-in. cover was probably not much, if any, more than one-third as severe on the pipe."

82. Strength of Pipe.—The theoretical analysis of stresses in culvert pipe is that of thin elastic rings and is similar to the general method employed for arches. The difference in the intensity of the load at the crown and at the extremities of the horizontal diameter, due to the difference in the depths of the earth, is considered negligible, and the pressure and its distribution on the lower half of the ring is assumed to be the same as that on the upper half.

Theory gives the following values of the bending moment at the top and bottom sections of a pipe:

(I) For single concentrated load (top and bottom). . $M = 0.159\,Pd$.

(II) For total uniformly-distributed load over entire horizontal projection (top and bottom). . $M = 0.0625\,Wd$.

(III) For a uniformly-distributed load over the top fourth of the circumference and with the pipe supported on its bottom quarter circumference. . $M = 0.0845\,Wd$. Where

d = diameter of pipe.
P = concentrated load at top.
W = total uniformly-distributed load above horizontal diameter.
M = bending moment in pipe in a unit length.

The bending moments at the ends of the horizontal diameters under the above conditions of loading are:

$$\begin{aligned} \text{I}\,.\,.\,M &= -\,0.091\,Pd \\ \text{II}\,.\,.\,M &= -\,0.0625\,Wd \\ \text{III}\,.\,.\,M &= -\,0.077\,Wd \end{aligned}$$

The above moments will be reduced for any lateral restraint or lateral pressure. In fact, for equal uniform horizontal and uniform vertical forces (which may be considered equivalent to a uniform radial pressure) the moments due to the lateral forces have equal but opposite signs to those given for Case II above, and it can be proved that the total moments at all points are zero. It is not good practice, however, to rely on any lateral restraint or pressure in the analysis of the strength of pipes. Mathematical analysis shows that the weight of pipe causes only five-eighths as much bending moment at the lowest point of the pipe as does an equal weight of earth.

Since the exact load and the nature of its distribution over the pipe surface is usually uncertain, the probable range of bending moments under actual conditions of construction is all that laboratory tests can be expected to furnish. In a series of tests at the University of Illinois, reinforced concrete rings and circular pipe (48-in. internal diameter and 4 in. thick) were tested for concentrated loads at the top and bottom of the vertical diameter (Case I above), and for uniformly-distributed loads above and below the entire horizontal diameter (Case II above). This latter loading was obtained by placing the pipe in a tight box so as to be entirely encircled with sand and then applying a load to the top surface of the sand. The reinforcement for most of the rings tested consisted of ¼-in. corrugated rods placed near the intrados at top and bottom and near the extrados at the sides. The rings were only 24 in. long, but the pipe sections were from 102 to 104 in. in length with the usual bell and spigot joints. To allow the circumferential reinforcement in the pipes to be circular in shape, the pipe cross-sections (Fig. 311) were made with the vertical diameter 4 in. longer than the horizontal diameter, thus bringing the reinforcement at the points of tension in the loaded pipe. Using the yield point of the steel in the common formula for the bending resistance of a reinforced section, there was found a close agreement between the theoretical and experimental values for the strength of pipe under these two methods of loading.

Marston and Anderson, in the investigation referred to in the preceding article, came to the conclusion that "the typical field bedding and loading of pipes in ditches are such that their effect on the pipe can be reproduced with practical exactness in laboratory tests by bedding the pipes in sand during the tests for 90° of the circumference at the bottom and also for 90° at the top." This method of loading is Case III above. In the Iowa investigation the weight of the pipe as well as that of the backfilling was taken into consideration.

The manner in which the formulas given above may serve as a guide to the judgment is indicated in the following extract from Bulletin No. 22 of the Engineering Experiment Station of the University of Illinois:

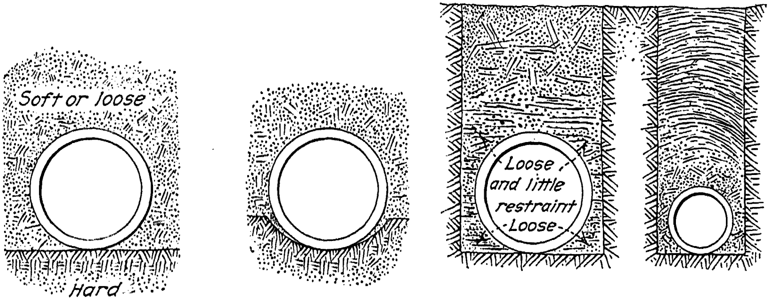
> "If the layer of earth immediately under the pipe is hard or uneven, or if the bedding of the pipe at either side is soft material or not well tamped, as indicated in Fig. 313(*a*), the main bearing of the pipe may be along an element at the bottom and the result is in effect concentrated loading. The result is to greatly increase the bending

---

[1] See Bulletin No. 22 of the Engineering Experiment Station of the University of Illinois —written by Prof. A. N. Talbot.

moment developed and hence the tendency of the pipe to fail. This condition may be aggravated in the case of a pipe with a stiff hub or bell where settlement may bring an unusual proportion of the bearing at the bell and the distribution of the pressure be far from the assumed condition. In bedding the pipe in hard ground it is much better to form the trench so that the pipe will surely be free along the bottom element, even after settlement occurs, and so that the bearing pressures may tend to concentrate at points, say under the one-third points of the horizontal diameter (or even the outer quarter points). This will reduce the bending moments developed in the ring.

"In case the pipe is bedded in loose material, the effect of the settlement will be to compress the earth immediately under the bottom of the pipe more completely than will be the effect at one side, as indicated in Fig. 313(*b*), with the result that the pressure will not be uniformly distributed horizontally. Similarly, in a sewer trench, if loose material is left at the sides and the material at the extremity of the horizontal diameter is loose and offers little restraint, as indicated in Fig. 313(*c*), the pressure on the earth will not be distributed horizontally and the amount of bending moment will be materially different from that where careful bedding and tamping give an even distribution of bearing pressure over the bottom of the sewer.



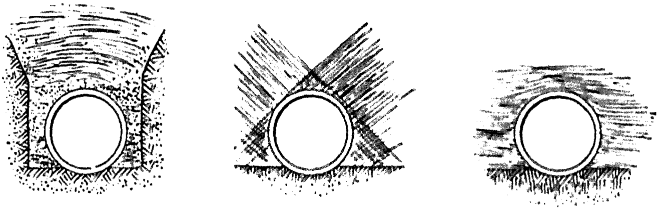
(a) (b) (c) (d)

FIG. 313.—Conditions of loading.

"In case of a small sewer in a deep trench, the load upon the sewer may be materially less than the weight of the earth above, as in the case shown in Fig. 313(*d*), where the earth forms a hard compact mass and is held by pressure and friction against the sides of the trench.

"In case a culvert pipe is laid in an ordinary embankment by cutting down the sides slopingly, as shown in Fig. 314(*a*), it is evident that the load which comes upon the pipe will be materially less than the weight of the earth immediately above it. If a culvert pipe replaces a trestle and the filling is allowed to run down the slope as shown in Fig. 314(*b*), the direction and amount of the pressure against the pipe will differ level filling, shown in Fig. 314(*c*). It is possible in the latter case that the smaller amount of settlement of the earth directly over the culvert pipe, due to the greater depth of earth on the adjacent sections, may allow a greater proportion of the load to rest upon the culvert pipe than would ordinarily be assumed.

"Attention should be called to the fact that the distribution of the pressure by means of earth under and over a ring assumes that the earth is compressed in somewhat the same way as when other material of construction is given compression. Unless the earth has elasticity, the distribution of pressure cannot occur. To secure the uniform distribution assumed, the ring itself must give enough to allow for the movement of the earth which takes place under pressure. This is especially true with reference to the presence and utilization of lateral restraint, and a ring which does not give laterally, as for example a plain-concrete ring, will not develop lateral pressure in the adjoining earth under ordinary conditions of moisture and filling to any great extent. As the conditions of earth and moisture produce mobility and approach hydrostatic conditions, the necessity for this elasticity and movement does not exist, but here the lateral pressure approaches the vertical pressure in amount and the bending moments become relatively smaller.



(a) (b) (c)

FIG. 314.—Conditions of loading.

"The discussion is sufficiently extended to indicate the importance of care in bedding culvert pipe and sewers and in filling over them, and to indicate the great difference in the amount of bending moment developed with different conditions of bedding and filling. Where there is any question of needed strength, it will be money well expended to use care and precaution in bedding the pipe and in filling around and over it. I am convinced that a little extra expense will add considerable stability, life, strength, and safety to such structures, far out of proportion to the added cost. It is possible that under careful conditions of laying, lighter structures may be used with a saving in the cost of construction."

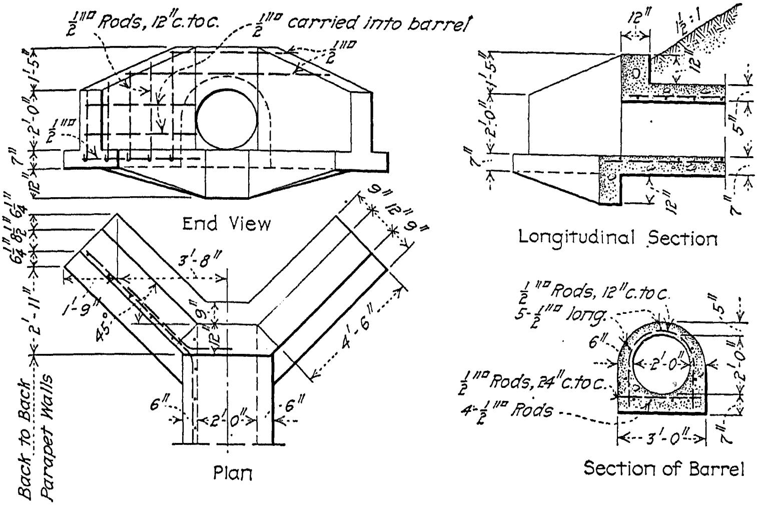
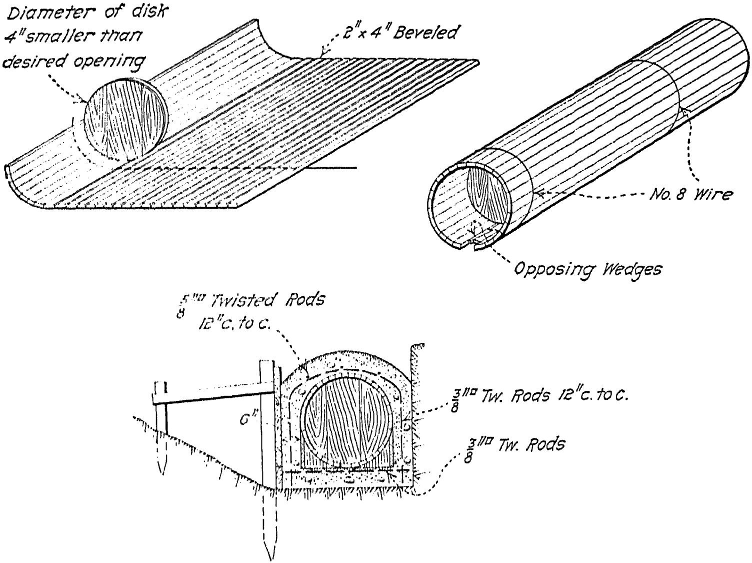


FIG. 315.—Standard design for 24-in. circular culvert, Iowa Highway Commission.



Diameter of disk 4″ smaller than desired opening
2″×4″ Beveled
No. 8 Wire
Opposing Wedges
⅝″□ Twisted Rods 12″ c. to c.
6″
⅜″□ Tw. Rods 12″ c. to c.
⅜″□ Tw. Rods

FIG. 316.—Collapsible, adjustable wood form for concrete culverts from 18 to 48 in. diameter.

83. Circular Culverts Cast in Place.—A cast-in-place culvert with circular opening is shown in Fig. 315. Fig. 316 shows an adjustable, collapsible wood form which can be very economically used for culverts of this type. It was originally designed by Mr. F. H. Meliza, Farmer City, Ill., and was mentioned in the Illinois Highway Commission report for 1907 and in a bulletin on highway improvement by Mr. W. S. Gearhart, Highway Engineer of Kansas, which was issued by the Kansas State Agricultural College.

The method of constructing and using this form is described as follows in a booklet entitled "Small Concrete Bridges and Culverts," published by the Universal Portland Cement Co.:

> "This form is constructed of two by four's beveled and strung on wires, as shown in Fig. 316. The number of staves to be used, varying with the size of the culvert, are placed side by side with a wire drawn through each end of the stave as shown. The form is then rolled around a circular head size of the proposed culvert and wire bands are tied tightly around it on the outside. Wedges are then driven as shown in Fig. 316 to hold the staves firmly in position. After the culvert has been built the wedges are removed and the circular heads knocked in; the staves will then collapse and are easily removed. This form can be used over and over again and Mr. Gearhart states that its cost should not exceed $15.00 or $20.00."

CHAPTER XXI

BOX CULVERTS

The box type of culvert is especially adapted to locations where the head-room is limited and, when planned for such locations, has a great advantage over the arch. A culvert of this type, for example, can be built with less excavation and less disturbance to the embankment and will give a greater distribution of load upon the foundation than the ordinary form of arch culvert. Also, the form work for this type is much simpler and the cost of construction correspondingly lower, except perhaps in some cases where the number and size of culverts to be con-

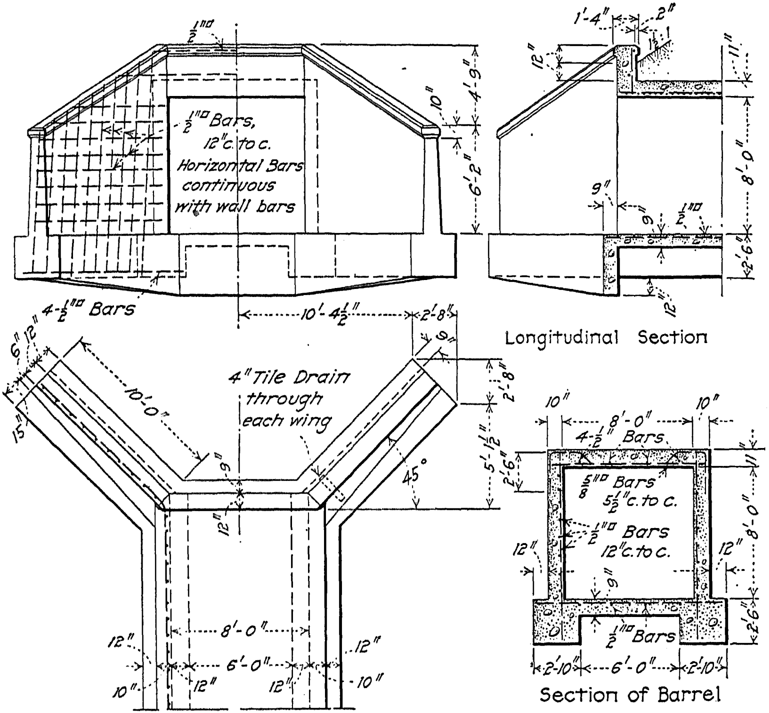


FIG. 317.—Standard design for 8 × 8-ft. box culvert, Iowa Highway Commission.

structed warrants the use of commercial arch forms of collapsible steel. Box culverts are not always employed only under shallow fills, as is evident from Figs. 320 and 321.

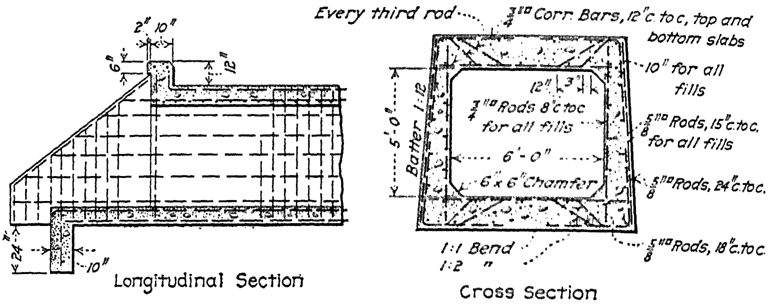
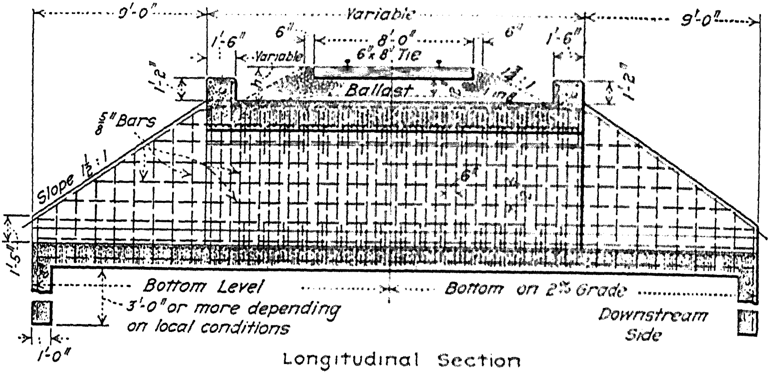


FIG. 318.—Standard culvert of 6-ft. span, Hampden R. R.



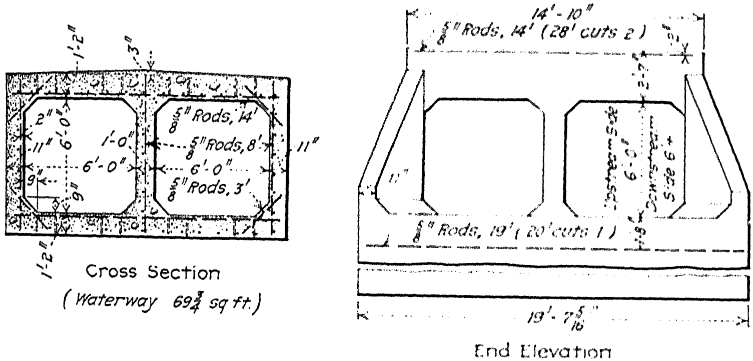
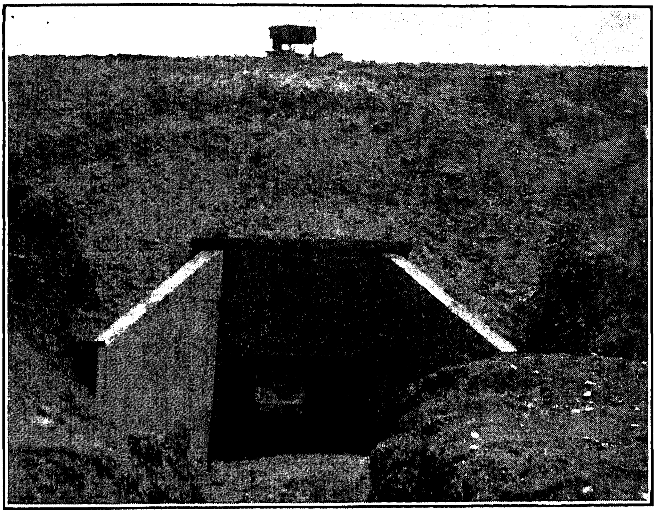


FIG. 319.—Double 6 × 6-ft. box culvert, A. T. & S. Fe. Ry. system.

84. Forms of Box Culverts.—There are two distinct forms of box culverts which may be distinguished by the terms *open box* and *closed box.* In the open box (Fig. 317) the side walls have dependent footings, while in the closed box (called simply box) the load is supported by the floor (Fig. 318). Double-box culverts (Fig. 319) are generally used where the span is equal to, or greater than, twice the height, as the use of a single box beyond these proportions greatly increases the cost.

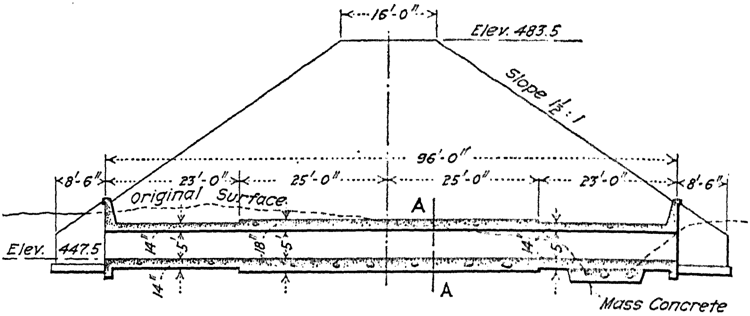
85. Loading.—For small box culverts built in trenches the load on the top slab may be approximately estimated by means of the formulas and tables in Art. 81. For large box culverts and for all culverts not built by trench construction, no allowance should be made for the arching action of the material, which means that such culverts should be proportioned to carry the entire weight of the fill above the cover slab. The lateral pressure of the earth (including live-load surcharge) is usually assumed as that due to a fluid weighing about 30 lb. per cubic foot—that is, a weight about one-fourth the weight of earth. It is obvious that the pressure due to live load does not spread out through the filling at the ordinary angle of repose of the material, but has a side slope, or line of zero stress, much more nearly vertical. It is frequently assumed that the live load is carried down at a slope of $\frac{1}{2}$ horizontal to 1 vertical. In railroad embankments this slope is taken from the ends of the ties.



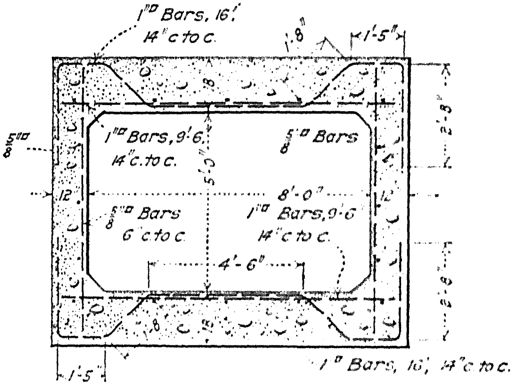
Courtesy of Mr. Wm. E. Beilharz, Architect and Engineer, Dallas, Texas.

FIG. 320.—Box culvert on lines of Eastern-Texas Traction Co.

An allowance is usually made for impact of the live load in the design of railroad culverts. Some designers allow 50 per cent for impact on all banks up to 40 ft. high. A more conservative plan often followed is to allow 100 per cent for fills of less than 2 ft., 75 per cent for fills between 2 ft. and 4 ft., and 50 per cent for all fills over 4 ft.



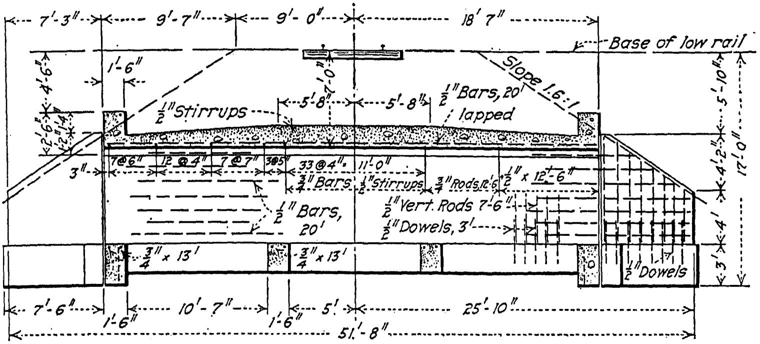
General Section


1" Bars, 16', 14" c to c.
8"
1'-5"
2'-8"
5/8"
1" Bars, 9'6", 14" c. to c.
5/8" Bars
5'-0"
12"
8'-0"
12"
5/8" Bars 6" c. to c.
1" Bars, 9'6" 14" c to c.
4'-6"
2'-8"
8"
8"
1'-5"
1" Bars, 16', 14" c. to c.

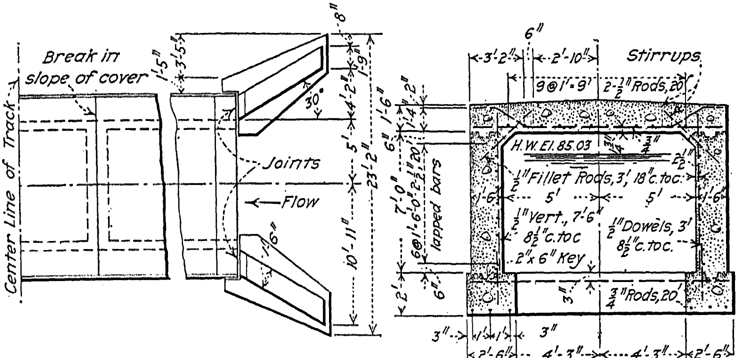
Section A-A

FIG. 321.—Details of box culvert on lines of Eastern-Texas Traction Co.

86. Design of Cross-section.—The top slab of a box culvert is partially fixed, but it is the general practice to design this slab as simply-supported and to reinforce at the corners against negative bending. Negative reinforcement, however, is not always provided (see Fig. 319), in which case the sides and top act as simple beams and more or less cracking occurs on the outside near the corners. The walls or sides of a box culvert are usually designed somewhat empirically, but are always provided with sufficient strength to support the lateral earth pressure, neglecting any outward bending due to the bending of the cover slab. In open-box construction, cross struts are often used to assist in holding the footings against the pressures on the walls and also to provide bearing area in addition to that furnished by wall footings (Fig. 322). The struts are designed as beams with a span equal to the width of the culvert, and the struts are so spaced and proportioned as to obtain a uniform soil pressure throughout. When a bottom slab is employed and assumed to support the load, its thickness is made the same as that of the cover slab since the load for both slabs is substantially the same. Longitudinal reinforcement should be provided on account of the possibility of beam action due to unequal settlement. The amount of this reinforcement should depend upon the char-



Sectional Elevation at Center Line of Culvert



Plan

Section at Center Line of Track

Fig. 322.—Single box culvert, C. M. & St. P. Ry.

acter of the foundation. Where the foundations are very bad, it is the practice of some engineers to figure the culvert as a whole to act as a beam, considering the length of beam as twelve times its depth. In long culverts under railroad tracks the load decreases beyond the ends of the ties and the cross-section should decrease whenever a material saving is found to result.

In Fig. 322, which illustrates a standard box culvert adopted by the Chicago, Milwaukee & St. Paul Railway, the cover is designed as a simply-supported slab with span equal to clear span when fillets are used and to clear span plus one-half the maximum cover thickness (but not to exceed 1 ft.) when no fillets are provided. Stirrups and bent rods are employed to take care of two-thirds of the shear when the shear exceeds 40 lb., bent rods being also used to care for any negative moments which may develop due to connection with side walls. Longitudinal steel is employed with a sectional area of about $\frac{1}{750}$ of the area of the entire concrete section. The side-walls, cross struts, and footings are proportioned in the same manner as above described. Keyways are formed in top of footings and side-walls so as to offer shearing resistance to movement of side-walls due to lateral earth pressure. For fills up to 40 ft., the load on the footings is assumed to include the live load and dead load of culvert, and the fill directly above the culvert for the width overall including footings. For fills over 40 ft. high, the total weight resting on the footings is considered as $62\frac{1}{2}$ per cent of the culvert weight plus $62\frac{1}{2}$ per cent of the fill above the footings. The live load is disregarded and, for ease in computation, the weight of culvert is taken as equal to the fill it displaces. Wing walls over 8 ft. in height or making an angle of less than 60° with the head-wall are made self-supporting cantilevers with a joint at connection with the barrel. Wing walls for all other conditions are made continuous with the main part of the culvert. The section of the culvert coming under the track is made approximately equal in length to the theoretical spread of the live load, which is equal to the distance out to out of ties (8 ft. for single track, 21 ft. for double track) plus the height of culvert plus one-half the height of fill above the cover slab. The invert is paved with a concrete slab in all spaces between struts so as to form a continuous concrete invert.

A somewhat rigorous analysis is sometimes made of the moments in box culverts when such culverts are reinforced so as to act as monolithic concrete boxes (Figs. 318 and 321). Of course, it becomes a question of whether such accurate analysis, comparatively speaking, is especially worth while. Unless a number of

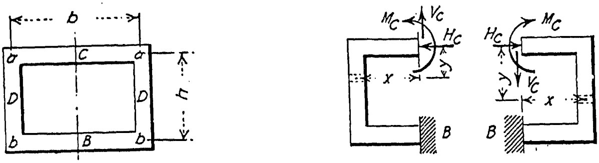
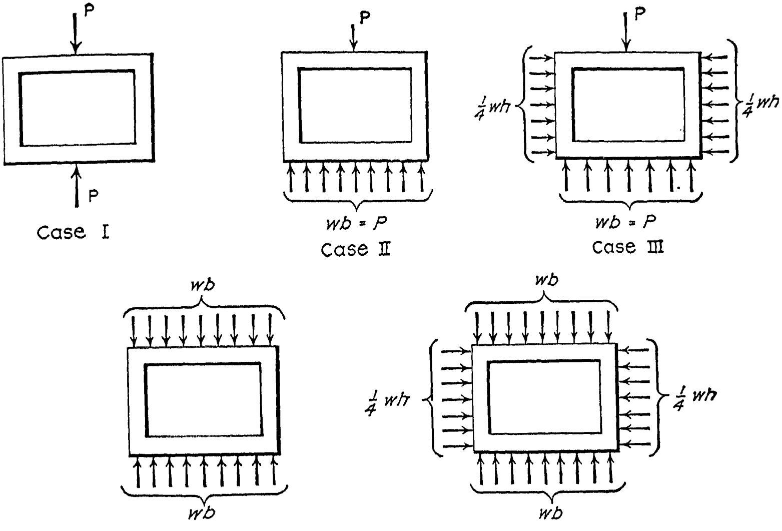


FIG. 323*A*.

identical culverts are to be constructed, the amount of material involved is likely to be too small to warrant any great refinement in design.



Case I

Case II

Case III

Case IV.

Case V

FIG. 323*B*.

Assume the rectangular culvert of Fig. 323*A* to be rigidly fixed at the center of the bottom slab. With this condition assumed, a unit length of the culvert may be treated in the same manner as a symmetrical arch with fixed ends. (See Fig. 323*A*.) The general formulas of Art. 14 will apply and the following expressions result:

$$H_c = \frac{\Sigma my \frac{s}{I} \Sigma \frac{s}{I} - \Sigma m \frac{s}{I} \Sigma y \frac{s}{I}}{\Sigma y^2 \frac{s}{I} \Sigma \frac{s}{I} - \left(\Sigma y \frac{s}{I} \right)^2}$$

$$M_c = \frac{\Sigma m \frac{s}{I} - H_c \Sigma y \frac{s}{I}}{\Sigma \frac{s}{I}}$$

in which

m = moment at any point on either half of culvert of all external loads between the point and the crown section —all values should be substituted in equations as positive.

Σ = summation referred to one-half of culvert only.

Let

b = width c. to c. of side walls.

h = height between centers of top and bottom slabs.

$R = \frac{b}{h}$

I_C = moment of inertia of cross-section of top and bottom slabs, considered alike.

I_D = moment of inertia of cross-section of side walls, considered alike.

$S = \frac{I_C}{I_D}$

P = concentrated load.

w = uniform load per unit area.

M_C = moment at center of top slab.

M_B = moment at center of bottom slab.

M_D = moment at center of side walls.

M_a and M_b = moment at the upper and lower corners, respectively.

H_C = thrust at the center of the top slab.

V_C = zero for symmetrical loading as is the case in the analysis of culverts.

M_L and M_R = moment at the center of any element to the left and right of the crown section, respectively.

The bending moment at any point may be expressed as follows:

$$M_L = M_C + H_C\, y + V_C\, x - m$$

$$M_R = M_C + H_C\, y - V_C\, x - m$$

The following formulas result:

Case I.

$$H_C = 0$$

$$M_C = M_B = \frac{Pb}{8} \cdot \frac{R + 2S}{R + S}$$

$$M_D = M_a = M_b = M_C - \tfrac{1}{4}\, Pb$$

For a square uniform section, $M_C = \frac{3}{16}\, Pb$, and $M_D = -\frac{1}{16}\, Pb$.

For a square uniform section and for equal vertical and lateral forces $M_C = M_D = \frac{1}{8}Pb$ and $M_a = M_b = -\frac{1}{8}Pb$, the same as in fixed beams.

Case II.

$$H_C = \tfrac{1}{8} P \cdot \frac{R^2}{3R + S}$$

$$M_C = \frac{Pb}{24}\left[\frac{6S^2 + 20SR + 9R^2}{(S + 3R)(S + R)}\right]$$

$$M_B = M_C + H_C\, h - \frac{wb^2}{8} = M_b + \frac{Pb}{8}$$

$$M_a = M_C - \frac{Pb}{4}$$

$$M_b = M_C + H_C\, h - \frac{Pb}{4} = M_a + H_C\, h$$

Case III.

$$H_C = (H_C \text{ for Case II}) + \tfrac{1}{8}\, wh$$

$$M_C = (M_C \text{ for Case II}) - \frac{wh^2}{48}\left(\frac{S}{S + R}\right)$$

$$M_B = M_C - \tfrac{1}{8}\, w\,(b^2 + h^2) - H_C h$$

$$M_a = M_C - \frac{Pb}{4}$$

$$M_D = M_a - \tfrac{1}{2}\, wh^2 - \frac{H_C h}{2}$$

$$M_b = M_a - \tfrac{1}{8} wh^2 - H_C h$$

Case IV.

$$H_C = 0$$

$$M_C = M_B = \frac{wb^2}{24}\left(\frac{R + 3S}{R + S}\right)$$

$$M_D = M_a = M_b = M_C - \tfrac{1}{8} wb^2$$

For a square uniform section, $M_C = \frac{1}{12} wb^2$ and $M_D = -\frac{1}{24} wb^2$.

For a square uniform section and for equal vertical and lateral forces $M_C = M_D = \frac{1}{24} wb^2$ and $M_a = M_b = -\frac{1}{12} wb^2$, the same as in fixed beams.

Case V.

$$H_C = \tfrac{1}{8} wh$$

$$M_C = M_B = (M_C \text{ for Case IV}) - \frac{wh^2}{48}\left(\frac{S}{S + R}\right)$$

$$M_a = M_b = M_C - \tfrac{1}{8} wb^2$$

$$M_D = M_a + \tfrac{1}{32} wh^2$$

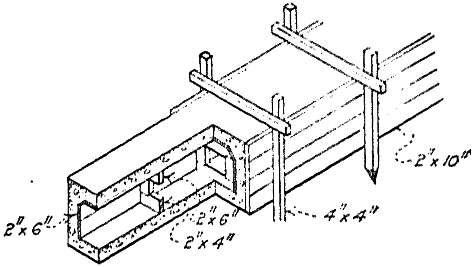


Fig. 324.—Forms for 2-ft. box culvert.

87. **Construction.**—The inside forms for small box culverts are usually arranged to be collapsible or at least so that the frames against which the lagging is placed may be easily knocked out after the concrete has properly set. In Fig. 324 the inner forms consist of boards placed against frames made of three pieces of 2 × 4-in. and one piece of 2 × 6-in. joists, notched as shown.

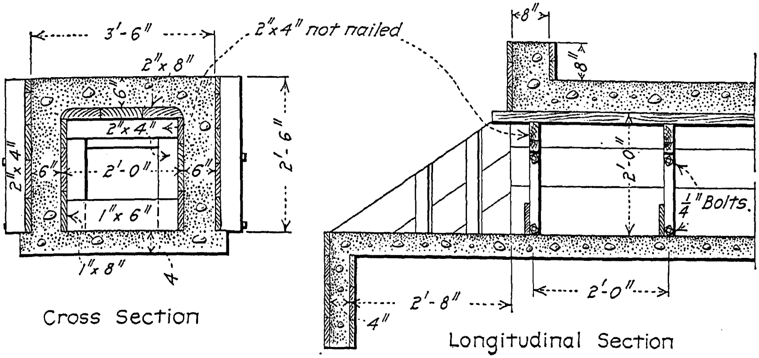


FIG. 325.—Forms for small box culverts, Iowa Highway Commission.

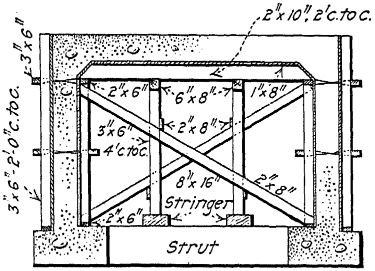


FIG. 326.—Box-culvert forms, C. M. & St. P. Ry.

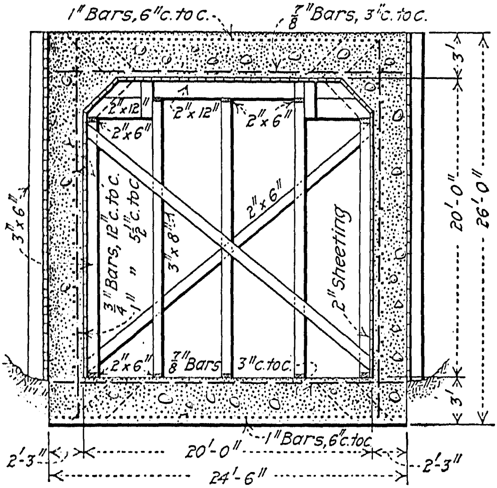


FIG. 327.—Box-culvert forms, C. B. & Q. R. R.

The 2 × 6-in. piece at the top is not nailed in order that it may be readily knocked out after the concrete has hardened. The removal of this piece allows the 2 × 4-in. joists to be withdrawn, thus releasing the boards. Where the soil is hard and compact, the outer forms may be omitted and the concrete deposited directly in the trench made to exact size of culvert cross-section. Another type of small box culvert form is shown in Fig. 325.

The forms for large box culverts need no special consideration as they are similar to the forms for walls and floor slabs in ordinary construction. Standard form details adopted by two prominent railroads are shown in Figs. 326 and 327.

If running water is encountered which cannot be temporarily diverted or dammed, the water in the case of small culverts should be carried in a new trench around one side of the back forms. In the case of the larger structures the trench excavated for the culvert should be arranged to flume the stream through between the abutments.

CHAPTER XXII

ARCH CULVERTS

The arch type of culvert should be employed where an artistic design is especially desirable, and should also be used in all cases where the fill to be supported is excessively high and the foundations suitable. High fills over box culverts necessitate a slab of considerable thickness and the arch becomes the more economical under such conditions because of the fact that an increase in fill does not produce a corresponding increase in ring stress. Arch culverts of reinforced concrete are not usually designed for spans less than about 8 ft., as plain concrete seems to answer the purpose for such small structures.

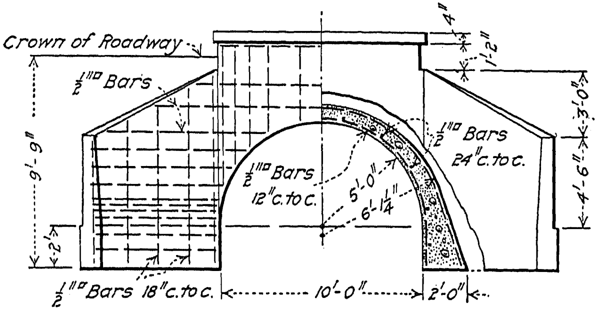


FIG. 328.—Standard design for 10-ft. arch culvert, State of Missouri Highway Department. (Actual dimensions and shape of foundations governed by conditions of soil at location of site.)

88. Design of Cross-section.—In the ordinary type of arch culvert represented in Fig. 328, the arch ring may be analyzed in the same manner as described for arch bridges in Part I of this volume. A uniform live load only is considered and this is placed over the whole span. Although steel is used, the line of pressure is usually kept everywhere within the middle third. In determining the dead load on the arch no allowance is made for the arching action of the fill, but the horizontal components of the earth pressure are taken into account. Longitudinal reinforcement is needed to prevent objectionable cracks caused by the shrinkage of concrete in hardening and the contraction due to a lowering of the temperature, and also to distribute the load.

Inverts are employed in the designs shown in Figs. 329 and 330 in order to provide additional bearing area and thus reduce the large abutments which would otherwise be needed in order to bring the pressure on the soil to a safe value. An invert also tends to prevent any possibility of water undermining the structure and the foundation need not always be carried to the same depth as when the abutments are independent of each other.

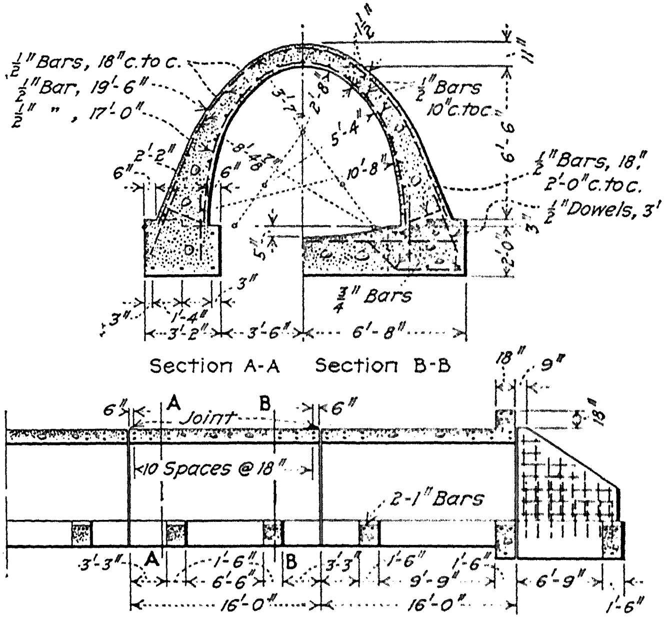
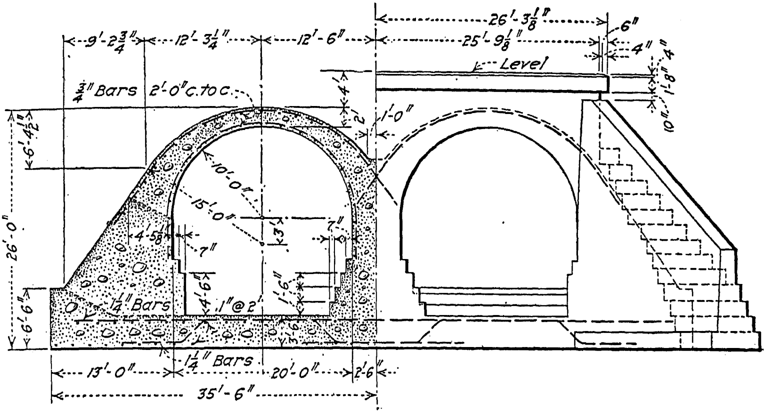


FIG. 329.—Standard arch culvert for fills up to 40 ft. high, C. M. & St. P. Ry.

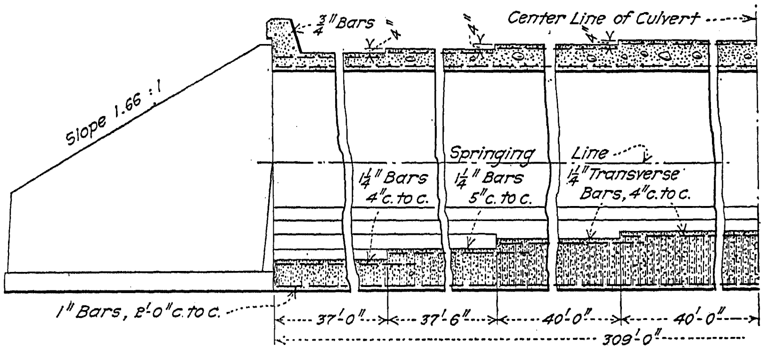
When the arch ring and invert are thoroughly tied together so as to act as a monolith (a rare case except in masonry sewer design), the entire culvert may be analyzed in a similar manner to that of the monolithic box (see preceding chapter). For a complete treatment of arch structures of this type, see Chapter XIII of American Sewerage Practice, Vol. I, by Metcalf and Eddy.



End Section

End Elevation

Maximum Fill over Crown · 90 ft.



Section of Barrel

Fig. 330.—Double-barrel culvert, D. L. & W. R. R.

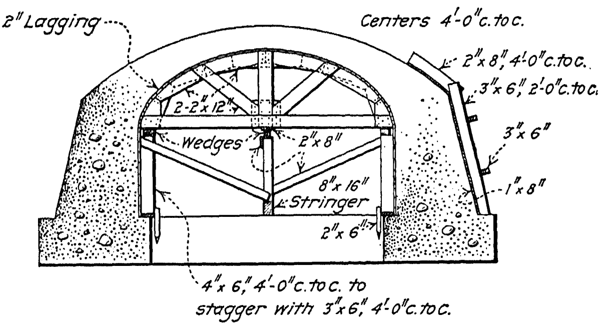


Fig. 331.—Centering for arch culvert, C. M. & St. P. Ry.

89. Forms.—The centers and forms for the arch culvert are similar to those required for the arch bridge of small span. Where inverts are employed, the concrete for the footings and floor in the stream bed should be deposited first and then the centers for the arch erected on this concrete. Fig. 331 shows the type of arch culvert centers and forms used by the Chicago, Milwaukee and St. Paul Railway. Other types of centers are shown in Figs. 332 and 333.

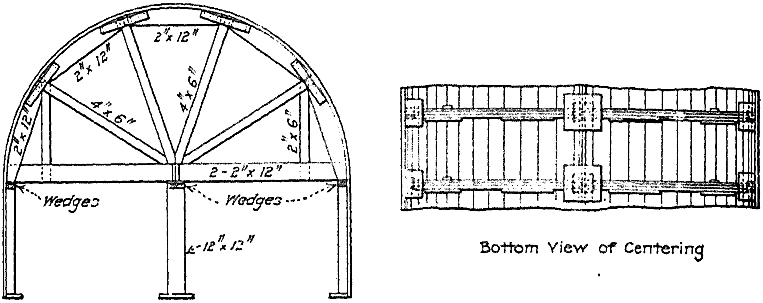


Fig. 332.—Centering for arch culverts.

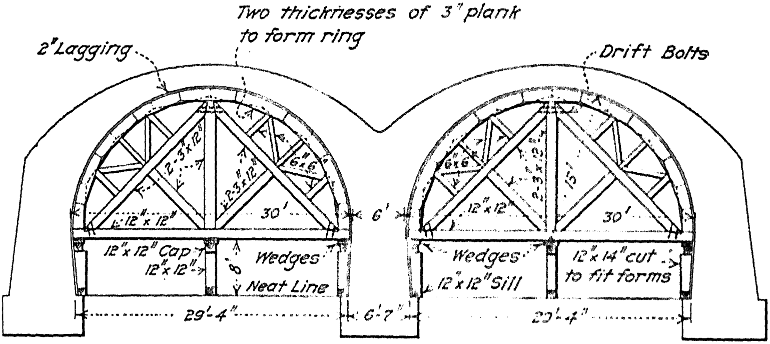


Fig. 333.—Details of centering, Canoe Creek culvert, near Cloe, Pa., B. R. & P. Ry.

PART IV

NOTES ON CONSTRUCTION PLANT

By A. W. Ransome

Vice-President of the Ransome Concrete Machinery Co.

General rules governing the selection of plant have been given in the preceding volume. Inasmuch as the rules there set forth will be found applicable to the selection of plant for bridges and culverts, the reader is advised to refer to these rules in Volume II in connection with the considerations which follow.

The primary consideration that plant is but a substitute for labor, and that no more should be invested in plant than will yield a good return on the investment, is so important as to bear repetition. This relation is apparently lost sight of in many cases. Perhaps it is too obvious, too axiomatic, and is consequently ignored; but then, again, it may be probable that this mistake is the result of lack of reliable information on which to base an accurate comparison of costs between hand labor and the various types of construction plants.

In the treatment of plant in this volume all phases of plant operation will be considered, avoiding as far as possible a repetition of the subject matter treated in Volume II. A certain amount of repetition is unavoidable, however, lest the reader be kept looking from volume to volume to the degree of discouragement.

CHAPTER XXIII

HANDLING MATERIALS

90. Aggregates.—Plant will be considered to include the cars in which the aggregate may be delivered by the railroad. The type of car used often exercises considerable influence on the cost of handling, so, where such is the case, it is important to specify the manner in which delivery is to be made. Care in this respect will be found to work a decided economy in carrying out the average contract, and the conditions of delivery may even determine the placing of the material order at a higher price.

Take, for example, two quarries located on different railroads, one railroad prepared to supply only hopper-bottom cars, the other to furnish flat-bottom cars. Assume now that the plan of operation involves unloading the stone by shoveling, and assume also that one quarry, on railroad No. 1, quotes $1.30 per cubic yard, and the other, on railroad No. 2, quotes $1.32 per cubic yard. It will be found cheaper in the end to order your material at $1.32 per cubic yard, for the reason that you can more than save the difference in cost in unloading.

A good man, under efficient superintendence, can unload 2 cu. yd. per hour from a flat-bottom car, as against 1½ cu. yd. per hour from a hopper-bottom car—a saving of at least 3 cents per cubic yard—and this proportional saving increases as less efficient labor is employed. These figures are based on the writer's personal experience and represent average results over a number of years. A good man under a yardage or carload system will perhaps average 50 per cent more than the above, but the ratio will not change between the two types of cars.

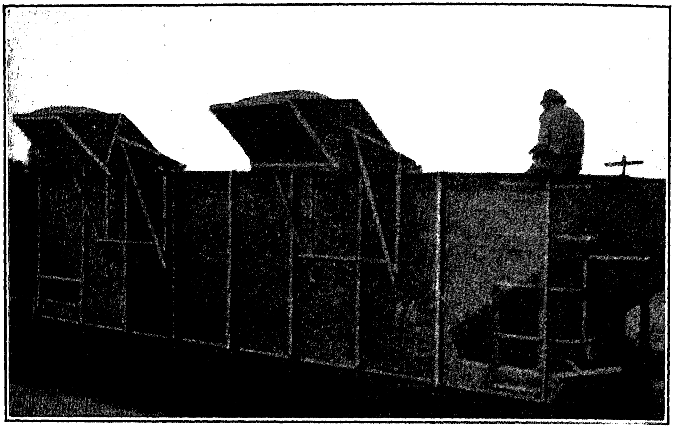


FIG. 334.

It will also be found that the low-side cars will be more economical for unloading than the high-side, except where a wagon loader (Figs. 334 and 335) is to be used. Where a loader of this type is employed, the car side to which it is attached should either be of sufficient height to give wagon clearance, or stakes should be provided to raise the hopper sufficiently for the purpose. The use of a hopper of this type expedites unloading the car and, in these days of demurrage charges, this is an item for consideration, aside from the saving of team hours. One, two, or three hoppers may be attached to a single car and one or more men put in the car for each hopper, depending upon the number of teams available. The hoppers should be ready as the teams range alongside the car, and the material should be instantly discharged into the wagons to avoid all delay at the car.

Where a siding extends to the construction site, sand and stone may be shoveled directly from the cars onto the ground, which should have been previously smoothed. That portion of the ground designed for the stone should first be spread with a layer of sand at least 1 in. thick, this layer serving to keep the stone clean and also working economy in subsequent shoveling. In order that materials may be piled high and the track be kept clear, a bulkhead may be built of a double row of 2 × 12-in. plank with 1 × 3-in. cross ties, having stops as indicated in Fig. 336. This method requires no fitting or cutting of the plank.

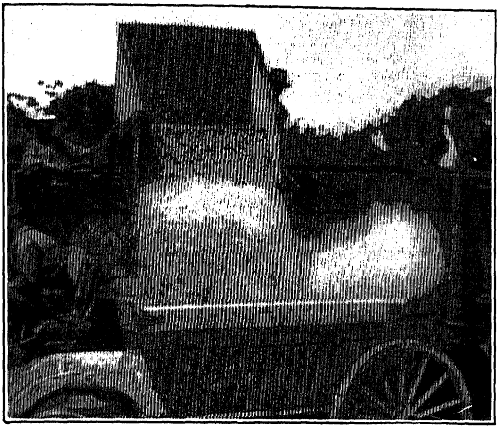


FIG. 335.

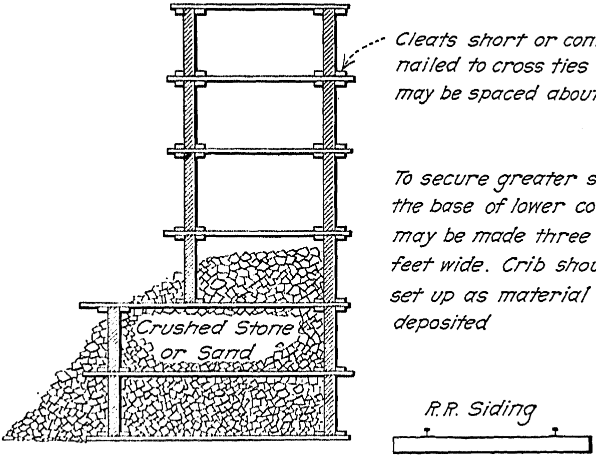

Cleats short or continuous nailed to cross ties which may be spaced about 3′c.to c.
To secure greater stability the base of lower courses may be made three to four feet wide. Crib should be set up as material is deposited
Crushed Stone or Sand
R.R. Siding

FIG. 336.

The type and condition of shovel used in unloading materials will have an important bearing on the results obtained. The writer has noticed a tendency to regard the shovel as a very unimportant matter, but it is not so regarded by the workman himself. A good man will always look for a good shovel, and under a task or piece-work system he would perhaps use a larger shovel than the one with which he would ordinarily be supplied. Throughout South America it has been the general practice for each man to bring his own shovel, and this practice leads to the exploitation of tools wonderfully adapted to reduce to the minimum the actual work done while going through the motions. The practice was followed very generally in the West in early days. Our western contractors, however, early found it economy to supply their men with good "workmanlike" tools.

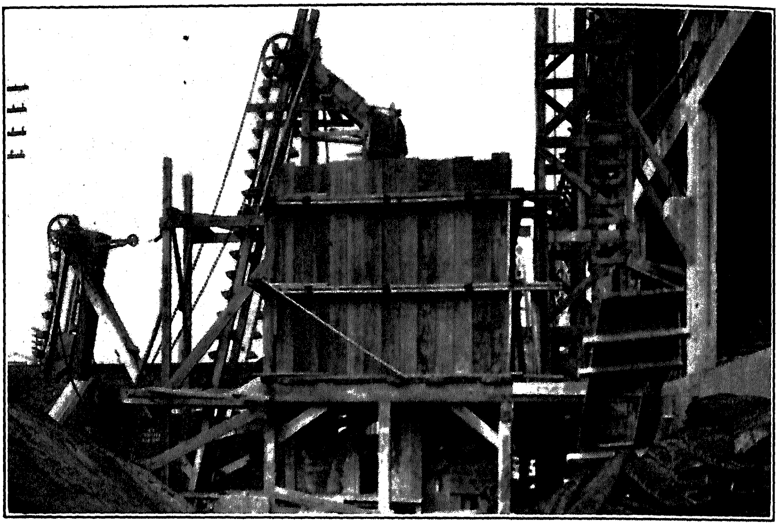


FIG. 337.

Mr. Frederick W. Taylor in his investigations found that a shovel adapted to a load of 21 lb. gave the best results. Such a shovel for general concrete work will correspond more nearly to the standard No. 4 size shovel than to the No. 3 size most generally supplied when your order calls simply for one dozen shovels. Why not add to your order the size of shovel wanted? Why not further specify the make and grade? Why not go to the maker and learn something of shovels and then specify intelligently?

For an implement which figures so largely in nearly all concrete work, the shovel used has perhaps been accorded the least consideration, whereas it is deserving of a great deal. Contractors ordinarily buy machinery by specification—they specify a maker known to them; but, more often than otherwise, shovels are purchased without any regard to the nature of the work for which they are to be used.

There are shovels of different sizes for various materials. For general concrete work a No. 4 shovel should be used to get the best results, and, as a general rule, the higher the grade of shovel, the better the result. No. 4 shovels of A-grade, costing $9.00 per dozen, will prove cheaper in the end than D-grade shovels, No. 3 size, costing $5.00.

Where the volume of material warrants the installation of an unloading device, a bucket unloader of the type shown in Fig. 337 may be used, or a clam-shell bucket as shown in Fig. 338 may be employed and operated from a derrick or locomotive crane. With a bucket unloader the labor cost, according to Mr. Turner of the Turner Concrete Steel Company of Philadelphia, will be 3.5 cents per cubic yard. With a clam-shell bucket it will be 4.5 cents per cubic yard.

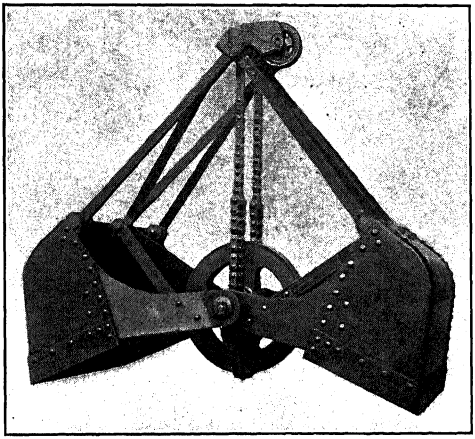


Fig. 338.

In connection with hopper bottom cars, a trestle may be built or any of the many types of wagon loaders may be used. Fig. 339 illustrates a type of loader sometimes employed. Wagons may be loaded directly by the device shown in Fig. 339A.

In Fig. 387, page 564, of Volume II is shown a hopper placed beneath the track to serve a feeder for an ordinary bucket elevator. Any of the above types of unloaders may be used to deliver the materials into wagons or bins, and the latter may be designed to discharge into wagons or direct into mixer, as conditions may require.

91. Cement.—Cement is ordinarily ordered in cloth bags, owing to greater ease in handling. Since the price of the cloth bag will be refunded by the cement mills only for bags returned in good condition,

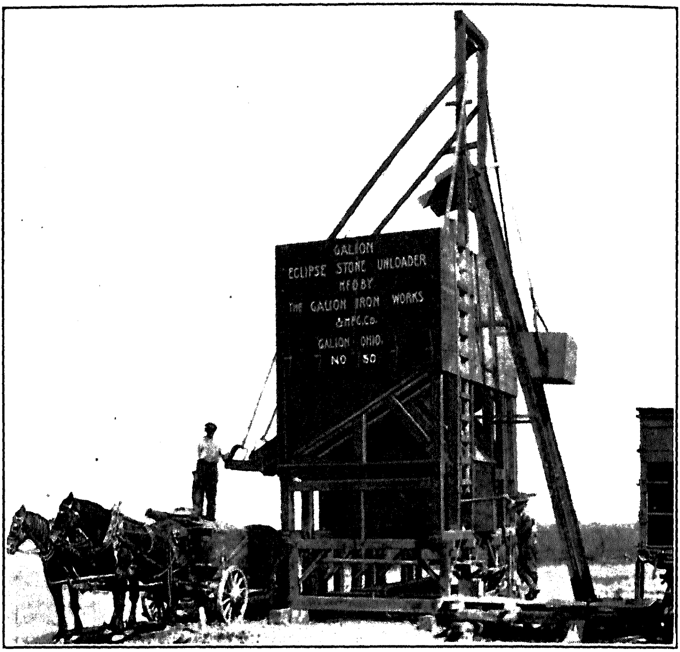


FIG. 339.

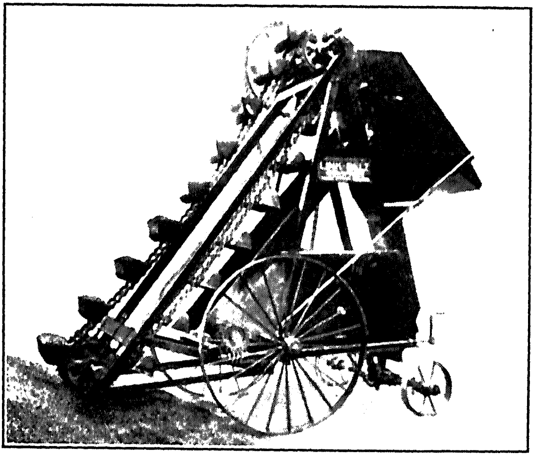
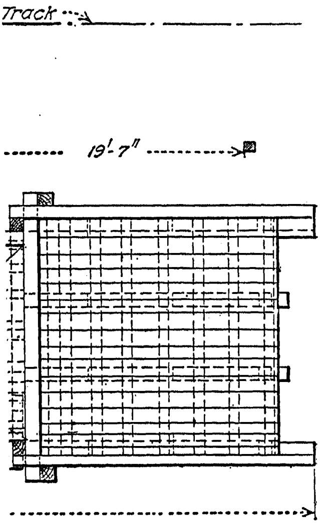
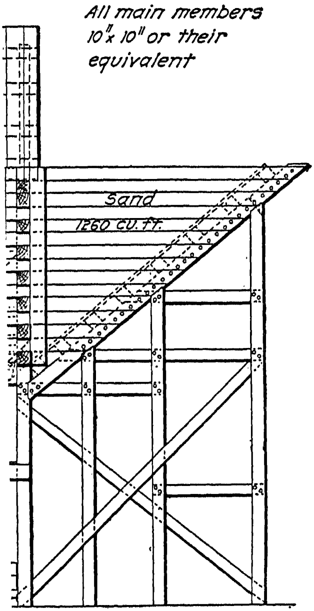


FIG. 339A.



oof



(*Facing Page* 458.)

t damage
nent bags
of bulk
In Fig.
ment.
e. Aside
r-present
nt of the
ximating
tions set
eed with

neasuring

ne above
h gradual
low pipe;
the full
s full the
ne mixing
of attain-
objection
capacity.
w of the
aches the
and shifts
ne proper
rmits the

provision should be made not only against loss but also against damage by water. This troublesome problem of caring for empty cement bags has been an important consideration in promoting the use of bulk cement, which is now being extensively used on large work. In Fig. 340 is shown a plant arrangement involving the use of bulk cement.

92. Water.—The problem of water is often a serious one. Aside from questions of fitness or unfitness, cost, etc., there is the ever-present problem of handling water fast enough. Probably 75 per cent of the plants operating at high efficiency—that is, at a rate approximating the theoretical maximum as determined by physical limitations set forth in Art. 108 of Volume II—have their limit set by the speed with which the water can be measured and fed.

Figs. 341, 342, and 343 illustrate the ordinary type of measuring tank. There are three principal objections to tanks of the above type: (1) The obstruction offered to the inflow of water through gradual closing of valve; (2) the practical limitation set to size of outflow pipe; and (3), but most important, the impossibility of utilizing the full capacity of the feed pipe, since in all cases when the tank is full the water is automatically cut off, requiring perfect timing of the mixing operation with the rate of water flow, an ideal impossible of attainment. Fig. 344 illustrates a duplex tank in which this latter objection is removed and provision made for utilizing the feed line to full capacity. The by-pass *A* set at the proper level provides for the overflow of the water from one tank to the other, as the water in the former reaches the proper height. A movement of the lever *B* opens the valve *C* and shifts the feed pipe into the second tank which in turn fills to the proper point and overflows into the first tank. This tank further permits the

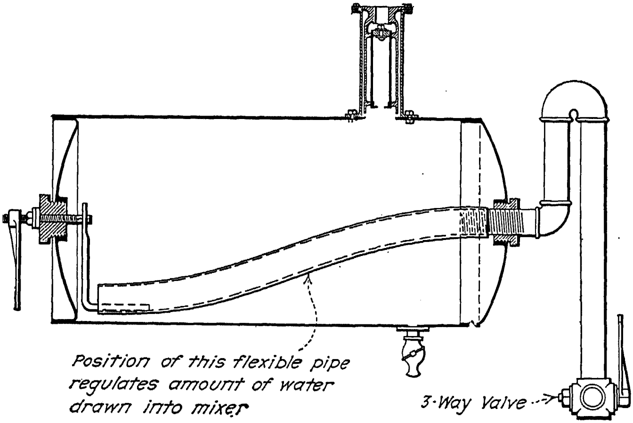

Position of this flexible pipe regulates amount of water drawn into mixer
3-Way Valve

Fig. 341.

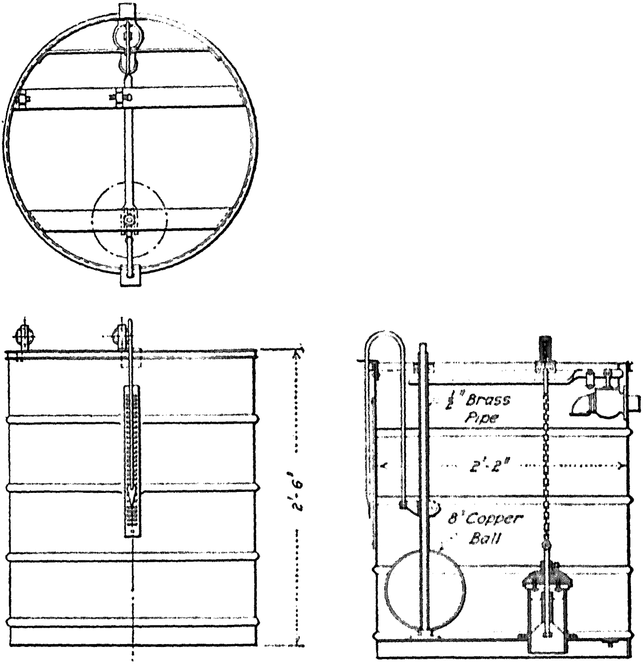


FIG. 342.

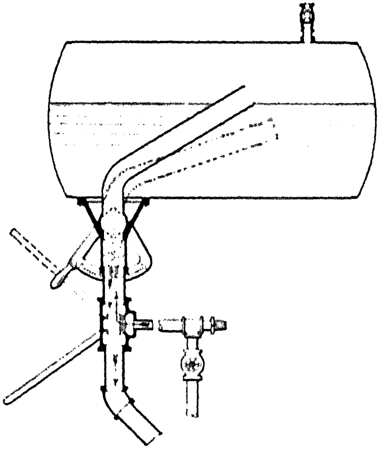


FIG. 343.

use of a large outflow pipe unobstructed by valves, etc. Water under a light head flows slowly (approximately 1 gallon to the second through a 2-in. pipe), and since common practice of today requires 1½ to 2 gallons of water per cubic foot of material, the time of flow becomes an important matter, especially so since it is very desirable that all the water for the batch be fed into the mixer in advance of the aggregates.

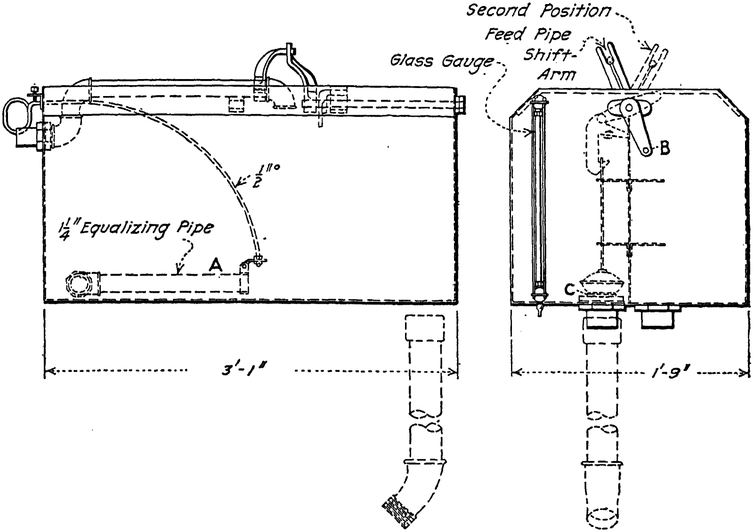


FIG. 344.

CHAPTER XXIV

MIXING

93. Hand Mixing.—The present tendency in every line of activity is toward the substitution of machinery for hand labor. To this tendency the contracting and engineering field is no exception. There are, however, many small jobs on which it is more economical to mix concrete by hand. Even in large work it is sometimes advisable to resort to hand mixing; for example, at the beginning and end of a job or where the concrete is to be placed in positions widely scattered. While the cases where hand mixing is economical are rapidly disappearing for reasons to be later set forth, it is important that due consideration be given to the process of hand mixing wherein the plant consists mainly of the shovel, the wheelbarrow, the rake, and the mixing board.

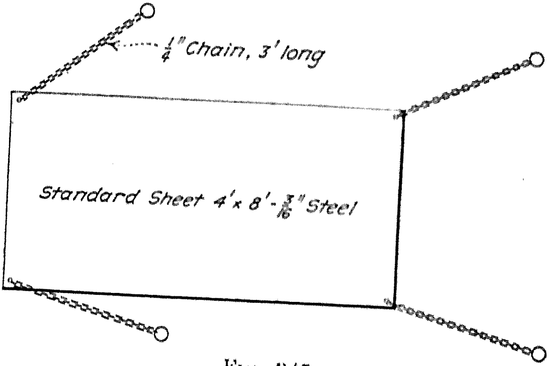


FIG. 345.

The cost of hand mixing will vary with local conditions; but with a full knowledge of conditions, the cost may be readily and accurately estimated, for which purpose the reader will find Taylor and Thompson's "Concrete Costs" of value.

For efficient mixing by hand it is of prime importance that a suitable mixing platform be provided. This platform should be at least 48 in. × 96 in. for single batches, or 96 in. × 96 in. for double batches, and may be of wood or steel. The writer has a decided preference for a steel plate (Fig. 345) of size suitable for a single batch, using two or more as the number of batches to be mixed simultaneously is increased. This type of platform is readily moved, gives an excellent shoveling surface, and if it is always kept close against the toe of the stone pile, minimum handling of materials is the result.

For measuring the aggregates, straight-topped barrows are strongly recommended as being more accurate than measuring boxes, and the barrows should be of such sizes, or assortment of sizes, as to permit struck measure rather than rounded or heaping measure. Where bottomless boxes are used, they should preferably be smaller in plan and deeper than those ordinarily employed. Also struck measurement should be insisted upon, as it is only in this way that accuracy may be obtained.

The cost of hand mixing may usually be accurately estimated by an analysis of the various operations entering into the process. The following is taken from page 278 of Taylor and Thompson's "Concrete Costs."[1]

" A gang on a large subway tunnel, materials close at hand, the cement and sand being mixed into a mortar and spread on the gravel before turning it; the concrete shoveled directly to place.

"One foreman with 2 gangs, consisting in all of 19 men, each gang divided as follows:
2 men mixing mortar.
4 men shoveling stone, mixing concrete, and shoveling concrete directly to place.
1 man assisting to shovel concrete, washing gravel, and other odd work.
2 men in excavation receiving concrete and ramming it.
1 extra man working with both gangs carrying water and on odd work.

"This double gang mixed about 64 batches per day of 10 hours. The proportions were 1 : $2\frac{1}{2}$: 4, and, since for gravel concrete (gravel being assumed to have 40 per cent. voids), we find from Table 22, on page 151, that 1.40 barrels cement are required per cubic yard, this quantity divided into 64 batches per day gives an output of about 46 cu. yd. per day of 10 hours."

Now let us assume for comparison that it is proposed to use a machine such as the "Bantam" Fig. 346. For each batch there must be loaded into barrows, wheeled, and discharged into the mixer, 1 barrow of sand and 2 barrows of stone, together with water and cement for each batch. An average man will load his own barrow, wheel a distance, say 50 ft., and return every 3 minutes, allowing for average delays and rests. Taylor and Thompson's "Concrete Costs" set the figure lower than we have assumed above. If, therefore, we put on a double crew consisting of 6 men, we shall be able to deliver material for a 1-bag batch of the proportions 1 :2 : 4 every $1\frac{1}{2}$ minutes. One man will be required at the to handle cement. If now we assume the mixer so placed that the concrete can be discharged directly into place, we need only provide the men in the excavation to receive and ram the concrete (say 6 men, by reason of the greater amount we shall handle) and our organization is approximately as follows, allowing for 2 extra men:

---

[1] Copyright 1912 by Frederick W. Taylor.

1 foreman.
6 men handling sand and stone.
1 man handling cement.
1 mixer attendant.
6 men placing concrete.
2 extra men.
—
17 total.

With these men, 3 less than required for hand work, we can turn out 40 batches of 1 : $2\frac{1}{2}$: 4 concrete per hour. Assuming 40 per cent voids in the stone, our day's out put would be 71 cu. yd. of concrete. We would thus save 3 men and do approximately 45 per cent more.

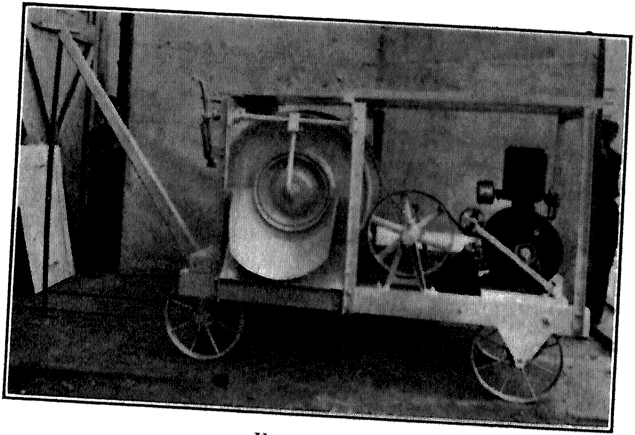


FIG. 346.

Since the mixer utilized for the above comparison can be purchased for $275.00 complete, it will quickly pay for itself and one need not have many days' work in sight to warrant the investment of the amount involved, provided of course that maintenance charges are not going to counterbalance.

94. Machine Mixing.—Of late the field of machine mixing has been materially broadened by the introduction of small machines, such as the "Bantam" type referred to in the preceding article. These machines are low in cost, efficient in operation, and are easily transported and set up. The T. L. Smith Company's "Mascot" (Fig. 347) is another machine of this class. The Standard Scale and Supply Company also turn out a reliable machine, and doubtless there are others. In making a purchase, however, it should be borne in mind that the more simple the construction of the machine, the more satisfactory will the investment prove; but on these lighter machines which have low first cost and lightness for their primary requisities, the details will vary materially from similar details on more elaborate and costly machines.

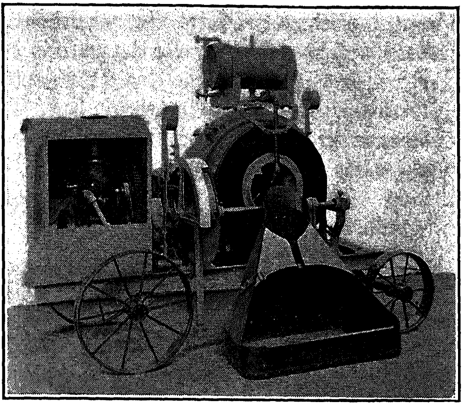


Fig. 347.

To the end that the machine may be as light as possible, rolled steel and wood should replace cast steel and cast iron wherever the change can be made without serious sacrifice of efficiency; but steel plates of lighter weight than No. 10 should not be used, save for protection against weather. Sprockets and chain should have preference over gearing on account of lightness. The carrying rollers for a similar reason should

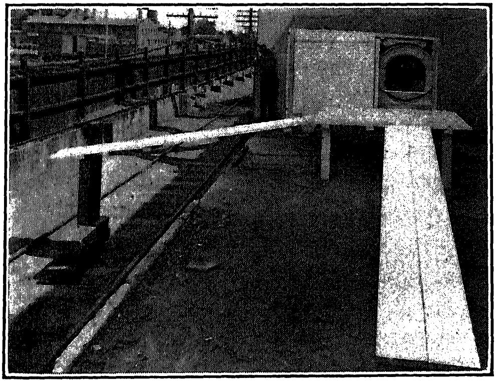


Fig. 348.

revolve loose in fixed shafts, thus saving the weight and expense of journals. It must be conceded that loose rollers of this type are not quite so durable as rollers fixed to shafts revolving in proper journals, but they are lighter and the arrangement is cheaper. The service they give will also be found fairly satisfactory.

In the development of the "Bantam" the considerations mentioned have been given due weight and, in order to increase the capacity to the maximum, a door has been perfected which permits the effective closing of the discharge opening with the result that 10 cu. ft. can be crowded into a mixer of size 1700 lb. lighter than is the case where the ordinary open-end type of drum is used.

The writer is credited with rather radical views as to the efficiency of small machines, holding that where a mixer must be charged wheelbarrow at a time and the mixed material similarly handled, a mixer designed to handle a 1-bag batch will prove most efficient, and furthermore that the simplest type of machine arranged for low charging and comparatively high discharge (see Figs. 348 and 349) will prove the most efficient of its class.

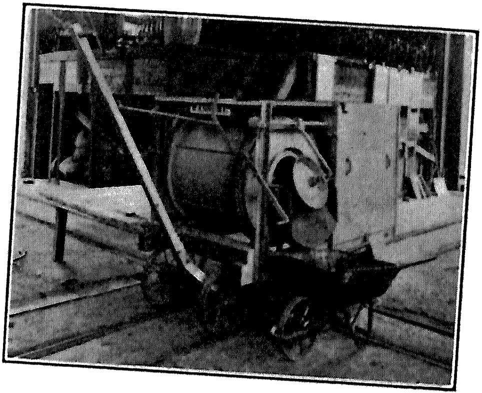


Fig. 349.

	Low charger	Power loader
Assembling batch—in mixer	24 seconds	In hopper 24 seconds
Hoisting hopper		10 seconds
Flow of material, hopper to mixer		10 seconds
Lowering hopper		5 seconds
Mixing	30 seconds	30 seconds
Discharging into 4 barrows	28 seconds	28 seconds
	82 seconds	107 seconds
Allow for overlapping of operations		24 seconds
		83 seconds

The above comparison is based on actual operations, and unit times given are not exceptional. It will be further noted that, as the assembling time is reduced, the comparison becomes more favorable to the direct charged mixer, since this operation is the only one which can overlap. Conversely, as the time of assembly increases, the comparison becomes more favorable to the mixer with the power loader. In the above the time required to load the barrow has not been considered, since this is identical in both cases. The operation alone has been considered starting from the point where the barrows are loaded ready for the mixing operation.

A second comparison will be made between a 1-bag-capacity low-charging mixer and a 2-bag-capacity machine with loader, materials to be handled in barrows on both sides of the mixer. For the purpose of illustration, average men will be assumed who will load a wheelbarrow, wheel a distance of 5 ft., discharge their barrow, and return in 3 minutes. This is a conservative figure. Upon this basis the organization for the two plants will be as follows:

1-bag mixer	Operation	2-bag mixer
6 men	Assembling batch	12 men
1 man	Handling cement at mixer	1 man
3 men	Wheeling mixed concrete	6 men
1 man	Extra man	2 men
11 men, total		21 men, total

The above crews will operate as follows:

24 seconds	Assembling batch	48 seconds
	Hoisting hopper	10 seconds
	Time of flow of materials hopper to mixer	15 seconds
	Lower hopper	5 seconds
30 seconds	Mixing	30 seconds
28 seconds	Discharge of mixer into barrows	56 seconds
82 seconds		164 seconds
	Less overlap of assembling time	48 seconds
		116 seconds

Reducing these operations to 1-man time gives 902 seconds per batch for the 1-bag mixer as against 2436 seconds for the 2-bag mixer, the respective batches yielding say 6 and 12 cu. yd. of finished concrete, with a time per cubic foot of respectively 151 and 203 seconds.

It must be borne in mind that, in making the above comparisons, conditions have been assumed somewhat unfavorable to the large machine; but the conditions so assumed are such as must prevail in many cases, and the comparison given merely outlines the method to be pursued in coming to a determination of the type of machine to be used.

If the entire batch is discharged directly from the mixer to place, into a car or bucket, the result at once becomes favorable to the larger machine both directly and indirectly in overheads, superintendence, etc. This point is one often overlooked and the advantage of the large machine is thrown away through failure to properly appreciate the importance of limiting the time consumed in the operations of charging and discharging. The writer observed, on a large building operation in Chicago, a case of flagrant violation of first principles in plant operation.

The organization on the work was as follows:

15 men loading barrows, wheeling, and charging.
1 man handling cement.
1 man operating discharge chute.
8 men distributing mixed concrete in barrows.

25 men, total

This crew was mixing concrete in a ½-yard mixer at the rate of one batch in 4½ minutes, this limit being set by the delays incident to the manner of charging and discharging. The mixing cycle was as follows:

For a 3-bag batch of 1: 2½: 5 concrete:

Assembling batch in mixer:		
Charging 3 bags cement	21 seconds	
Charging 10 pails of water	30 seconds	
Charging 9 barrows of aggregate	63 seconds	114 seconds
Mixing		30 seconds
Discharging into 10 barrows		70 seconds
Delays in discharging due to incidental shifting of runways, etc		56 seconds
		270 seconds

It was pointed out to the superintendent of this work that he was wasting time and labor by using 6 extra men to load barrows with sand and gravel, since his wheelers were forced in every instance to wait for the discharge of materials from the mixer. He was shown that men could load their own barrows, wheel to mixer and return in 3 minutes. Installation of a batch-charging hopper was recommended. The delays on the discharge side were pointed out and a suggestion made that the batch be discharged on a platform, whence it could be pulled off into barrows with a stroke of a hoe and the number of wheelers be reduced, since they could work steadily rather than spasmodically. A water tank was recommended. These suggestions were carried out, with the results that a speed of 3 minutes per batch was easily maintained with an organization reduced to:

9 men loading barrows and charging.
1 man handling cement and water.
1 man operating discharge chute and loading barrows.
6 men wheeling to place.

17 men, total.

The above instance is not an isolated case. It is more or less typical of results which follow close upon failure to observe elementary principles, and to obtain an accurate knowledge of costs.

So far only the simplest types of mixing plants have been treated, but it is believed that a knowledge of the possibilities in this direction will lead to intelligent selection of the more highly specialized plants. The more or less standard types of equipment were covered at some length in Volume II of this series, and will not be considered here.

CHAPTER XXV

TRANSPORTING OF CONCRETE

The mixed materials may be handled in barrows, carts, or cars, or in buckets operated in connection with derricks, cableways, or spouts. For the scope of the present chapter it will be sufficient to treat of effecting distribution only by barrows, carts, and spouts, leaving the derrick and cableway out of consideration. The use of derricks, on the one hand, is ordinarily limited to work confined to limited area, or to work wherein derricks are in any case a necessary part of the equipment. Cableways, on the other hand, are high in first cost which renders their adoption inadvisable except in exceptionally large work.

95. The Use of Barrows or Carts.—A question deserving of very serious consideration is the cost of handling material in barrows or carts. This method involves less original outlay than any other and in many cases which have come under the writer's observation, the cost of installation of a more elaborate plant would cover the costs of distribution by barrow or cart.

Using the ordinary barrow, a man will handle an average of 19 cu. ft. per barrow and, including time of loading by shovel, will require (according to Taylor and Thompson) 25.4 minutes per cubic yard of concrete, with an additional 3.9 minutes per cubic yard of concrete for each additional 25 ft. that it is necessary to wheel the material.

If two-wheeled carts of 6-ft. capacity are substituted for the smaller barrow, a time is obtained of 11.28 minutes per cubic yard of concrete per 100 ft. of wheel, with an additional 1.66 minutes per cubic yard for extra distance. The above figure is based on an average load for the cart of 4½ cu. ft., which can and should be maintained.

From the above comparisons it will be seen that, at 20 cents per hour, the cost of wheeling concrete in barrows should not exceed $0.084 per cubic yard with an additional $0.013 for each 25 ft. of additional wheel. For carts, results will be $0.0376 per cubic yard per 100 ft. of wheel, with an extra $0.0055 per 25 ft. additional wheel. There are, however, certain offsets to be made. The wheelbarrow requires less in the line of runways or scaffolding than does the heavier cart, and the cost of these runways should be carefully estimated. Average results show a cost for 200-ft. runs of 2-in. plank as being approximately $0.03 per cubic yard of concrete for wheelbarrows and $0.074 for carts, assuming the cost of runways as distributed over 1000 cu. yd. of concrete.

It must, however, be borne in mind that the above prices are based upon runways laid on the ground. Where the runways must be elevated, the showing becomes more favorable to carts, for the reason that the bents or supports for the wheelbarrows must be practically the same as for the carts.

96. Spouting Plants.—While there has been a tendency to exaggerate the advantages of spouting concrete, and to use it on small jobs, the present state of the art limits the use of spouting plants practically to the placing of 2500 yd. of concrete or upward. It is being recognized more and more widely that the extension of this system to smaller jobs is not a money-saving proposition, particularly when the contractor is not well acquainted with all the problems involved.

The installation of a spouting system is somewhat expensive. Under proper conditions it will make a definite saving, but it must not be rated too high by those undertaking it, for the saving will rarely reach the abnormal figures sometimes expected. Where the work is light and scattered, any attempt to spout concrete into place is likely to prove disappointing. It is principally on heavy, solid construction that it is of advantage, and each case must be considered carefully on its own merits before a decision is made to use this method.

Concrete may be conveyed a considerable distance by means of chutes supported in a variety of ways, but care must be taken that these chutes have a workable inclination. In all cases it is important to maintain a uniform pitch throughout the entire line. Unless the pitch is kept uniform, the concrete will flow faster in some parts of the chute than in others, with the result that it will pile up at some point of slow flow, and will overflow and fall to the ground. The pitch, also, must be greater when the material is to be carried a considerable distance than when it is to be carried only a short distance. If given a slight start, fairly wet concrete will flow 50 ft. with a pitch of about one in six. As the distance increases, the friction of the concrete in the chute overcomes its initial momentum. This makes it necessary to increase the pitch for greater distances, so that a pitch of one in four is needed for 100 ft. while a distance of 300 or 400 ft. will require a slope of one to three. These slopes are based upon a chute rigidly supported and of uniform pitch. Where the spouts are supported from guy lines, the slope must be a little steeper, say 4 to 6 in. to the foot for long lines.

If the pitch is too steep for the distance to be covered, the flow of concrete may be so rapid as to separate the rock from the sand and cement. The concrete should be kept in a continuous flow, moving slowly and uniformly, if a well-mixed mass is to be delivered into the forms. Crushed-slag concrete is sluggish, while washed gravel concrete is fairly rapid in its speed of travel down the chute. All of these things have to be taken into account in preparing the chute concrete.

For successful operation of any spouting system, the aggregates must be mixed properly. There must be just as much water as the material can carry without separation, so that the stone particles will be carried in suspension in the mass. The proper amount of water is from 1¼ to 1½ gallons to the cubic foot of material. The concrete materials must

Fig. 350.

slide down the spout, without any inclination to rolling. With a carefully designed system and properly mixed concrete, large quantities can be handled by a small stream, and placed where desired, without

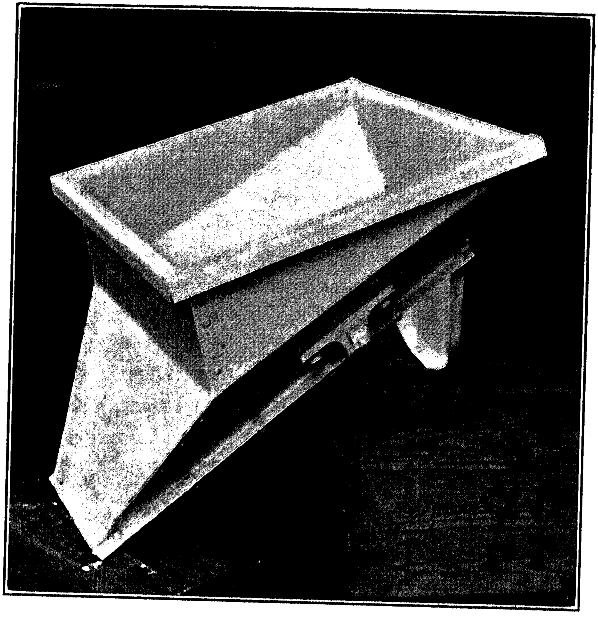


Fig. 351.

trouble from air pockets, without separation of particles, and without any tendency to produce laitance.

Spouting lines are carried, in general, in one of three ways: (1) by booms hung from the concrete tower; (2) by a guy line, one end of which is attached to the concrete tower; (3) by tripods or other supports from the ground, located at the ends of successive sections of spouting. Considerations of space, and particularly of the distance to which spouting is to be carried, determine generally which method of support will be used.

In a boom spouting plant the spout is mounted upon a swiveled bracket at the tower end, while the outer end, supported by the boom, is moved freely about the work as required. A second length of spout usually completes this unit. One great advantage of this type is the flexibility and freedom of movement obtained in placing the concrete.

In guy-line plants the spouting is suspended by ordinary blocks and manila falls, from steel wire rope guy lines or special cables set up for the purpose. The principal advantage of this type lies in its ready adaptation to handle concrete over long lines. Its disadvantages are mainly those involved in difficulty of lateral movement.

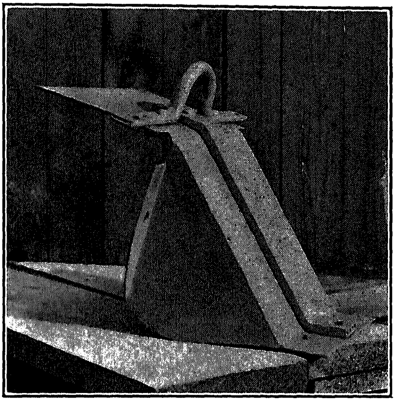


Fig. 352.

The tripod spouting plant uses movable towers to support the ends of successive spouting sections. While this is less flexible than the boom plant, it is much more flexible than the guy-line plant, for the various supports in the line may be moved in succession, thus covering a very wide area from a single tower. In order to move the guy-line plant, and thus cover additional area, the whole line must be dismantled and set up again in the new location.

In certain special cases the combination of two of these methods might be used to advantage, carrying the material across obstructions with the guy-line plant, and then distributing from the end of that section by means of one of the other methods. This would give some of the flexibility of the other methods while retaining, for a special purpose required, the peculiar features of the guy-line plant.

esirable that concrete spouting be based on a series of units which may be assembled in various combinations. Continuous line spouting should be changeable to swivel-head, or swivel-head to continuous line, as the conditions of the work require, it being necessary merely to have in stock a supply of the necessary units. This interchangeability is of great value in service, for spouts wear at the head and foot of each unit of length. By reversing a trough section, end for end, when showing heavy wear at one point, a new, unworn surface may be put at point of greatest wear.

A standard trough section, Fig. 350, is made of No. 14[1] gauge steel, forming a trough 8½ in. deep by 10 in. wide on top. The bottom is curved to practically a semicircle of 4-in. radius, the upper part of the sides being straight and tangent to the curve. Each section is punched with standard spacing, arranged for connecting all of the various attachments.

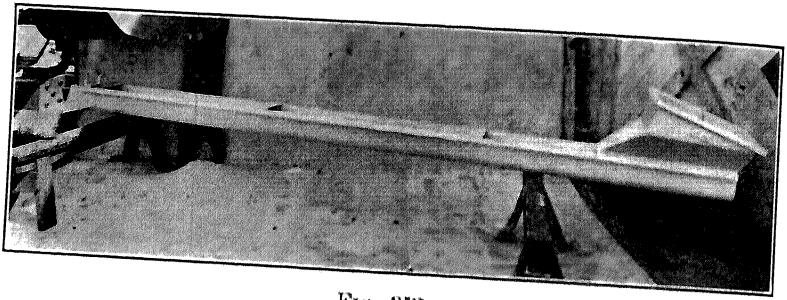


FIG. 353.

The hopper head, Fig. 351, attached at one end for receiving the concrete from the bin, or from an upper trough section, forms one point of support of the next trough section. At the other end is the splash hood, Fig. 352. By fastening the hopper head to the trough section at one end, and the splash hood at the other, we have the complete trough section, Fig. 353. These 24 by 24-in. hopper heads, as well as the splash hoods, can be bolted to either end of any standard trough section.

Standard trough sections are joined for continuous line spouting by bolting together their angle-iron yokes or flanges and bolting on the compression plate part. Thus, several sections are joined together, with a hopper head at one end of the entire group, and a splash hood at the other end, as shown in the assembly of the 48-ft. chute, Fig. 357.

Fig. 354 shows the swivel hook used in making the flexible joint between successive trough sections for swivel-head spouting and shows one of these joints, in which the upper line of spouting is supported by a fall and tackle attached to the bail on the splash hood; while the lower line is supported by the swivel-hook, connecting the lower hopper head with the splash hood of the upper line. The swivel hook is kept clear of the path of the concrete. The effectiveness of this method of support is shown clearly in Fig. 355, where the swivel hook on the splash hood is supporting a weight of 1500 lb.

---

[1] Where desired, trough sections may be furnished in No. 12 or No. 10 gauge steel plates. For all ordinary conditions, however, the standard section of No. 14 gauge will be found most satisfactory. The writer, however, strongly favors the lighter plate, and is now conducting experiments to the end that the plate used may be of special high carbon steel (0.22). This should give longer life and greater stiffness. The steel ordinarily used is 0.10 carbon.

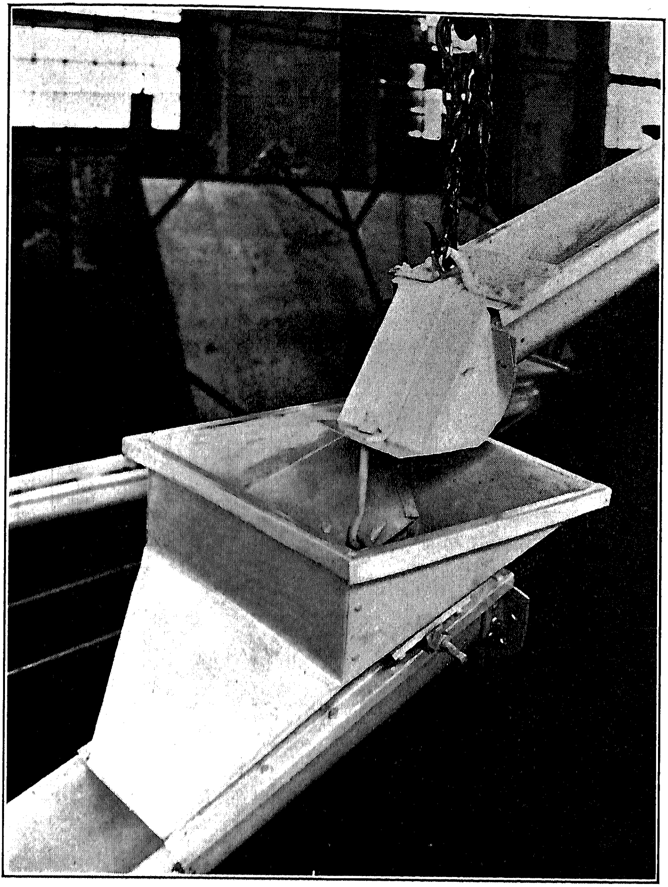


Fig. 354.

In some cases it is desirable to have a flexible joint in continuous line spouting. In this case the two sections are put together in a different manner, Fig. 356, where both the hopper head and the splash hood plate is here used in conjunction with a special yoke, after one of the angle-iron yokes has been removed. This allows a slight movement sideways, without requiring the attachments for the swivel-head operation.

The stiffness of the section is well shown by Fig. 357 which is a photograph of a 48-ft. assembled unit, heavily loaded. The sand was piled in the trough, 40 lb. per running foot, and then three men, weighing together 500 lb., stood on the center unit of the section.

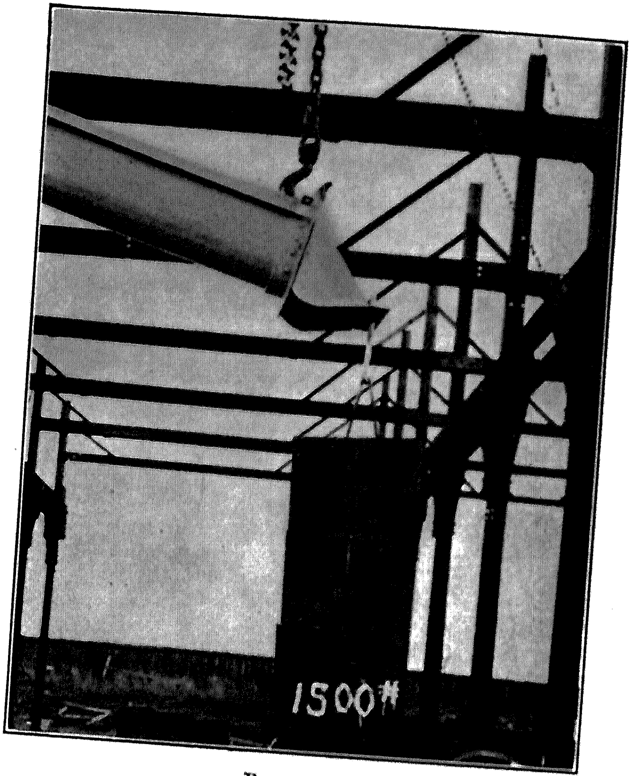


Fig. 355.

97. Hoists.—Whether the distribution is by spouts, by carts, or by barrows, it has become general practice on all work extending above ground to hoist the concrete. For this purpose a tower is practically indispensable.

Towers are constructed of steel as well as wood, the writer's preference being decidedly in favor of the wood tower. A concrete bucket is usually constructed on the simplest lines without catches or trips. A substantial bail made of two 3-in. Z-bars back to back, is arranged to operate between two $5\frac{3}{4}$-in. wooden guides, and is fitted at the lower end with journals in which rests the bucket trunnion. In setting up the tower and bucket, it is advisable in all cases to set the bucket so that it is practically balanced, and to this end the front guide should be so set as to be practically in contact with the nose of the bucket when the latter is pushed back to a point where the load in the bucket will tend slightly to press the stops on the sides of the bucket backward against the bail. Friction of the nose against the guides is, by this means, practically eliminated. By removing the front guide at any point in the height of the tower, and placing a block on the back of the latter,

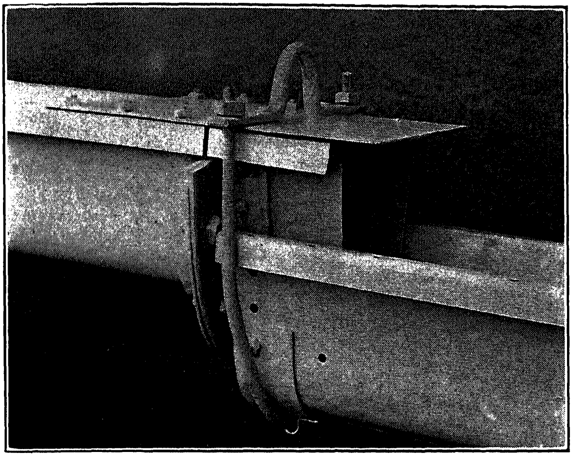


Fig. 356.

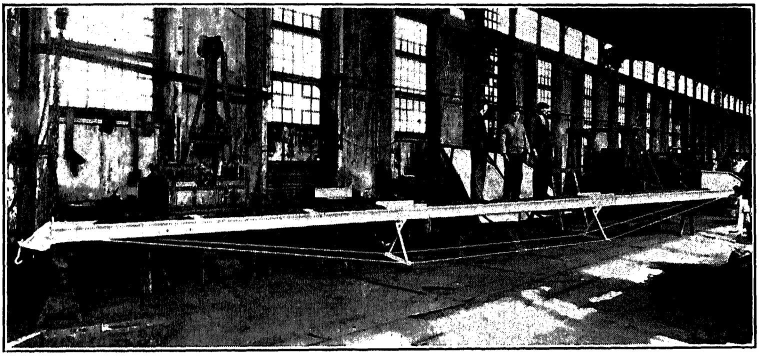


Fig. 357.

the bucket is canted forward so that it will drop its contents out through the opening made by the removal of the front guide. The bucket automatically rights itself, and is pulled back into position by the weight

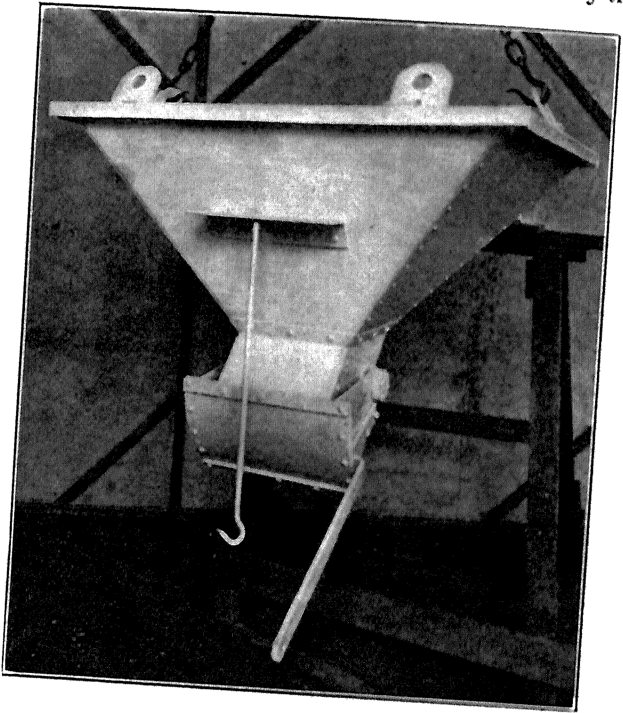


FIG. 358.

of the bail when the operator releases the brake. The concrete bin shown in Fig. 358 forms the upper end of the spouting system, the gate of the hopper controlling the flow.

PART V

NOTES ON ESTIMATING

By Leslie H. Allen

Of the Aberthaw Construction Co., Boston

The estimating of quantities and unit costs has been treated in a general way in Volume II. The chapters on estimating in this volume will deal mainly with the application of the general principles, already set forth, to the work of estimating bridges and culverts.

CHAPTER XXVI

ESTIMATING BRIDGES

98. General Considerations.—Large bridges are sometimes built in places that are very difficult of access and in consequence the cost of teaming materials may be much greater than in building work. In some cases it may be found necessary to set up a crushing plant for the supply of stone. The contractor, however, always avoids this, if possible, as the cost of operating a small temporary plant is always greater than that of running a large permanent plant, and it pays to buy crushed stone from such a plant, even if rock from the excavations is available.

The cost of plant, also, will vary greatly according to the nature and size of the work. As a general rule, the plant required in bridge construction is more costly than that for building construction. This is particularly the case where cableways are used. It is best to calculate separately the cost of labor and depreciation on each item of plant in each case and add the cost for supplies, fuel, and small tools. In general, however, it will be found that the total cost of plant for bridges, as in buildings, will vary pretty closely with the yardage of concrete. In the writer's practice he has found that figures of $6000 for plant on a 5000-yard job, $7000 on a 6000-yard job, $5000 on a 4000-yard job, and so on, check up quite closely with actual costs. These figures, of course, do not include plant for excavating, drilling, etc.

The labor cost of mixing and placing concrete will vary in the different parts of the structure. The abutments and piers of a bridge are usually very massive and the cost of placing concrete in same is low. In arch ribs, on the other hand, the concrete requires very careful spading and the cost of placing is considerably higher.

plant, the cost of mixing concrete should be about 20 to 25 cents per yard, and the cost of transporting and placing same in abutments and thick piers should be about 35 to 45 cents per cubic yard additional.

It is usual to specify that large stones may be embedded in massive concrete work to reduce the cost of same. These stones are generally placed not less than 6 in. apart and are kept at least 12 in. away from the face of the work. Some specifications will allow stones that one man can handle; others will allow any stone that the derricks can lift. It will be found that from 25 to 50 per cent. of the volume of a massive pier can be composed of large stone in this way. The cost of placing these stones, or "plums" as they are commonly called, should not exceed $1.25 per cubic yard. If the rock has first to be excavated for the purpose, the cost of the rock excavation must be added.

As an illustration the cost of concrete on a large typical bridge job might be estimated as follows:

Abutments and piers—1:2½:5 mix:

Cement	1.4 bbl.	@ $1.45 net	$2.03
Sand	0.5 cu. yd.	@ $1.00	0.50
Crushed stone	1.35 tons	@ $1.50	2.03
Labor, mixing and placing			0.65
Plant			1.10
Total			$6.31 per cu. yd.

Abutments and piers—1:2½:5 mix—with 30 per cent. of large stones:

7 cu. yd. concrete as above	@ $6.31	$44.17
3 cu. yd. placing large stones	@ 1.25	3.75
Cost of 10 cu. yd. of concrete and rock in place		$47.92
Average cost per cu. yd.		$4.79

Arch ribs and deck slabs—1:2:4 mix:

Cement	1.66 bbl.	@ $1.45 net	$2.42
Sand	0.5 cu. yd.	@ 1.00	0.50
Crushed stone	1.35 tons	@ 1.50	2.03
Labor, mixing and placing			1.00
Plant			1.10
Total			$7.05 per cu. yd.

The above costs do not include forms, steel, or finishing of surfaces.

The most uncertain and difficult item to estimate in bridge construction is the form work. It is best to estimate by the square foot of surface contact, as in building work, and to this add for the staging required for long arch spans.

No general rules or instructions can be given for estimating bridge form work, except to say that the labor should be estimated at not less than 10 cents per square foot, and the lumber, nails, and oil, etc., at 4 cents per square foot. Further discussion in regard to the cost of form work is given in the examples of typical estimates which follow.

It is not possible to give the cost of reinforcement accurately, any more than that of form work, but the following are approximations:

Assume, steel at mill	$1.50 per 100 lb.
freight	0.25 per 100 lb.
	$1.75 per 100 lb. or $35.00 per ton
Unloading, teaming, and piling	0.60 per ton
Labor, bending and placing	12.00 per ton
Tools, wire, and sundries	0.75 per ton
Total	$48.35 per ton

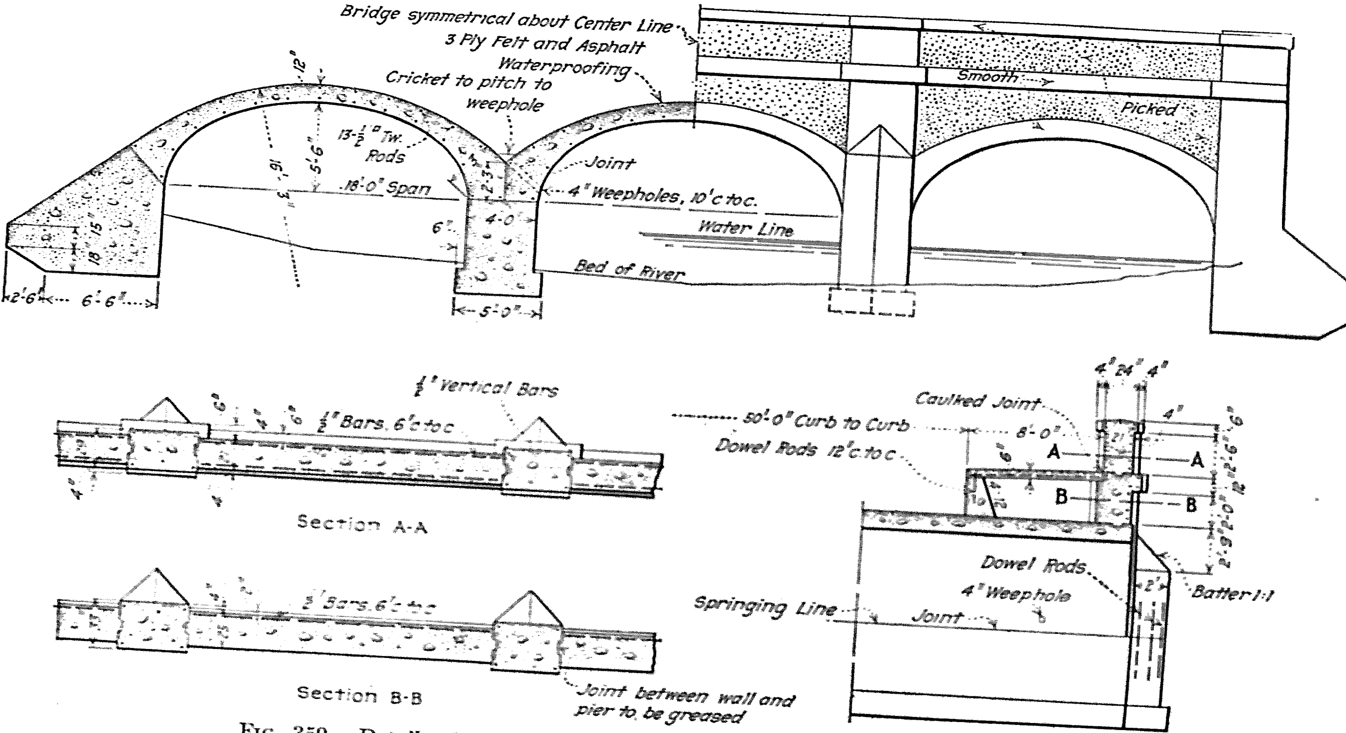
The cost of surface finish varies very little from that given in Volume II, Art. 119.

99. Examples of Typical Estimates.—Examples of typical estimates for bridges are included in the following paragraphs:

Arch Bridge Estimate.—Fig. 359 shows a small highway bridge built in 1912. The bridge was built in two sections, so as to cause no interruption to the street car service, which was maintained with one or two short interruptions at 30-minute intervals during the progress of the work.

The cost of the concrete work was as follows:

Labor, mixing and placing concrete in foundations and abutments	$0.91 per cu. yd.
Labor, mixing and placing concrete arches	1.25 per cu. yd.
Labor, mixing and placing concrete parapet and spandrel walls	1.31 per cu. yd.
Labor on form work to foundations and abutments	0.10½ per sq. ft.
Labor on form work to arch soffits	0.10 per sq. ft.
Labor on form work to parapet and spandrel walls	0.19 per sq. ft.
Labor, bending and placing steel reinforcement	$15.78 per ton
Labor, rubbing surfaces of parapet walls with carborundum three times (once measured)	0.06½ per sq. ft.
Labor, picking panels of same	0.07¾ per sq. ft.



Section A-A

Section B-B

Part Section at Crown

FIG. 359.—Details of New Broadway bridge over Spicket River, Methuen, Mass.

Cement cost $1.50 per bbl., f.o.b. cars to nearest siding.
0.40 credit for bags.

1.10
0.09⅔ unloading and teaming.
0.05 testing.
0.01 freight on empties.
0.05¼ loss on empties.

$1.30 9/10 price of cement ready for use in concrete.

Sand was dug and screened and teamed to the job at a cost of $1.28 per cubic yard.

Crushed stone cost $1.00 per ton, f.o.b. cars at quarry.
0.48 per ton, freight to nearest siding.
0.47½ per ton, unloading and teaming.

$1.95½ per ton, price of stone ready for use in concrete.

The plant consisted of mixer and derrick operated by electric motor, the power being obtained from a local power station. The cost was as follows:

Labor, erecting and moving derrick	$230.00	
Labor, erecting and dismantling mixer	89.47	
General labor around plant	157.50	
Freight and teaming	153.02	
Rental of mixer and derrick	360.00	
Small tools and supplies	527.50	
Electric current	210.00	$1727.49
Deduct part cost of derrick, tools, etc., charged to excavation and coffer dams		620.00
Cost of plant chargeable to concrete work		$1107.49

Divided by 826 cu. yd. equals $1.34 per cu. yd.

Form lumber cost for 57 M ft. b.m.	$1612.10
Nails, wire, and sundries cost	65.00
Cost of lumber for forms	$1677.10

Divided by 16,970 sq. ft. equals $0.099 per sq. ft.
or practically ....$0.10 per sq. ft.

Steel reinforcement cost $33.20 per ton on cars nearest siding.
Unloading and teaming, 1.23 per ton

$34.43 per ton

work would, therefore, be as follows:

Concrete foundations and abutments—1:2½:5 mixture:

Cement, 1.4 bbl. @ $1.31 per bbl.	$1.83
Sand, 0.5 cu. yd. @ 1.28 per cu. yd	0.64
Stone, 1.35 tons @ 1.95½ per ton	2.64
Labor, mixing and placing	0.91
Plant	1.34
Total	$7.36 per cu. yd.

Concrete arches—1:2:4 mixture:

Cement, 1.66 bbl. @ $1.31 per bbl.	$2.17
Sand, 0.5 cu. yd. @ 1.28 per cu. yd	0.64
Stone, 1.35 tons @ 1.95½ per ton	2.64
Labor, mixing and placing	1.25
Plant	1.34
Total	$8.04 per cu. yd.

Concrete parapet and spandrel walls—1:2:4 mixture:

Cement, 1.66 bbl. @ $1.31 per bbl	$2.17
Sand, 0.5 cu. yd. @ 1.28 per cu. yd	0.64
Stone, 1.35 tons @ 1.95½ per ton	2.64
Labor	1.31
Plant	1.34
Total	$8.10 per cu. yd.

Forms to foundations and abutments:

Labor	$0.10½ per sq. ft.
Lumber, etc.	0.10 per sq. ft.
	$0.20½ per sq. ft.

Forms to arch soffits:

Labor	$0.10 per sq. ft.
Lumber, etc.	0.10 per sq. ft.
	$0.20 per sq. ft.

Forms to parapet and spandrel walls:

Labor	$0.19 per sq. ft.
Lumber	0.10 per sq. ft.
	$0.29 per sq. ft.

Steel reinforcement:

Reinforcement	$34.43 per ton
Labor	15.78 per ton
Total	$50.21 per ton

SUMMARY

Concrete foundations and abutments, 408 cu. yd. @ $7.36	$3003
Concrete arches, 268 cu. yd. @ 8.04	2155
Concrete parapet and spandrel walls, 150 cu. yd. @ 8.10	1215
Forms to foundations and abutments, 3800 sq. ft. @ 0.20½	779
Forms to arch soffits, 8100 sq. ft. @ 0.20	1620
Forms to parapet and spandrel walls, 5070 sq. ft. @ 0.29	1470
Steel reinforcement, 5½ tons at 50.21	276
Rubbing faces of parapet walls with carborundum, 1600 sq. ft. @ 0.06½	104
Picking panels of same, 650 sq. ft. @ 0.07¾	50
Total	$10,672

To this total must be added the cost of excavation, cofferdams, pumping, road surfacing, waterproofing arches, watchman, temporary fences, liability insurance, and contractor's profit. These items do not come within the scope of the present work.

Beam and Girder Bridge Estimate.—Figs. 360 and 361 show a reinforced-concrete viaduct which was planned to carry highway traffic across a freight yard. The total length of the bridge was about 700 ft. The figures given below are from a contractor's bid submitted in competition.

The chief problem to be considered was the cost of carrying on work expeditiously without interruption to the freight traffic below. The method contemplated was the construction of a substantial temporary working platform about 3 ft. below the spring of the arch, the platform being constructed of steel I-beams on hard pine posts and floored with 3-in. plank. This would afford a good working platform for the carpenters and a firm base for the support of the centers and wet concrete.

It will be noted that the piers are not parallel to each other in most cases. This adds considerably to the cost of the form work, as no forms could be re-used without a good deal of alteration or re-making.

The estimated costs of the work were as follows:

Cement quoted	$1.75	per bbl., f.o.b. cars to nearest siding.
	0.40	credit for bags.
	$1.35	
	0.03	tests.
	0.06	unload, team, and store.
	0.03	freight on empties and loss.
Net	$1.47	bbl.

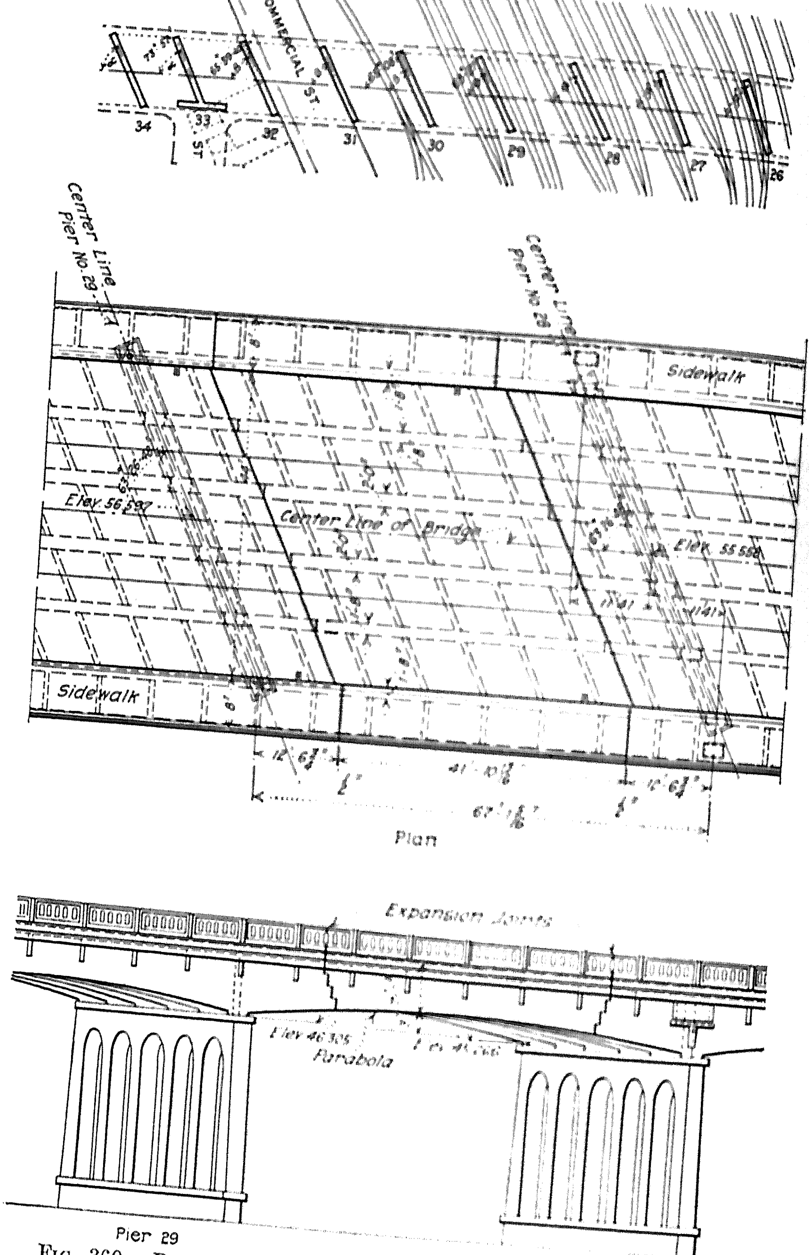


FIG. 360.—Details of Portland bridge, Cumberland County, Me.

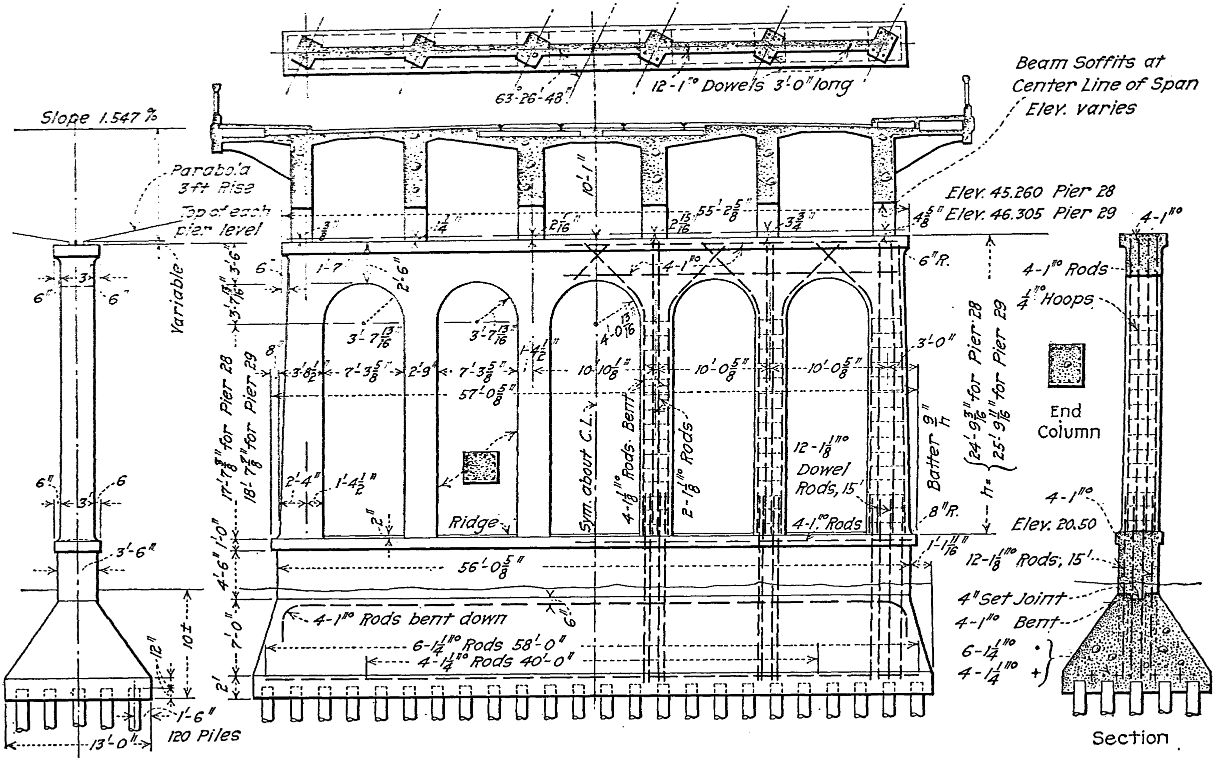


FIG. 361.—Details of piers 28 and 29, Portland bridge, Cumberland County, Me.

Sand quoted, 90 cents per cu. yd. delivered in teams at job. Screened gravel quoted $1.15 per ton delivered in auto trucks at job.

Labor, mixing and placing concrete:

in foundations	$0.80 per cu. yd.
in piers	1.00 per cu. yd.
in arch ribs and deck slabs	1.20 per cu. yd.

Plant—labor, depreciation, supplies, and power $1.00 per yard, giving a total of $7500 for the job.

Price of concrete in piers—1:2:4 mixture:

Cement, 1.66 bbl. @ $1.47	$2.44
Sand, 0.5 cu. yd. @ 0.90	0.45
Gravel, 1.35 tons @ 1.15	1.55
Labor, mixing and placing	1.00
Plant	1.00
Total	$6.44 per cu. yd.

Price of concrete in foundations—1:2½:5 mixture:

Cement, 1.4 bbl. @ $1.47	$2.06
Sand, 0.5 cu. yd. @ $0.90	0.45
Gravel, 1.35 tons @ $1.15	1.55
Labor, mixing and placing	0.80
Plant	1.00
Total	$5.86 per cu. yd.

Form work:

Labor on forms to foundations	$0.07 per sq. ft.
Piers	0.10 per sq. ft.
Curved soffits to arches	0.20 per sq. ft.
Sides of beams and girders	0.10 per sq. ft.
Deck (or floor)	0.10 per sq. ft.
Belt course below railing	0.50 per lin. ft.
Form lumber, nails, etc	0.04 per sq. ft.

Steel reinforcement:

Reinforcement quoted	$1.20 per 100 lb. f.o.b. mill 0.18 freight. $1.38 = $27.60 per ton *base*
Teaming, unloading, piling	$ 1.00 per ton
Labor, bending and placing	$11.00 per ton
Extra cost of small sizes and small amount bought from stock	1.00 per ton
Wire and sundries	0.40 per ton
	$41.00 per ton in place

Cost of working platforms, average size 57 ft. 0 in. × 51 ft. 0 in. × 28 ft. 0 in. high:

60 pcs. 12 in. × 12 in. H.P. 28 ft. long = 20 M @ $32.00	$640
500 lin. ft. 8 in. × 12 in. H.P. = 4 M @ $31.00	124
3-in. spruce plank for platform, 3000 sq. ft. = 9 M @ $24.00	216
Nails, dogs, bolts, etc.	50
Sundry lumber, 5 M @ $30.00	150
630 lin. ft. of 12-in. 40-lb. I-beams = 25,200 lb. @ $0.02	504
	$1684
Probable salvage on I-beams and hard pine at close of job	700
Net cost of material for one platform	$984
Cost of material for three platforms (to be used four times each)	2952
Labor, erecting and taking down platforms twelve times @ $450	5400
Estimated total cost of working platforms	$8,352

SUMMARY

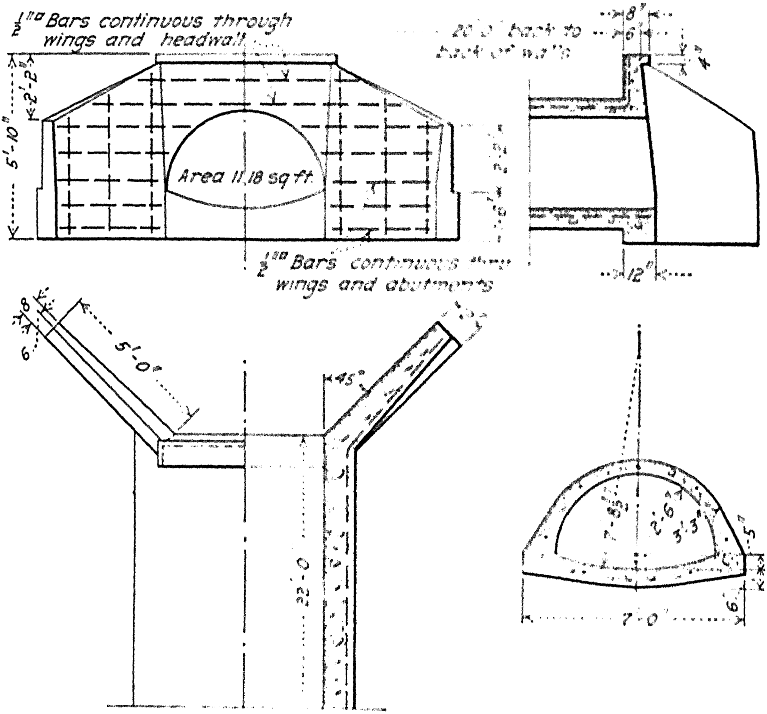
Concrete foundations, 1:2½:5, 2520 cu. yd. at $5.86	$14,767
Concrete piers, 1:2:4, 1692 cu. yd. @ $6.44	10,896
Concrete arch ribs and deck slabs, etc, 1:2:4, 3289 cu. yd. @ $6.64	21,839
Forms to foundations, 29,560 sq. ft. @ $0.11	3,252
Forms to piers, 45,600 sq. ft. @ $0.14	6,384
Forms to curved soffits to arches, 8870 sq. ft. @ $0.24	2,129
Forms to sides of beams and girders, 66,440 sq. ft. @ $0.14	9,302
Forms to deck or floor slabs, 35,600 sq. ft. @ $0.14	4,984
Forms to belt course below railing, 1332 lin. ft. @ $0.75	999
Steel reinforcement, 392 tons @ $41.00	16,072
Clinton wire cloth reinforcement in sidewalks, 16,120 sq. ft. @ $0.02½	403
4-in. concrete sidewalk on cinder fill and finished in granolithic, 10,050 sq. ft. @ $0.14	1,407
Concrete curb to same with metal nosing, including forms 5520 lin. ft. @ $0.55	3,036
Rubbing outside faces of piers and belt course, etc., with carborundum, including staging, etc., 6000 sq. ft. @ $0.06	360
Concrete railings cast by unit method and hoisted and set in place, including forms and reinforcement, 1306 lin. ft. @ $2.00	2,612
Cost of working platforms over railroad tracks	8,352
Total cost of concrete work	$106,794

Excavation and piles, wood block paving, conduits for cables, etc., and contractor's profit not included in above items.

CHAPTER XXVII

ESTIMATING CULVERTS

100. Typical Estimate.—Fig. 362 shows a concrete culvert of a common type designed for use in highway construction. The unit cost of a single culvert of this type built independently of any other work would be very high. It is usually the case, however, that in a highway construction job of 10 miles in length there is found a large number of such culverts and they are ordinarily built by a large, well-organized contractor's force.



½"□ Bars continuous through wings and headwall
2'-2"
5'-10"
Area 11.18 sq ft.
½"□ Bars continuous thru wings and abutments
20'0" back to back of walls
8"
6"
4"
12"
8
6
5'-0"
45°
22'-0"
7'-8½"
2'-6"
3'-3"
5"
6"
7'-0"

Fig. 362.—Standard 5-ft. arch culvert, State of Missouri Highway Department.

The analysis of the cost of cement, sand, and stone would follow along the same lines as in the preceding examples. The concrete would probably be hand mixed, and this labor of mixing and placing may be estimated at $2.00 per cubic yard. The plant would consist of shovels, mixing boards and tarpaulins, barrows, etc., which would be used again on other work, but it is fair to charge about $7.00 for use of these, including teaming, which is equal to about $0.65 per cubic yard.

The price of concrete — 1:2:4: mixture — would probably be as follows:

Cement, 1.66 bbl	@ $1.60	$2.66
Sand, 0.5 cu. yd	@ $1.00	0.50
Crushed stone, 1.35 tons	@ $1.30	1.76
Labor		2.00
Plant and tools, etc		0.65
Total		$7.57 per cu. yd.

Form lumber, nails, etc., for one culvert would cost about $60.00, which is equal to 6 cents per square foot, but if several culverts were built this cost would be divided between them.

SUMMARY

Concrete—1:2:4	11 cu. yd	@ $7.55	$83
Forms to vertical faces, labor only	310 sq. ft.	@ $0.10	31
Forms to arch labor only	660 sq. ft.	@ $0.13	86
Form lumber for one culvert			60
Steel reinforcement	360 lb.	@ $0.02½	9
Total cost of concrete work			$269

Excavation and road surfacing not included.

PART VI

THE ARTISTIC DESIGN OF CONCRETE BRIDGES

BY WM. J. TITUS

ASSOC. M. AM. SOC. C. E.

CHAPTER XXVIII

PRESENT STATUS OF THE ART

101. City Bridges.—Volumes have been written on the beautification of city bridges. Municipal Art Commissions and City Beautiful Commissions have been appointed by the score in an effort to obtain bridges of pleasing outline and artistic ornamentation. In fact, it is felt by many that the character of the bridges, streets, and public buildings of a city bespeak the character and enlightenment of the inhabitants of that city. This desire to secure beautiful bridges is to be commended, but unfortunately, we have tended to load down many of our supposedly beautiful city bridges with ginger-bread ornamentation and every imaginable kind of inappropriate detail. It is better that we should be content with plain, honest, solid, useful bridges than that they should be adorned according to the standards that have governed the ornamentation of dwellings and public buildings, proper as these standards may be in such places.

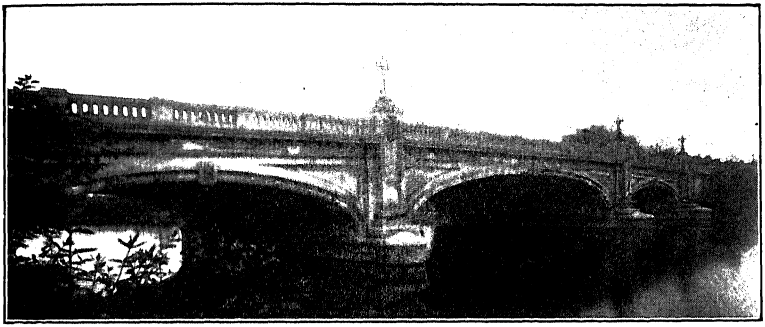


FIG. 363.—Excess of details; too elaborate panels; lack of balance in arch spans, resulting in ginger-bread effect.

102. Bridges in Country Districts.—Only with the coming of the automobile have we begun to realize that the roads and bridges of the suburban and more remote country districts are as deserving of careful and artistic design as those in our cities and other thickly populated districts. This has come about largely through the fact that our farmers are providing for themselves and their families a broader social and educational life. But even at the present time one finds in the literature on the subject many statements to the effect that "Few people would consider spending money for purely artistic features in a rural or remote district that is little frequented." The truth of the matter is, however, that the bridges in the country districts are more often designed to suit their surroundings than bridges in the city.

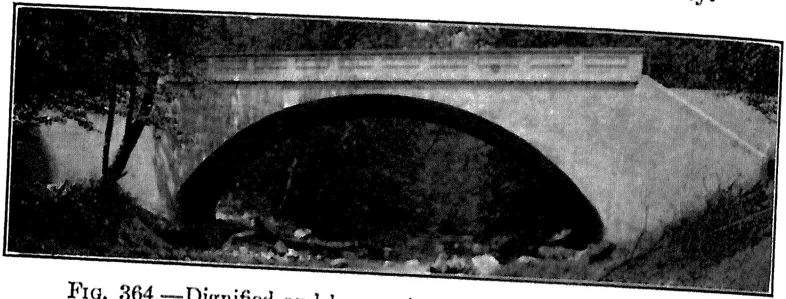


FIG. 364.—Dignified and harmonious structure, yet with little ornamentation.

103. Influences Affecting Design.—There have been many influences which have tended to prevent the construction of any considerable number of really beautiful bridges in America. Chief among these hindrances has been the insistent demand for more economical designs. This hindrance, however, has been largely removed by the use of concrete in bridge construction. Another great difficulty has been that most of our highway bridges (especially the earlier ones) have been built on designs inherited from the railroads, where the most popular bridge was formerly the one most easily designed and most easily and quickly erected with little or no regard for appearances.

Competition in design and the practice of allowing contractors to submit their own plans have been the reasons given for the existence of many of our flimsy and homely bridges. These are not valid reasons, however, in spite of the fact that the cheapest bid on the cheapest structure was most often selected. On the contrary, competition in design, with proper engineering supervision, has proved to be a most effective means of securing adequate and beautiful bridges. In fact, competitive designs have been received on a large proportion of the notable bridges of America.

There are today a number of consulting engineers who specialize in or devote all their time to concrete bridges, and many beautiful, well-designed bridges throughout the country bear witness of the increasing ability of these specialists. The engineer in direct charge of the building of a bridge often feels that he is admitting his own inability by allowing experts to submit designs, but in the large majority of cases he will be held in increased esteem by his associates for having secured for his clients at a minimum expense a structure that is very nearly the ideal one for the location.

CHAPTER XXIX

MAIN FEATURES IN DESIGN

Concrete bridges today are more easily made attractive than those of any other material, for even to the average mind they have the appearance of solidity coupled with strength. Concrete is a molded material and there should be no attempt to disguise this fact. Imitation of stone arches, for example, is not to be desired. Expressiveness to most people is the chief source of beauty and when the characteristics of a material are destroyed rather than emphasized, the effect is not only flat and characterless, but may even be untruthful and insincere. Stepping the upper surface of a wing has no excuse from an artistic standpoint.

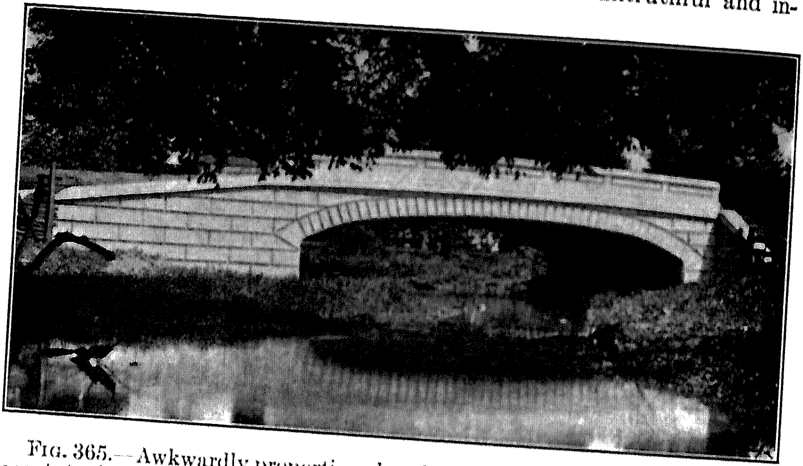


FIG. 365.—Awkwardly proportioned arch with objectionable surface treatment to imitate stone. Other bad features are the high springings and uniform thickness of arch ring.

104. Principles of Design.—There are a number of principles which must be observed in the general design of a concrete bridge in order that the structure shall have a pleasing appearance. These principles pertain to the correct use of material, a symmetrical arrangement of the structure as a whole, and the securing of proper style and harmony.

105. The Proper Use of Material.—One of the essential features of good design is clearness of purpose. The mind of the observer should be impressed with the visible strength of the structure and the proper use of material should be apparent. These requirements are satisfied by the use of steel in the suspension bridge and of masonry in the arch, so that bridges of this type are sure to command attention and admiration. They possess a boldness and an inherent beauty of line that it is not possible to approach in a truss or girder bridge. In the case of a masonry girder, with the main strengthening element, the steel reinforcement, entirely hidden and difficult of representation in any way, it is scarcely possible to hide an appearance of weakness and obtain a truly artistic effect.

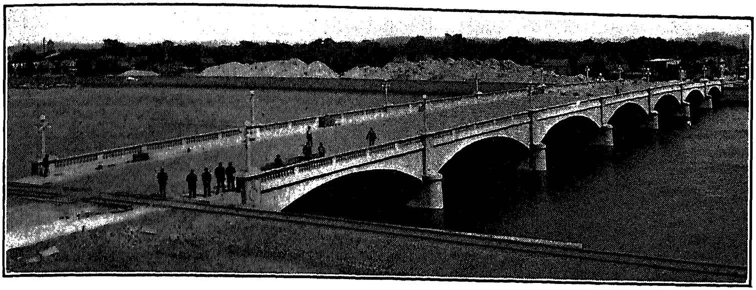


FIG. 366.—Arch bridge with beautiful lines and handsome railing and lamp posts, all of concrete.

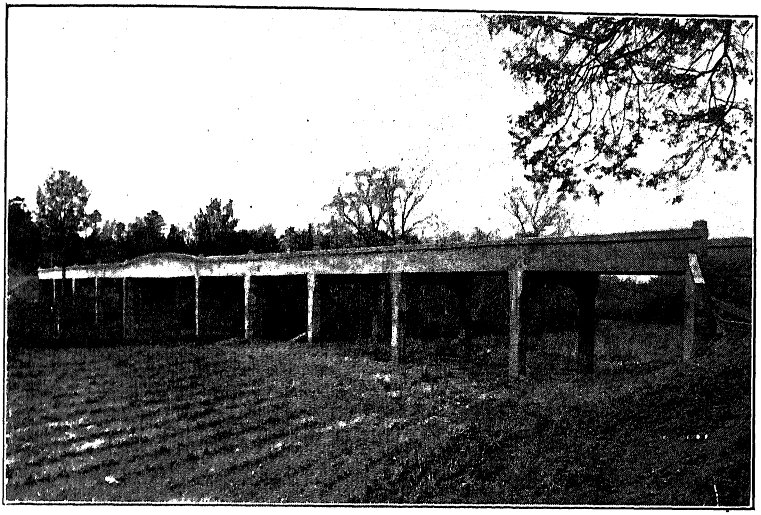


FIG. 367.—Girder bridge, well proportioned structurally, but inherently unsightly.

106. Symmetry.—Symmetry is another of the qualifications of a beautiful bridge and is an elementary idea in esthetics. A bridge should

CHAPTER XXIX

MAIN FEATURES IN DESIGN

Concrete bridges today are more easily made attractive than those of any other material, for even to the average mind they have the appearance of solidity coupled with strength. Concrete is a molded material and there should be no attempt to disguise this fact. Imitation of stone arches, for example, is not to be desired. Expressiveness to most people is the chief source of beauty and when the characteristics of a material are destroyed rather than emphasized, the effect is not only flat and characterless, but may even be untruthful and insincere. Stepping the upper surface of a wing has no excuse from an artistic standpoint.

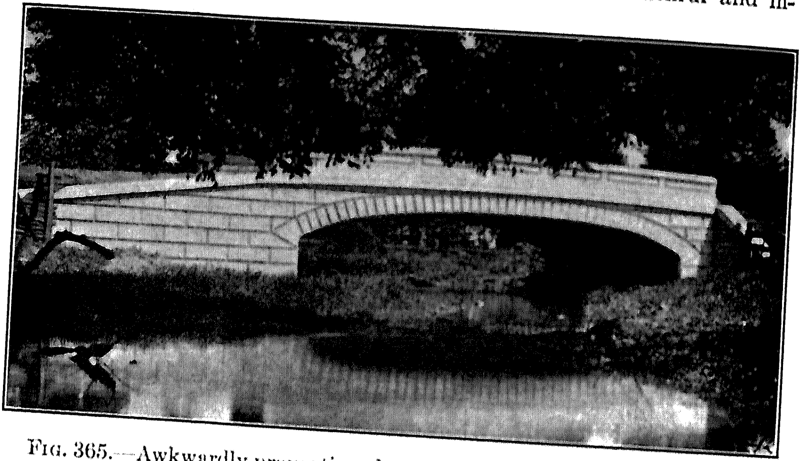


FIG. 365.—Awkwardly proportioned arch with objectionable surface treatment to imitate stone. Other bad features are the high springings and uniform thickness of arch ring.

104. Principles of Design.—There are a number of principles which must be observed in the general design of a concrete bridge in order that the structure shall have a pleasing appearance. These principles pertain to the correct use of material, a symmetrical arrangement of the structure as a whole, and the securing of proper style and harmony.

105. The Proper Use of Material.—One of the essential features of good design is clearness of purpose. The mind of the observer should be impressed with the visible strength of the structure and the proper use of material should be apparent. These requirements are satisfied by the use of steel in the suspension bridge and of masonry in the arch, so that bridges of this type are sure to command attention and admiration. They possess a boldness and an inherent beauty of line that it is not possible to approach in a truss or girder bridge. In the case of a masonry girder, with the main strengthening element, the steel reinforcement, entirely hidden and difficult of representation in any way, it is scarcely possible to hide an appearance of weakness and obtain a truly artistic effect.

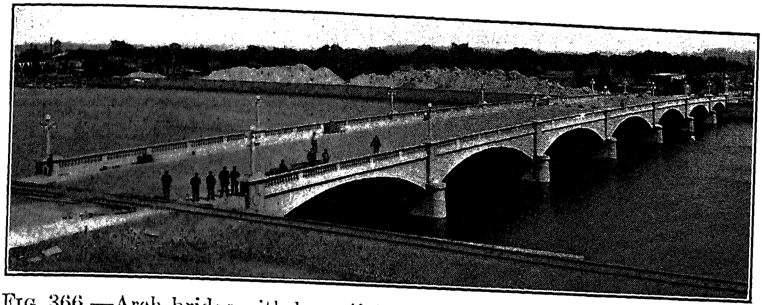


FIG. 366.—Arch bridge with beautiful lines and handsome railing and lamp posts, all of concrete.

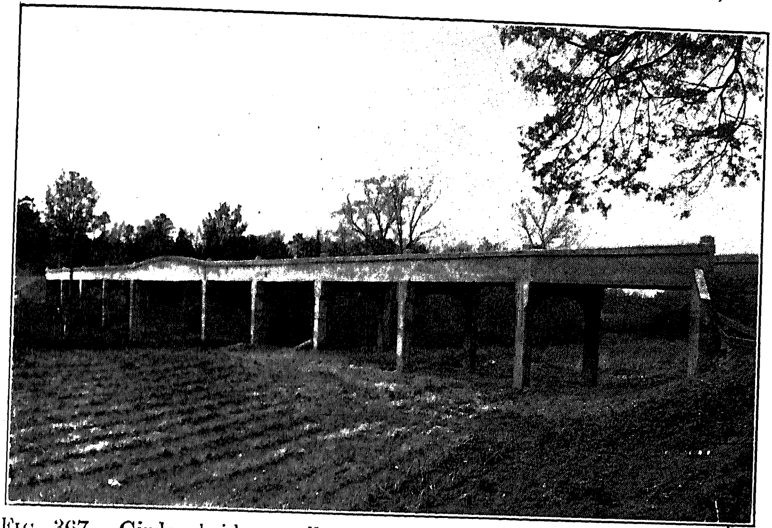


FIG. 367.—Girder bridge, well proportioned structurally, but inherently unsightly.

106. Symmetry.—Symmetry is another of the qualifications of a line unless the contour or other conditions are such that it is impossible to make a symmetrical design. If the latter condition be the case, the arrangement should clearly show an adaptation to the natural profile. That the line of symmetry should, if possible, be at the middle of a span and not at the middle of a pier is a feature of good design. This is true, probably, because the current of the river is ordinarily at the middle of the bridge and because one dominating span, rather than two, gives to the bridge a more graceful appearance.

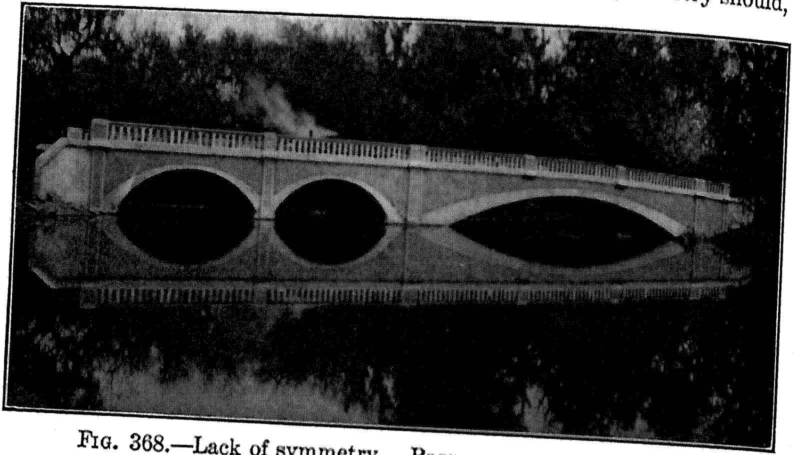


FIG. 368.—Lack of symmetry. Poor arrangement of spans.

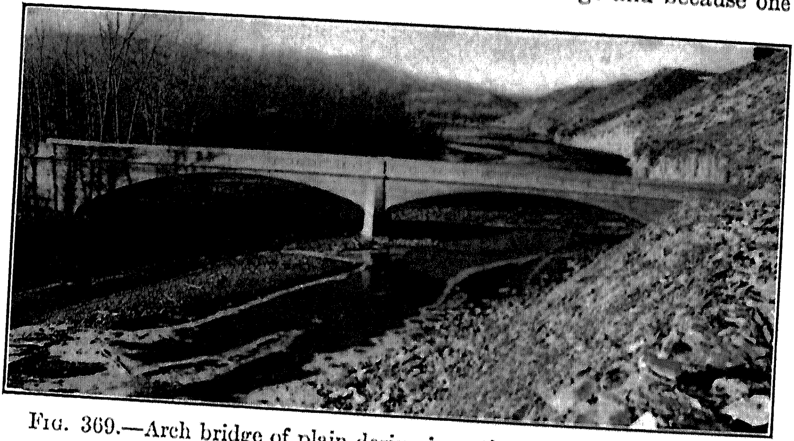


FIG. 369.—Arch bridge of plain design in entire harmony with rugged surroundings.

107. Style.—The style of a bridge should be in conformity with the environment, though a beautiful and appropriate bridge may often be the means of improving its surroundings. It is only because they may be considered to harmonize with their surroundings that many of the over-decorated bridges of Europe can be called beautiful, so that the same standards of design and ornamentation applied in the United States would often be entirely out of place. However, a bridge should always dominate as well as conform to its surroundings. A bridge among rugged hills or rocky cliffs should have a simple and bold outline so that it may not be dwarfed by its surroundings. A bridge in a low level country should have long sweeping lines and be quite simply designed. On the other hand, a bridge in a park should be more graceful, ornate, or even quaint in appearance. All the details of the latter type of bridge should be worked out very minutely, since they will be subject to much closer inspection than if the bridge is in any other location. A bridge in the city can usually carry considerable ornamentation, but must possess dignity commensurate with its importance. If there are buildings of a permanent character in the immediate vicinity of such a bridge, it should be designed in harmony with them.

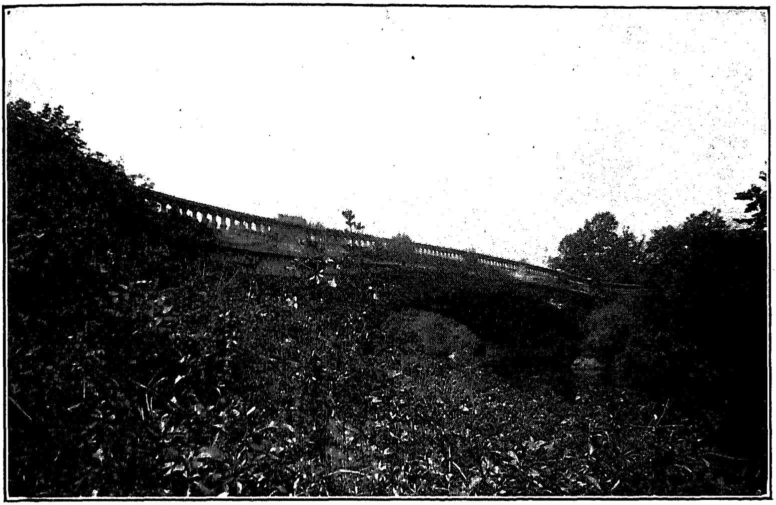


FIG. 370.—Park bridge of graceful lines and harmonious treatment.

108. Harmony.—In addition to harmony with surroundings, a bridge should have harmony of material, harmony of form, and harmony of dimension.

109. Harmony of Material.—Harmony of material is best secured with the use of but one kind of material. It is very difficult to obtain an appearance of balance when different materials are used, as there will be no agreement in the sizes of members performing similar duties. If it is necessary to make adjacent spans of different materials, the piers should be considerably enlarged and extended above the roadway as posts or pedestals in order to emphasize the separation of the two types of construction. Ornaments made of different material from the main structure often appear quite out of place, though a pleasing contrast may sometimes be obtained.

110. Harmony of Form.—Harmony of form is an important element to be considered in obtaining an esthetic design. Concrete bridges have been built in three general forms: (1) arches, (2) girders, and (3) trusses. There are so few examples of concrete trusses that they are scarcely more than a novelty and will not be discussed here.

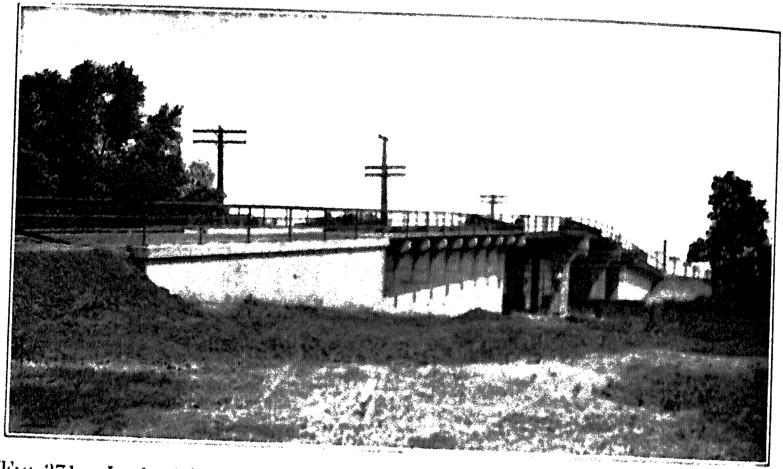


Fig. 371.—Lack of harmony due to combinaton of steel girders with concrete girders in the same structure.

The arch form cannot in general be combined with girders or trusses in a single structure with harmonious results. If variety is desired, it must be secured in some other way. Even the various forms of arches must be associated with great care when used in a single structure. For example, an arch tangent at the springings should not be placed adjacent an arch meeting the pier at a sharp angle; and springings at different elevations must at least conform to some reasonable order of arrangement.

Arch Bridges.—The arch is one of the most beautiful of engineering forms, but is probably the most difficult to design. There are two classes of concrete arch in general use: the earth-filled and the open-spandrel types. These types are each suitable for certain conditions and require entirely different shapes of arch ring. If the bridge be high and the spans long, a reduction in the weight of the structure may be desirable from both structural and artistic standpoints. An open-spandrel type will probably be most suitable for this location, with the roadway supported on columns or transverse walls resting on the arch ring. With this class of construction the parabola is the form of arch ring which most closely follows the true pressure curve of equilibrium.

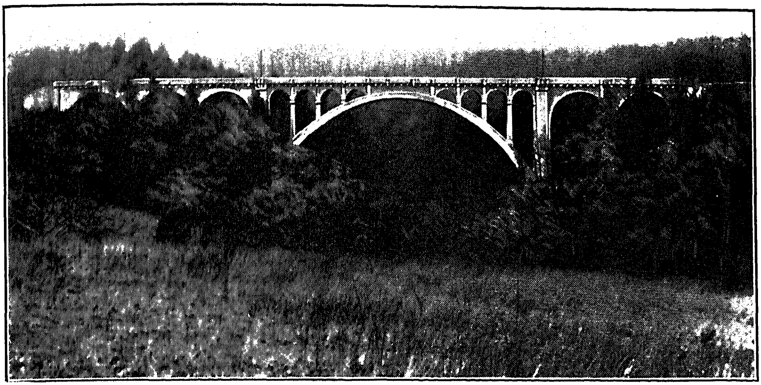


FIG. 372.—Appropriate design for great height and long span. Unfortunately the arches of the approaches do not harmonize with the spandrel arches.

But if the bridge is an ordinary one, with the springings of the arches below the high water line, it will be necessary to construct full spandrel walls in order to prevent the catching of débris. The addition of the weight of the fill will be desirable and the curve of equilibrium will approximate an ellipse. This is probably the most graceful form for an arch, and fortunately is very nearly the proper form for the large majority of cases in America. In any case, different variations should be tried until the best result is obtained, for upon the careful selection of

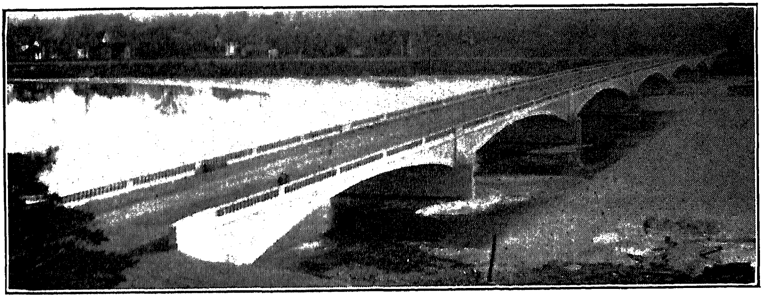


FIG. 373.—Oval arches approximating ellipses, with solid earth-filled spandrels, suitable for locations subject to floods.

the curve of the arch ring depends largely the final appearance of the bridge.

Girder Bridges.—The possibilities for esthetic treatment of concrete girders are very limited. The ends may be corbeled or made to resemble the arch in order to get curved lines. If further artistic treatment seems desirable, reliance must be placed on ornamentation. From an esthetic standpoint at least, the concrete girder is not suitable for bridges of very great height, because the engineering limitations are such that the span cannot be increased in proportion to the height and therefore it is not possible to secure a clean, bold design. A series of short heavy girders supported by tall and apparently inadequate columns cannot be made to present a pleasing appearance.

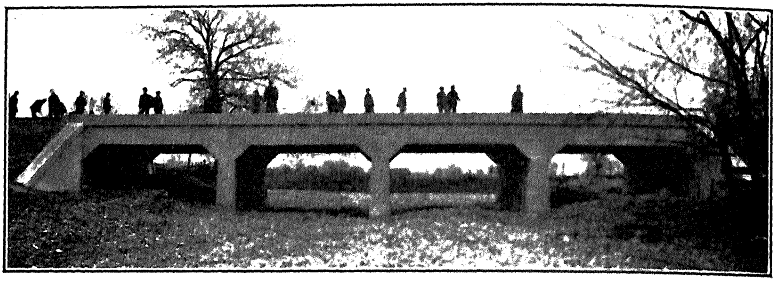


FIG. 374.—A well-designed girder bridge showing clearly the limitations of the girder.

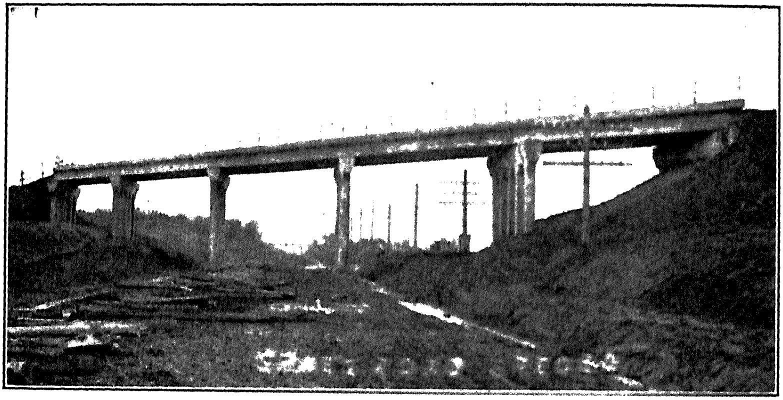


FIG. 375.—Pile trestle in concrete. Supports appear to be inadequate, and the entire effect is one of cheapness accentuated by the gas pipe railing which is not even panelled in harmony with the spans. This structure is a wooden trestle moulded in concrete.